A Forest Service
ch Paper NC-41

Relation between the National Fire Danger Spread Component and Fire Activity in the Lake States

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Relation Between The National Fire Danger Spread Component And Fire Activity In The Lake States

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The National Fire Danger Rating System consists of four descriptive components: risk, ignition, spread, and energy release. Each component is intended to provide useful information for judging a potential fire situation. Risk indicates the probable number of firebrands landing on receptive fuels. The ignition component indicates the probability of ignition if a firebrand lands on receptive fuels. The spread component indicates the forward movement of surface fires. The energy release component indicates the driving energy from the combustion process that maintains the fire.

Until recently, only the preliminary indices of the spread component had been applied operationally (USDA Forest Service 1964). Because it was the only developed portion of the total system, the spread component was used (or perhaps abused) not only as an indicator of fire spread but also as an indicator of fire occurrence and burning severity. However, there has been only a limited effort to determine the statistical relationships between it and fire activity.

With the exception of risk, conceptual models for the other components of the System are now developed. Although these components are based on physical laws governing fire behavior, it is necessary to show how well and in what form they serve the purpose for which they were designed. Empirical relationships between various indices or index combinations and fire activity provide such a test and are also a basis for developing operational guides for fire control. Because preliminary indices of spread are familiar, they are used here to develop a flexible system of translating fire danger rating into commonly used measures of fire activity.

Other investigations have tried to determine such things as "normal" class frequency of the spread component. For example, Barney (1967, 1968) graphed the normals and frequency distribution of the buildup index and fine fuel spread index in Alaska, and the Minnesota Department of Conservation (1965, 1966) compiled class frequencies for that State. Nelson (1964) compared cumulated days, fires, and C, D, and E fires by spread indices and the 8 and 8-100 burning index (previously used in some regions). He concluded the timber spread index was superior to the 8 and 8-100 burning index as both an indicator of the probability of fire occurrence and rate of spread.

Little study of the spread component has been made beyond the development of frequency classifications. Since 1964 the Georgia Forest Research Council has published yearly information categorizing Georgia fire activity by spread component classes. Fairly close relationships were found between acres per fire and timber spread index (Ryan and Pachence 1965). Bruce[1] attempted to identify parameters that seemed most useful in accounting for variation in number of fires, and examined spread component indices as input.

No one has attempted to establish the many possible relationships between the National Fire Danger spread component and fire activity. A number of questions about spread component have been asked: Does its reliability as an indicator of fire activity vary during the year? How important is it to include a vegetative stage? Does the system show more meaningful fire activity relationships in conifers than in hardwoods or grasslands? We will give objective answers to some of these questions by presenting various empirical relationships between indices of the spread component and fire activity records in the Lake States. These relationships may also form a base from which comparisons may be made between National Fire Danger Rating System components recently developed and undergoing refinement.

The use of historical fire records in this type of study presents problems. When indices are high, fire protection units try to caution the public, alert fire crews, and maximize suppression ability. Although these factors tend to bias evaluation, they should not affect such things as seasonal variation in fire activity or the relationship between spread indices and conifer versus hardwood fires. Hopefully, sufficient data will reduce the influence of other variables or at least show systematic bias. Therefore, if treated cautiously, fire activity records can be a valuable tool.

THE DATA BASE AND COMPUTATIONAL PROCEDURE

Daily weather records and fire reports for April 1 to October 31 were collected from nine areas over the Michigan, Minnesota, and Wisconsin region (fig. 1) for the years 1957 to 1962. Each of these nine areas contains a reliable weather observation station near its center. Fire information was included in the data only when the occurrence was within a 35-mile radius of one of these weather sites. Pertinent data were taken from fire and weather forms, tabulated, and placed on punched cards. The data record contains a tota of 11,324 observation days, of which 1,958 days had at least one fire. A total of 4,288 fires burned 126,095 acres in the nine selected areas during this period.

The various indices of the spread component were calculated by a computer program developed at the North Centra

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[1] *Bruce, D. Development of man-caused fire occurrence index. USDA Forest Serv., Pac. Northwest Forest and Range Exp. Sta. Unpublished manuscript. 1965.*

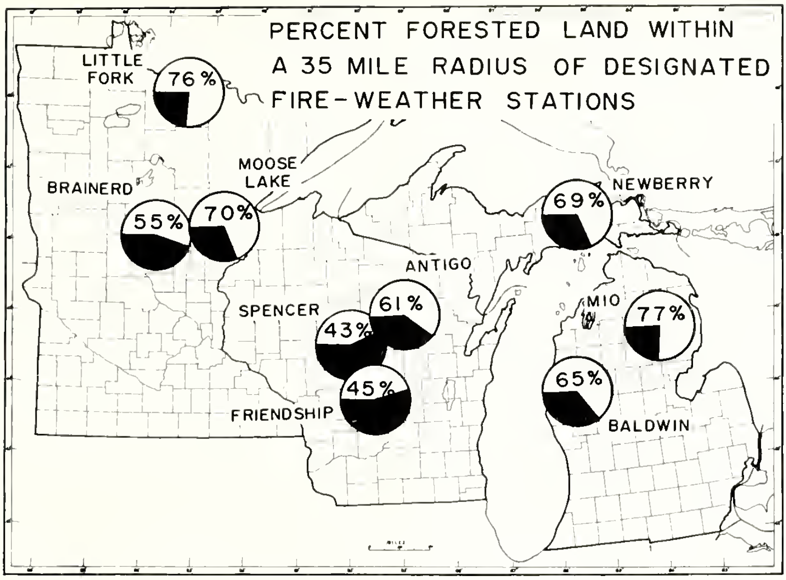

Figure 1.—The nine areas from which fire and weather data were gathered for the years 1957 to 1962.

Forest Experiment Station (Main 1969). The program computes the Buildup Index (BUI), the Fine Fuel Spread Index (FFSI), the Timber Spread Index (TSI), nd the Fire Load Index (FLI)[2] (fig. 2). The BUI gives a measure of the progressive rying of fuels (excluding fast-drying fine uels) and is related to the moisture conent of standardized 10-day timelag fuels. FFSI is based on the moisture content of ast-drying fuels coupled with windspeed. TSI is based on the same factors as the FFSI, but the BUI is also included. FLI vas developed to indicate the number of nan-hours necessary to control an average surface fire in litter-type fuels. It is a composite of the TSI and the BUI. These indices are calculated on a daily basis for vegetative conditions (always green, alvays transitional, always cured, or chosen). Vegetative stage refers to the physiological condition of the lesser vegetation and not to deciduous trees and shrubs. Chosen vegetative stage was determined by the observer at an individual station and

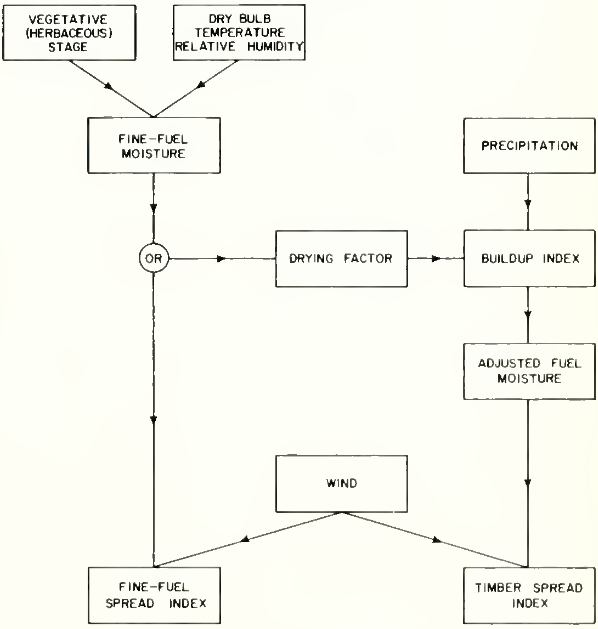

Figure 2.—Indices and input factors of the spread component.

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[2] *Keetch (1967). (Fire Load = 1.75 log TSI + 0.32 log BUI – 1.640)*

entered each day on the station form. If chosen, the cured stage prevails when vegetation is 75 percent or more dead or dormant, transition when 25 to 75 percent is dead or dormant, and green when less than 25 percent is dead or dormant.

The program also breaks the information into season (spring, summer, fall, and all seasons) and cover type (grass, hardwood, conifer, other, and any). The program then takes the various indices of the spread component and compares them with fire data by regression analysis. This is done on a linear basis as well as with logarithmic transformations. The calculated spread-component values can be compared with such measures of fire activity as fires per fire-day, fires burning more than 10 acres, probability of a fire-day, and others. Computation was done by using mean values over two-unit index-increments and was restricted to a scale range from 0 to 65. The 0 to 65 restriction was imposed because only a few cases occur at the higher end of the scale. Also, we suspect that control action is more intense when the indices are in the upper ranges.

THE SPREAD COMPONENT OVER THE TOTAL FIRE SEASON

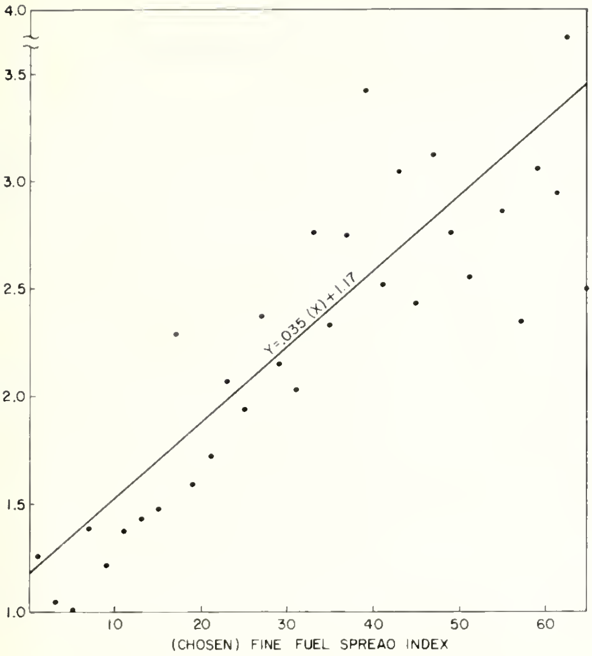
The spread component was developed as an indicator of the forward movement of surface fires; therefore, increasing component values should indicate an increasing number of acres burned per fire. Control action along with natural factors would certainly bias this relationship, but the trend should still be apparent. Table 1 gives the coefficients of determination (R^2), using various vegetative-stage values of the indices as predictors of four measures of fire activity, for the entire fire season. The highest R^2 obtained by linear analysis or after transformations is listed. It appears that an R^2 value below about 0.2 is not worth considering.

Table 1.—Coefficient of determination (R^2) values of index by vegetative conditions versus four fire activity measures (the data include the entire fire season for any cover type)

Index, by vegetative condition	Probability of a fire-day	Probability of a C, D, or E fire-day	Number of fires per fire-day	Acres burned per fire
Fine Fuel Spread				
Cured	0.94	0.89	0.64	0.40
Transitional	.93	.93	.42	.26
Green	.91	.94	.72	.42
Chosen	.97	.91	.74	.61
Timber Spread				
Cured	.86	.82	.33	.44
Transitional	.89	.55	.20	.25
Green	.92	.91	.28	.54
Chosen	.95	.57	.59	.36
Buildup	.92	.65	.13	.10
Fire Load	.68	.44	.21	.11

The R^2 values for acres per fire, although meaningful, are still relatively low for all possible vegetative conditions (table 1). Generally, the amount of variation explained with the BUI is low, as is also the case with the FLI. Burned-area criteria do not produce exceptionally high R^2 values, and this is also often the case with another measure of activity — fires per fire-day (table 1). The R^2 values for fires per fire-day are low for the TSI, BUI, and FLI. They are, however, much higher when the FFSI is used. Also, the scatter along the regression line is acceptable (fig. 3).

The FFSI appears to be a fair predictor of fires per fire-day for any vegetative condition except transitional (table 1). However, it does a better job as a measure



igure 3.—The relation between the chosen FFSI
and expected fires per fire-day.

the probability of a fire-day. The total-
eason R^2 value produced by the FFSI for
iosen vegetative stage is almost unbeliev-
oly high — 0.97. As pointed out by Fahne-
ock (1965) and others, fire occurrence is
rgely influenced by the same weather
ctors as fire size, although the relation-
ips are somewhat different. Therefore,
ie might expect significant relationships
etween indices of spread component and
re occurrence. Crosby (1954) and Bruce
1963) also recognized that basically al-
ost all fire-danger meters sort days into
asses with general levels of fuel moisture.
oth devised methods, with good results,
at could employ other fire danger meters
s predictors of the probability of fire oc-
irrence or the number of fires that might
e expected in sections of the central Unit-
l States. Consequently, it should not be
irprising that these data produce the
ame close relationships.

Probability of fire-day tabulations shows that scatter is at a minimum along the regression line (fig. 4). Here, for instance, one might expect a fire on 1 day out of 10 when the value of the FFSI is 10. On the other hand, that fire will probably be class A or B as the C, D, E fire regression line intersects the x-axis at a FFSI value of 10. When the FFSI reaches 50, there is a 60-percent chance of a fire, and a 40-percent chance of a large fire (class C, D, or E).

The probable number of fires can be determined from the data in figure 3. As an example, on any given fire-day, if the fine fuel spread index is near 25, an area averages two fires. If the same index is at 50, an average of three fires occurs.

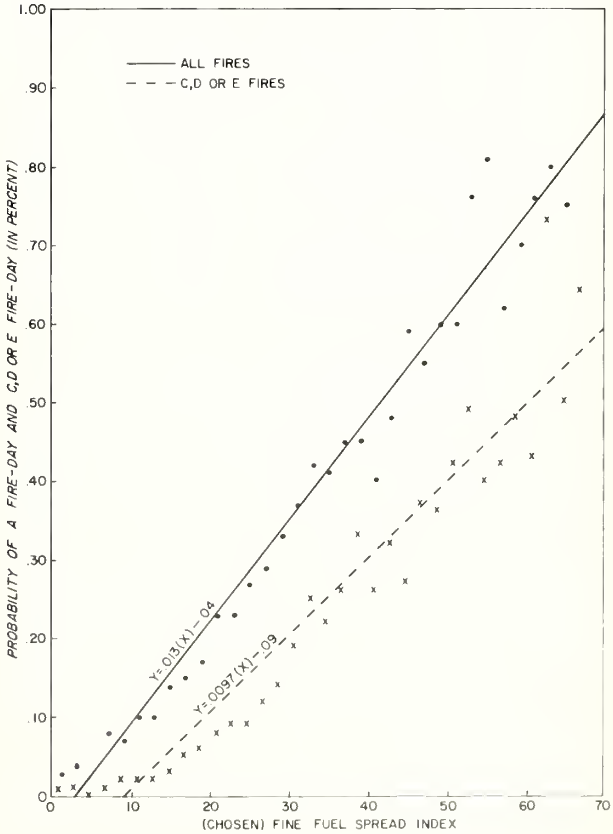


Figure 4.—The relation between the chosen FFSI and both the probability of a fire-day and the probability of a C, D, or E fire-day.

COVER AND SEASONAL CONSIDERATIONS

Thus far we have not considered seasonal and cover-type differences. Of the 4,288 fires used in this sample, 3,075 were spring fires, 948 were summer fires, and only 265 were fall fires. The fact that almost half were grass fires would tend to negate the usefulness of such things as the BUI. The BUI is built upon 10-day drying lag characteristics, and grasses may have a drying factor of a few hours.

Precipitation fits into the TSI scheme through the BUI. Obviously, precipitation influences fire spread in heavier fuels, but this fact isn't readily apparent when the measure is acres per fire (table 2), as R^2 values for the BUI are very low. Also, FFSI gives better acres-per-fire R^2 values than TSI (table 2) for hardwood, grass, and shrub cover, while the TSI gives the highest R^2 for conifer. But the FFSI does not use precipitation amount or frequency directly in its computation, while the TSI does. This result then would be expected for conifer cover, but would seem to be the reverse of the expected for hardwood cover.

The probability of a fire-day is in close agreement with the chosen FFSI in the spring (table 3), and additional computations showed that this applies in the spring for all cover types. During summer the relationship is poor, but it is somewhat better in the fall, with further computations showing this was especially true for hardwood cover. In the summer the R^2 value increases dramatically if we assume the cover is always in cured stage instead of choosing the stage. The same general comments hold with the probability of C, D, or E fire-day. When we consider the fire activity to be number of fires per fire-day at each danger level instead of the probability of a fire-day, we find the R^2 for FFSI

Table 2.—*Coefficient of determination (R^2) values of index by cover type versus fou fire activity measures (the data include various cover types over th entire fire season using chosen vegetative stage)*

Index, by cover type	Probability of a fire-day	Probability of a C, D, or E fire-day	Number of fires per fire-day	Acres burned per fire
Fine Fuel Spread				
Crass	0.82	0.87	0.54	0.45
Hardwood	.89	.88	.47	.37
Conifer	.84	.83	.22	.23
Other	.87	.88	.22	.32
Timber Spread				
Crass	.53	.19	.10	.03
Hardwood	.61	.47	.28	.16
Conifer	.36	.50	.08	.38
Other	.28	.15	.11	.06
Buildup				
Crass	.82	.17	.27	.03
Hardwood	.84	.55	.10	.06
Conifer	.83	.49	.00	.10
Other	.79	.27	.00	.05
Fire Load				
Crass	.10	.12	.04	.04
Hardwood	.86	.40	.10	.11
Conifer	.09	.16	.07	.05
Other	.37	.34	.01	.09

Table 3.—Coefficient of determination (R^2) va ues of index by season versus four fi activity measures (the data inclu three seasons, any cover type, usi chosen vegetative stage)

Index, by season	Probability of a fire-day	Probability of a C, D, or E fire-day	Number of fires per fire-day	Acres burne per fire
Fine Fuel Spread				
Spring	0.95	0.91	0.53	0.15
Summer	.32	.06	.22	.18
Fall	.57	.60	.26	.33
All season	.97	.91	.74	.61
Timber Spread				
Spring	.87	.80	.32	.27
Summer	.46	.14	.13	.21
Fall	.44	.32	.22	.31
All season	.95	.57	.59	.36
Buildup				
Spring	.89	.73	.08	.15
Summer	.91	.47	.45	.42
Fall	.54	.26	.06	.04
All season	.92	.65	.13	.10
Fire Load				
Spring	.54	.26	.04	.33
Summer	.46	.09	.04	.32
Fall	.23	.22	.08	.09
All season	.68	.44	.22	.35

osen vegetative stage is highest in the ring and much lower in the summer.

There is a problem in interpreting the portance of the varying R^2 values in e tables. Are we justified in assuming, r example, that a superior-inferior relaonship holds between the probability of fire-day and the FFSI in grass as against ardwood ($R^2 = 0.82$ and 0.89 respective-) (table 2)? Fisher's z' transformations ethod (Brooks and Carruthers 1953) ields confidence limits (at the 0.05 level) r correlation coefficients and may help lve the problem.

Results of the transformation of corretion coefficient for the regression data ow that if one of the corresponding R^2 alues is in the 80's, there would have to be a difference of roughly 0.15 between R^2's before the larger R^2 yields a superior relationship. When the R^2's are lower, the difference between values that indicate a superior-inferior relationship would have to be even greater. If we apply these methods to the stated problem example, we see that the FFSI apparently does not give differing results in differing cover types for the criterion, probability of a fire-day.

During the summer a special problem develops with all fire activity measures. The data tend to group at the lower end of the spread index range unless the vegetative stage is considered to be always cured. Cured choice will cause the data to spread over the scale in a better distribution.

SUMMARY AND CONCLUSIONS

The various indices did not produce ceptionally high R^2 values when comared with the activity measure, acres per e, although the coefficient of determinaon values are statistically important in any cases. This may be the result of field ethods employed in measuring acres urned, or because burned acreage is not good way to judge spread, or because of mission of important variables that result om control action. As Countryman 1966) states, no danger rating system ies to make a complete evaluation of fire anger, and all, therefore, are partial, not tal, systems. There are just too many ctors that affect fire danger to include l in an operational system. No usable ting method, consequently, explains total ariation. Also, the present design of the read index may not adequately predict re spread; however, because the criterion sed here was burned acreage and not rate f spread measurement, no firm conclusion an be drawn.

On the other hand, when we use the spread component to measure another form of fire activity — probability of a fire-day — we find excellent associations on a total-season basis and good relationships for many seasons and cover types. What this implies is that spread component indices are a good approximation of ignition.

The inclusion of vegetative stage in the indices does not always appear to produce significantly better relationships. If a single vegetative condition is used, the cured stage is probably best for various forms of spread. This stage produces a more normal distribution over the scale range. A continuous choice of green vegetative condition is especially poor because it gives low-scale, skewed distributions.

The various forms of the spread component consistently show the best results during the spring season in Michigan, Minnesota, and Wisconsin. The FFSI and TSI produce the poorest results during the summer. The R^2 values for the BUI are lowest in the fall.

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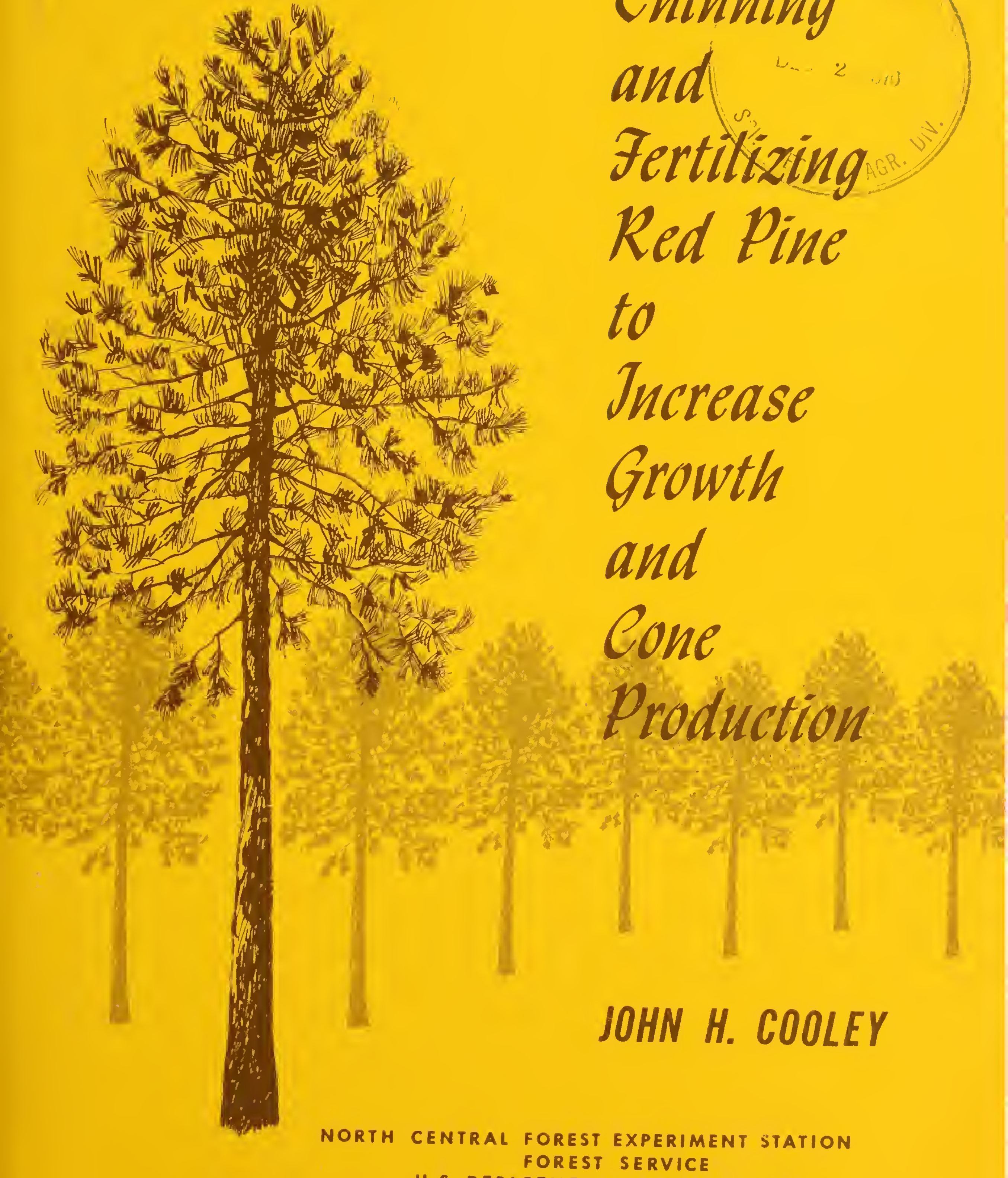
USDA FOREST SERVICE
RESEARCH PAPER NC-42
1970

Thinning and Fertilizing Red Pine to Increase Growth and Cone Production



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Thinning and Fertilizing Red Pine to Increase Growth and Cone Production

John H. Cooley

Manipulating red pine (*Pinus resinosa* Ait.) ands to increase seed production has several vicultural implications. For example, stands at are to be regenerated naturally often require eharvest treatment to assure an adequate seed pply, and the large quantities of seed needed r artificial regeneration necessitates manage- ent of some stands for maximum seed produc- on. It has been shown that heavy thinning is ie effective way of increasing the number of ature cones (Godman 1962), and there is some ggestion that fertilization might also be effec- ve (Cayford and Jarvis 1967). Both treatments n also increase tree growth rate.

To further test the effects of thinning and rtilizing on red pine seed production and owth, two natural stands and one plantation ere studied in Lower Michigan. Results showed at the age and site of red pine stands strongly fect their response to treatment. For instance, a 20-year-old plantation on a good site, thin- ng increased the number of mature cones per ee in only 2 years out of 6, and fertilizer had effect. However, in 53- and 55-year old natural ands on medium sites, thinning increased cone ops every year, and fertilizer increased them 2 ars out of 6. Tree growth response also varied stand age and site. Thinning increased dia- eter growth during the first 5 years in the 20- ar-old plantation, while fertilizing decreased it itially and increased it later on. In the older tural stands, fertilizing had no effect on dia- eter growth, and thinning had no effect until e second growing season, after which differ- ces among thinning levels became more pro- unced each year.

METHODS

The 20-year-old plantation is located near the est side of Lower Michigan. It is planted on alkaska sand and has a site index of over 70. he basal area before thinning was 109 square et per acre; there were 969 trees per acre with average d.b.h. of 4.6 inches.

The 53- and 55-year-old natural stands, located near the east side of the Lower Peninsula, are separated by about half a mile. One is growing on Croswell sand, has a site index of 55, and was 53 years old when the study began. The original stand had 159 square feet of basal area and 581 trees per acre, with an average d.b.h. of 7.1 inches. The other is growing on Grayling sand, has a site index of 52, and was 55 years old when the study began. Originally it had 111 square feet of basal area and 454 trees per acre, with an average d.b.h. of 6.7 inches.

It seems possible that the site indices do not adequately reflect the difference in productivity of the sites on which these two older stands are growing. The stand located on Croswell sand had more and bigger trees than the one on Grayling sand, and according to Van Eck and Whiteside (1963), Croswell sand averages about 15 site index units higher than Grayling sand. Neverthless, the response in these two stands was similar and differed markedly from the response in the younger stand.

All combinations of three thinning and fertilizer levels were applied to 1/10-acre plots arranged in randomized blocks. In the younger stand there were three contiguous blocks; in the older stand growing on Grayling sand there was one block; in the stand growing on Croswell sand there were two. The 3 thinning levels were: 4 uniformly spaced trees per plot, 16 uniformly spaced trees per plot, and no thinning. The 3 fertilizer levels were:

Treatment	*Pounds per acre*
No treatment	0
Treatment 1:	
Nitrogen:	
Urea	169
Ureaform	131
Potassium	60
Phosphorus	195
Treatment 2:	
Nitrogen:	
Urea	338
Ureaform	262
Potassium	120
Phosphorus	390

Urea (45 percent N) and ureaform (35 percent N) were used in equal quantities for supplying nitrogen, potassium sulfate (40 percent K) for potassium, and superphosphate (13 percent P) for phosphorus. Thinning was done in the winter of 1960-1961 and fertilizer was broadcast during the first half of May 1961.

During late July or August of the next 6 years mature cones were counted on four sample trees in each plot. Other measurements were made with less regularity on the same trees. D.b.h. was measured the next 5 years, and height was measured the first, third, fourth, and fifth years. Needles from some plots in all stands were sampled after the first growing season, from the younger stand after the second, and from all stands again after the third. Samples were collected from midcrown position in late fall as recommended by White (1954). Needles collected in the younger stand were analyzed for mineral nutrients, both major and minor, after the first growing season and needles from all stands were analyzed after the third season. Dimensions were measured on cones collected from some of the sample trees in the older stands after the third season.

RESULTS

Cone Production Increased More By Thinning Than Fertilizing

In the younger stand, the number of cones per tree maturing 3 and 6 years after treatment was much greater on thinned plots than on unthinned plots (table 1). In these years, the proportion of sample trees producing cones was greater than at other times (table 2). In neither year was the difference between the two thinning levels statistically significant. In the third year, trees that bore cones on thinned plots had 17 times as many mature cones as comparable trees on unthinned plots. The sixth year thinned trees had about 30 times as many cones.

Trees in the older stands did not produce nearly as many cones, but there was some increase due to thinning each year. After the first year, when there was an apparent interaction between thinning and fertilization, the effect of thinning was independent of fertilizer (table 1). In the second year, the lighter thinning had no effect, but the heavier thinning did. In the fifth year there was no difference between the two thinning levels. In the other 3 years the heavy thinning increased cone yield more than the light thinning although the light one increased production to some extent. Trees on the plot thinned to 160 per acre had from 2.4 to 3. times as many cones as those on unthinne plots, and trees on plots thinned to 40 per acr had from 2.5 to 8.0 times as many.

Table 1.—*Number of cones per tree by thinnin treatment*

20-YEAR-OLD STAND

Years since treatment	Thinning treatment: No thinning	Thinning treatment: 160 trees per acre	Thinning treatment: 40 trees per acre
1	5	9	16
2	4	5	4
3	9	1/ 163*	171*
4	0	5	9
5	0	0	0
6	5	133*	202*
53- AND 55-YEAR-OLD STANDS2/			
2	3	3	7*
3	16	39*	82**
4	4	15*	32**
5	1	4*	4*
6	8	25*	48**

1/ For each year, treatment means that differed significantly at the 5-percent probability level have different numbers of asterisks.

2/ First-year means are not shown for thinning treatments because there was an interaction between thinning and fertilization (fig. 1).

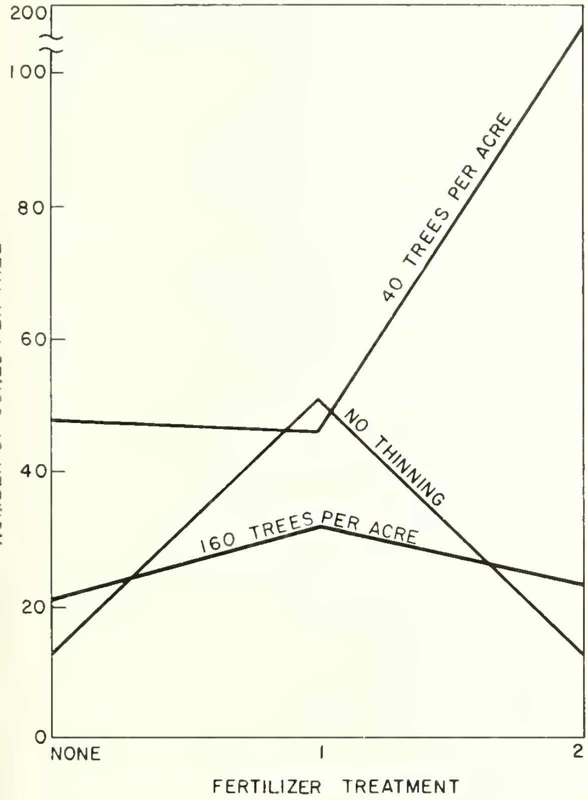
Table 2.—*Percent of trees with cones by thinnin treatment*

20-YEAR-OLD STAND

Years since treatment	Thinning treatment: No thinning	Thinning treatment: 160 trees per acre	Thinning treatment: 40 trees per acre
1	19	39	33
2	19	39	36
3	52	97	81
4	0	19	47
5	0	0	0
6	22	100	97
53- AND 55-YEAR-OLD STANDS1/			
2	33	9	67
3	64	9	100
4	75	86	100
5	11	17	58
6	61	86	100

1/ First-year means are not shown for thinnin treatments because there was an interaction between thinning and fertilization (fig. 1)

Fertilizer had no detectable influence on the umber of mature cones in the younger stand. n the older stands there was an interaction etween thinning and fertilization the first year fter treatment (fig. 1). This immediate response is somewhat unexpected because the only processes that could have been influenced were fertilization of the female gamete and maturation. The former should have taken place soon after treatment. Possibly, the activity of insects that might have interfered with cone maturation was affected. In a Canadian study it was noted that "a smaller percentage of cones were damaged in fertilized than control trees" (Armson 1967). On the other hand, White (1968) did not note any increase in the number of mature cones in an Upper Michigan red pine stand until the third year. This response in the third year is more logical because all processes of cone development occurred after treatment. In the present study, the lower level of fertilization approximately doubled the number of mature cones in the third year, and the higher level more than tripled it (table 3). Fertilizing also increased the size of third year cones, but thinning had no effect on size.



igure 1. Effect of fertilization on cone production at different densities in the older stand 1 year after treatment.

In the fourth year there were cone production differences between the two older stands that might be associated with soil differences. Trees growing on Croswell sand averaged 23 cones each and those on Grayling only 6. There were also differences between the older and younger stands that could be at least partly attributed to soils. The average site index reported for Kalkaska sand, on which the younger trees are growing, is much higher than for either Croswell or Grayling (Van Eck and Whiteside 1963), and the site indices observed on the study plots bear this out. In 5 out of 6 years trees on unthinned plots in the older stands had more mature cones than comparable trees in the younger stand, but in the 2 years that the younger trees bore heavily, those on thinned plots had from 2.0 to 5.5 times as many cones as comparable trees in the older stands.

Table 3.—*Cone production and cone size in the older stand in the third year after treatment*

Fertilizer treatment	Trees bearing cones	Cones per tree	Cone length	Cone diameter
	Percent	Number	Inches	Inches
None	66.7	24	1.6	1.0
1	91.7	50	1.7	1.1
2	94.4	82	1.8	1.1

D.b.h. Growth Increased More in Younger Stand

In the first year after thinning, trees on thinned plots in the younger stand grew more than those on unthinned plots, and the difference became progressively greater during the next 2 years (table 4). The heavy thinning increased growth no more than the lighter thinning until the fourth year.

In the older stands the effect of thinning was delayed about 1 year. There was no detectable response the first year, and no significant difference between thinning levels until the fifth year. Otherwise, the pattern of response was much the same as in the younger stand, but the magnitude was much smaller. For the entire 5 years, thinning to 160 trees per acre increased d.b.h. growth by 172 percent in the younger stand, but only by 49 percent in the older ones. Thinning to 40 trees per acre increased growth by 208 percent and 77 percent, respectively.

In the older stands there were no differences in d.b.h. growth that could be attributed to fertilizer treatments. However, in the younger stand fertilization decreased d.b.h. growth significantly at all thinning levels in the second year after application, and increased it in the fifth year. The shift from adverse to beneficial response came about gradually (fig. 2), probably as a result of the tree's adjustment to higher nutrient levels and reduction of salt concentration in the rooting zone. This fertilizer response was independent of the thinning treatment.

Discounting the adverse initial effect, which lasted 2 years for the lower level of fertilization and 3 years for the higher, it would appear that fertilization increased d.b.h. growth by about 15 and 25 percent for the lower and higher levels, respectively. It is too soon to be positive about the duration of fertilizer effects, but the data imply that the lower level had its maximum effect 4 years after application, while the effect of the higher level was still increasing 5 years after application. Heiberg *et al.* (1964) reported that height growth on a fertilized potassium-deficient soil reached a maximum 5 or 6 years after application and continued to be greater than on unfertilized areas for 17 years. However, this was observed on a degraded agricultural soil where the sustained response was attributed to reestablishment of a stable level of potassium. Such a long-term response is not likely where nitrogen is the principal limiting element, because nitrogen tends to be more subject to attrition.

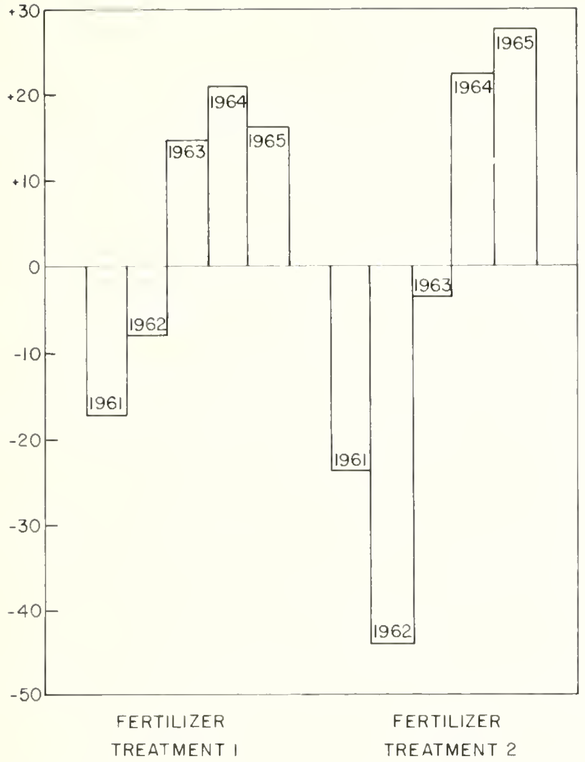
Several investigators have reported increased height growth in red pine after fertilization

Table 4.—*Annual d.b.h. growth after thinning*
(In inches)

20-YEAR-OLD STAND

Years since treatment	Thinning treatment		
	No thinning	160 trees per acre	40 trees per acre
1	[1] 0.18*	0.27**	0.28**
2	.16*	.34**	.31**
3	.17*	.57**	.66**
4	.19*	.49**	.56***
5	.10*	.50**	.65***
53- AND 55-YEAR-OLD STANDS			
1	.18*	.19*	.19*
2	.17*	.23**	.26**
3	.16*	.29**	.35**
4	.21*	.29**	.35**
5	.17*	.32**	.42***

[1] For each year, treatment means that differed significantly at the 5-percent level have different numbers of asterisks.



gure 2. D.b.h. growth in the younger stand during the 5 years following fertilization, as percent deviation from growth on unfertilized plots.

Ieiberg *et al.* 1964; Gagnon 1965; Leech 1965) it no such effect could be detected in this udy, partly because height was not measured curately and partly because the deformed ems of heavily fertilized trees in the younger and reduced height growth.

Treatments Increased Needle Size and Nutrient Content More in Younger Stand

Needles collected after the first growing sea- n from fertilized trees in the younger stand ere 24 percent longer and 39 percent heavier an those from unfertilized trees. Differences etween the two levels of fertilization were less an 1 percent. Fertilization increased foliar trogen concentration and decreased aluminum ncentration (table 5). These were the only ements that differed significantly among treat- ents. Potassium concentrations ranged from 5 to 0.9 percent, well above the deficiency vel reported by Heiberg and White (1951).

Some discoloration or wilting of needles was bserved on 17 trees in the younger stand that ad received the higher level of fertilization, on seven that had received the lower level, and on three that were unfertilized. Malformation of apical leaders was also observed on some of the fertilized trees (fig. 3), due to inability of the succulent new growth to support the additional weight of foliage that resulted from fertilization. In some trees it caused a permanent deformation of the main stem.

Table 5.—*Nitrogen and aluminum concentration in foliage collected from the younger stand*

Fertilizer treatment	Nitrogen		Aluminum	
	First year	Third year	First year	Third year
	Percent	Percent	Ppm	Ppm
None	1.43	1.21	606	587
1	2.20	1.48	285	447
2	2.83	1.70	214	292

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Figure 3. Apical leader malformation was observed on some of the heavily fertilized trees in the younger stand.

It would appear that a fertilizer response persisted into the third growing season, but in all samples there was less nitrogen than after the first season. Aluminum content decreased in unfertilized trees, but increased in trees at both fertilizer levels, more for the lower than the higher. Needles from plots fertilized at the lower level were 12 percent longer, but no heavier, than those from unfertilized plots; needles from the higher level were 10 percent longer and 83 percent heavier than those from the lower.

Thinning also increased needle size in the younger stand, and there was some indication that it increased nitrogen content of the needles; however, there was no discernible interaction between thinning and fertilization.

Needles collected after the first growing season from the older stands were neither weighed nor analyzed for nutrient content. Fertilization did not have a significant effect on their length, and wilting, discoloration, or terminal deformation were not observed. Increased needle length attributable to thinning was significant at the 99 percent level.

Needles collected in the older stands after the third growing season were analyzed more thoroughly, but neither fertilization nor thinning had a significant effect on their length, weight, or nitrogen content. The average nitrogen content of all samples was 1.24 percent — lower than fertilized trees in the younger stand, but not significantly different than unfertilized trees.

Potassium and phosphorus in most samples were within the range observed in the younger stand, but there were differences among samples that were attributable to soils, thinning, fertilization, and interaction between thinning and fertilization. Potassium concentration in samples from plots on Grayling sand averaged 0.55 percent while comparable plots on Croswell sand averaged 0.49. For unthinned plots and the light thinning, percent potassium increased with fertilizer level, but for the heavy thinning the trend was just the opposite (fig. 4). Both soils and thinning influenced the effect of fertilization on phosphorus concentration (fig. 5).

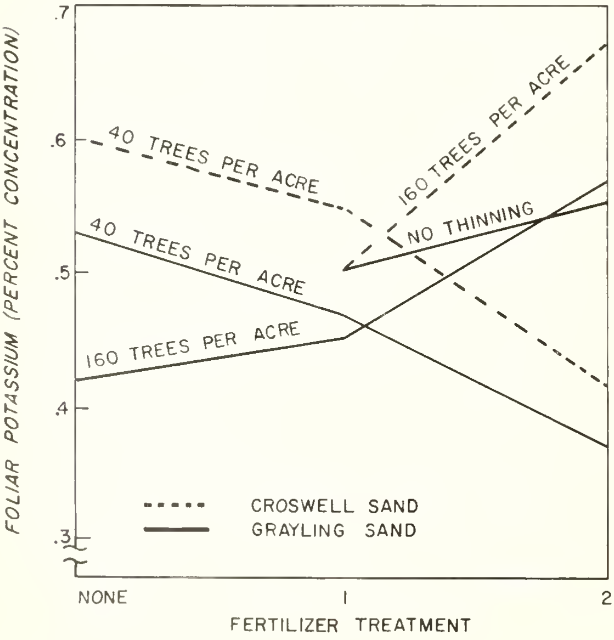

Figure 4. Percent concentration of potassium in foliage collected 3 years after treatment in the older stand.

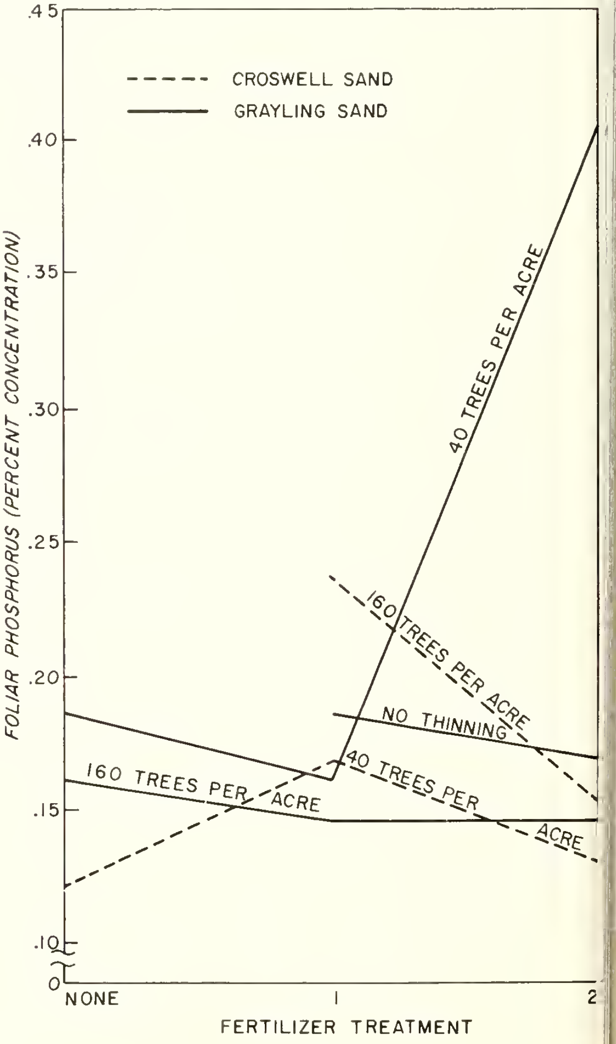

Figure 5. Percent concentration of phosphorus in foliage collected 3 years after treatment in older stand

DISCUSSION and CONCLUSION

Even though the stands in which this study vas carried out are referred to as "younger" and older," the reader should not assume that age vas the only important contrast between them.)ifferences between Croswell and Grayling sand uggest that soils are also important, and it is ntirely possible, in fact probable, that genetic nake-up and climate influenced response. It s difficult to generalize on the effects of treatment because the observed responses were unoubtedly the combined effects of all these facors and perhaps others.

The response to thinning seems consistent nd easily explained. It increased growth and one yield more in the younger stand partly ecause leaving the same number of trees per cre increased growing space per unit of biomass nore for smaller trees than for larger ones. There nay also be differences in physiological processes elated to age or site quality. Apparently the ffect of thinning on cone production can be ntirely nullified, and the frequency with which his occurs varies from stand to stand. Again, ge or site quality might be controlling factors, out it seems likely that weather is also implicated (Lester 1967).

The physiology of response to fertilization is nuch more complex; therefore, it is not surprising that its effect is more variable. Additions of mineral nutrients can increase growth rate nd cone yields even though there are no obvious leficiency symptoms, but response to the same ate of application will vary from stand to stand. n one stand, fertilizing may result immediately n toxicity symptoms and growth reduction, while n another it may increase cone yields but have no effect on growth. It seems logical to expect hat the fertilizer response would be conditioned by age and size of the trees as well as physical and chemical properties of the soil. Perhaps oliar analysis will indicate the probable response. At least in this study, increased foliar nitrogen vas associated with increased growth. In any event, the growth increase that can be achieved vith addition of mineral nutrients is likely to be small relative to the acceleration due to thinning. White (1968) points out that investment in fertilization to increase growth of lowvalue products, such as red pine pulpwood, generally are not profitable, but similar investments to increase seed production may be "quite reasonable."

Based on this study, then, there are three general recommendations that can be made:

1. Thinning for seed production should be limited to young stands on good sites, because response is much better than in older stands on poor sites.
2. Stands managed for seed production *must* be heavily thinned.
3. The effectiveness of fertilization should be determined for each prospective stand before extensive programs are begun.

In connection with the third recommendation it should be noted that the determination cannot be based on a 1-year observation. An immediate increase in foliar nitrogen is a strong indicator that fertilizer will increase growth over the long run, but the initial reaction to a heavy application may be reduced growth. The effect on cone yield cannot be assessed with any certainty until there is a good crop.

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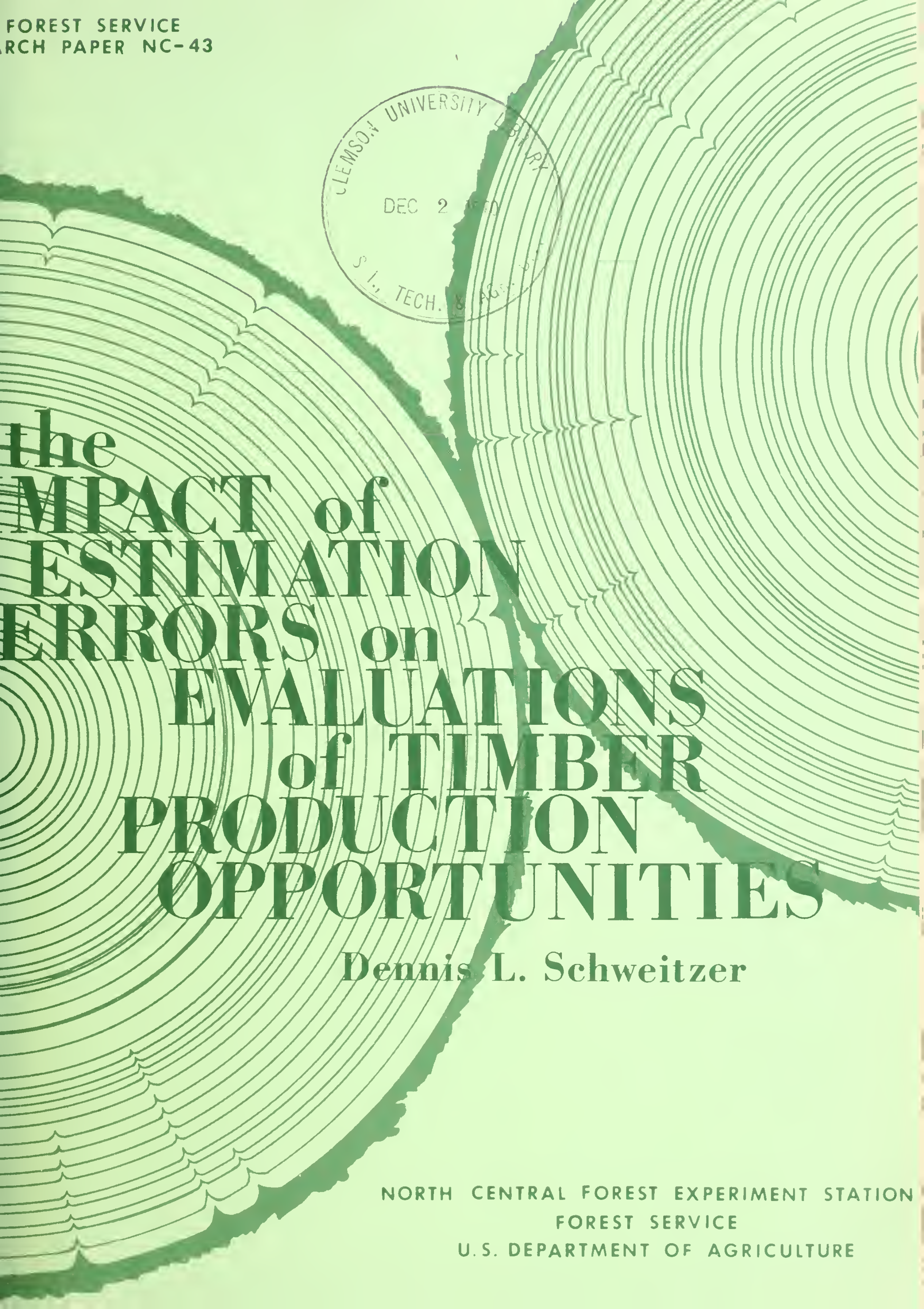

the MPACT of ESTIMATION ERRORS on EVALUATIONS of TIMBER PRODUCTION OPPORTUNITIES

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The Impact of Estimation Errors On Evaluations of Timber Production Opportunities

Dennis L. Schweitzer

Much attention has been paid to efficiently allocating funds to timber production. Alternative investments are usually ranked by their present worths or internal rates of return or benefit-cost ratios; investment priorities are then established to follow these rankings (Marty *et al.* 1966, Marty and Newman 1969, Webster 1960). Unfortunately, if the costs or returns that define investment opportunities are incorrectly estimated, such rankings may be worthless. This paper is concerned with measuring the impact of estimation errors on calculated present worths. These errors may occur in specifying costs, returns, the length of time between investments and harvests, and the costs of using funds in particular investments.

The importance of errors made yesterday in forcasting today's costs and prices can be readily determined. It is only necessary to find the difference in value between what was done, based on erroneous information, and what would have been done, given correct information. Unfortunately, the same kind of absolute value cannot be placed (now) on the importance of errors concerning tomorrow's costs and prices.[1] Instead, a variety of more-or-less satisfactory approximation techniques must be relied upon if anything is to be said about how much effort should be spent in refining data or about the likelihood that resulting decisions will be less than the best possible.

Perhaps the most common approach is to make a series of "sensitivity analyses." Essentially, the aim is to determine how much the present worth, or other decision criterion, is altered if input data are

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[1] *For a summary of probabilistic techniques that can be used to determine the odds that particular values will be realized, given less-than-perfect data, see Dennis L. Schweitzer. Evaluating forest investments under uncertainty: a synopsis of available techniques. (Unpublished report on file at Pacific Northwest Forest and Range Exp. Sta., Portland, Ore.)*

systematically varied. A general availability of investment analysis computer programs (Chappelle 1969, Forster 1968, Schweitzer *et al.* 1967, Row 1963) makes feasible repeated analyses of complex alternatives. "Canned" linear programming routines usually show the effects on the objective function of relaxing each constraint. Wikstrom and Alley (1967) suggest that a multiple regression analysis can lead to the same goal. In this paper, partial derivatives and a graphical analysis are used to illustrate the effects of various estimation errors on the present worths of individual investments in timber production.[2]

Measuring Impacts By Partial Derivatives

When present worth is used as a measure of the desirability of an investment, it is defined as:

$$(1)\qquad \text{Present worth} = x_0 + \frac{x_1}{(1+i)^1} + \ldots + \frac{x_j}{(1+i)^j} + \ldots + \frac{x_n}{(1+i)^n}$$

where x_j is the income or cost occurring in the j^{th} year, i is the (assumed constant) cost of using investment funds expressed as a decimal fraction, and n is the number of years in the investment period or the length of the rotation. We will consider timber production opportunities that are defined by establishment costs (EC), constant annual costs (AC), and returns (R) from clear cutting at the end of a rotation. The present worth of a single rotation can be calculated from:

$$(2)\qquad PW = \frac{R}{(1+i)^n} - AC\left[\frac{(1+i)^n - 1}{i\,(1+i)^n}\right] - EC.$$

Errors in Establishment Costs

The sensitivity of present worth to errors in estimating the values on the right side of this expression can be explored by varying each over some range while holding the others constant. A straightforward approach is to take partial derivatives of the expression for present worth with respect to each of these investment parameters and graph the results. Taking the partial derivative of present worth with respect to establishment cost yields:

$$(3)\qquad \frac{\delta PW}{\delta EC} = -1.$$

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[2] *This approach follows that of Canada who was concerned with applications of continuous discounting to engineering problems. (John Robert Canada. The effect of risk and uncertainty in economic analyses of investments in capital assets. Unpublished doctoral thesis on file at Georgia Inst. of Tech., Atlanta.) I acknowledge Allen L. Lundgren's major contribution in formulating the following partial derivatives.*

That is, if the establishment costs were underestimated by $1.00, present worth would be overestimated by the same amount.

The meaningfulness of such a derivative can be illustrated by considering how an opportunity to produce jack pine with clear cutting after 40 years might be evaluated. The investment costs and incomes have been estimated to be $10.00 per acre for establishment costs, $.40 per acre per year for annual costs, and $200.00 per acre income from harvesting at age 40. Further, a 6 percent interest rate must be charged against using funds in this investment. The anticipated present worth would be:

$$(4) \qquad PW = \frac{\$200.00}{(1.06)^{40}} - \$.40 \left[\frac{(1.06)^{40} - 1}{(.06)\ (1.06)^{40}} \right] - \$10 = \$3.42 \text{ per acre.}$$

That is, discounted returns are expected to exceed discounted costs by $3.42 per acre. The above derivative suggests that if establishment costs were underestimated by, say, $4.00, the investment would be a loss.

Errors in Harvest Returns

Another error might arise in estimating harvest returns. The partial derivative of present worth with respect to harvest returns is:

$$(5) \qquad \frac{\delta PW}{\delta R} = \left[\frac{1}{(1 + i)^n} \right].$$

When this expression is graphed, the impact of assuming returns to be, say, $180.00 rather than $200.00 can be quickly determined (fig. 1). For example, assuming the year of harvesting to be 40 and using 6 percent interest, present worth will change about $.10 for each $1.00 change in harvest returns. Therefore, returns reduced by $20.00 would lower present worth for this investment by about $2.00 to less than $1.50 per acre.

Errors in Annual Costs

Similarly, figure 2 can be prepared after determining that the partial derivative with respect to annual costs is:

$$(6) \qquad \frac{\delta PW}{\delta AC} = - \left[\frac{(1 + i)^n - 1}{i\ (1 + i)^n} \right].$$

Underestimating annual costs by, say, $.25 a year leads to an overestimate of present worth of about $3.75—enough difference to change an apparently profitable venture to one that will lose money.

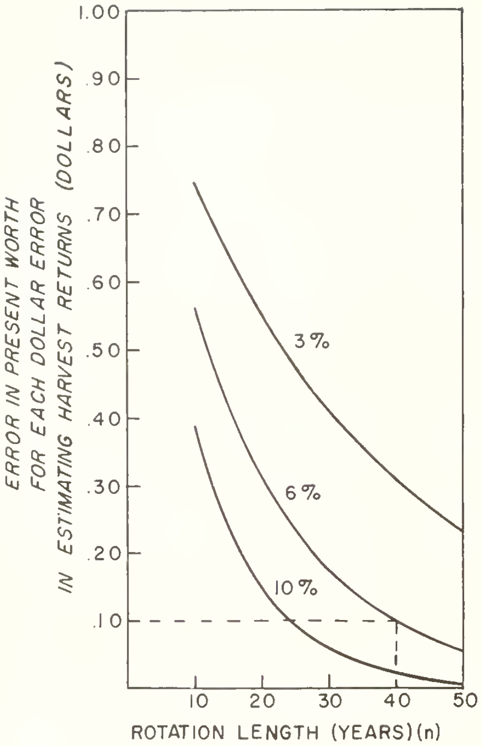

Figure 1.—Impact on present worth of erroneously estimating harvest returns.

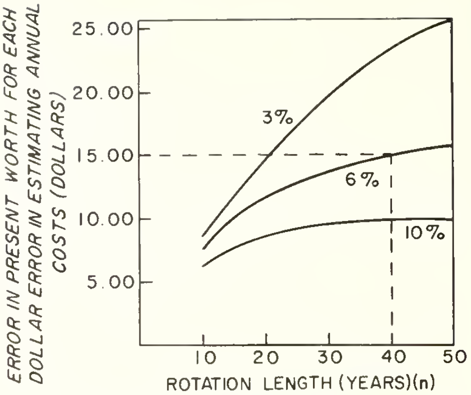

Figure 2.—Impact on present worth of erroneously estimating annual costs.

Errors in Length of Rotation

Considering the effect of poorly estimating the time of final harvest is slightly more complex. The partial derivative is :[3]

$$\frac{\delta PW}{\delta n} = -\left[R\left(\frac{\ln\ (1 + i)}{(1 + i)^n}\right) + AC\left(\frac{\ln\ (1 + i)}{i\ (1 + i)^n}\right) \right]. \tag{7}$$

Because a change in the rotation length, *n*, will affect the discounted values of both annual costs and harvest returns, both figures 3A and 3B must be used to determine the full effect. Receiving the income, *R*, after 39 years rather than after 40, will increase present worth about \$.006 for each dollar of *R* because that sum will be discounted for 1 less year (fig. 3A). Such a change is seen to further increase present worth by

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[3] *Recall that all variables in the present worth equation have been assumed correct except the length of the rotation. If, because* n *is incorrect, the clear-cut yield and return,* R, *is also wrong, a more complex expression is required:*

$$\frac{\delta PW}{\delta n} = -\left[R\left(\frac{\ln\ (1 + i)}{(1 + i)^n}\right) + AC\left(\frac{\ln\ (1 + i)}{i\ (1 + i)^n}\right) \right] + \frac{\delta R}{\delta n}\left[\frac{1}{(1 + i)^n}\right]. \tag{8}$$

If the new term is assumed to be roughly equivalent to the value of the mean annual increment, in the jack pine example $\delta R / \delta n$ *would be about* \$200 ÷ 40 *years or* \$5.00. *Discounting at* 6 *percent per year, then, would result in additional change of about* \$.50 *for each year change in rotation length.*

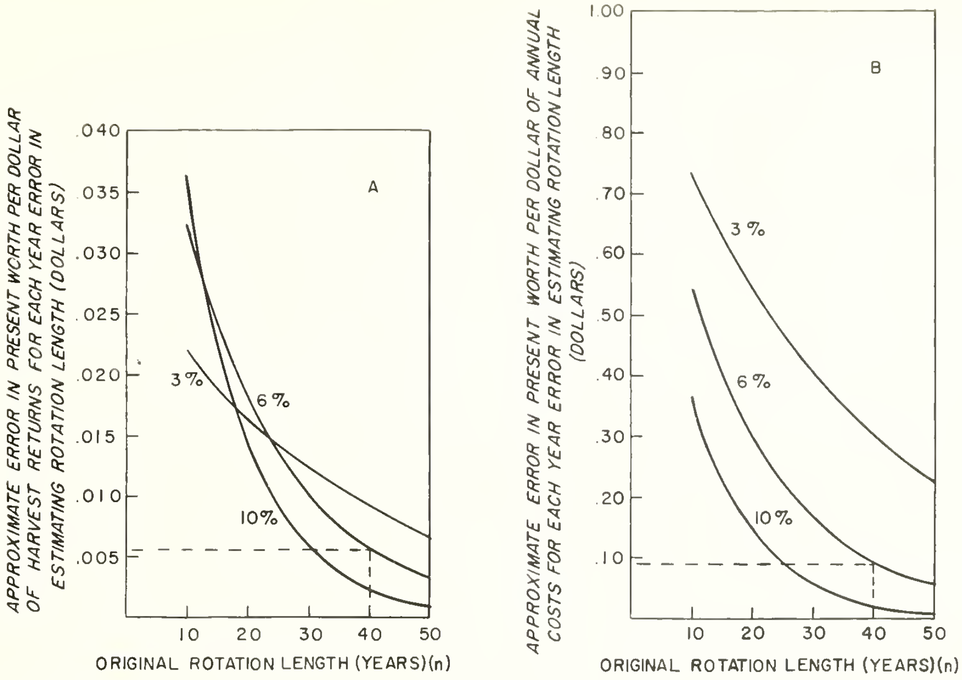

Figure 3.—(A) Approximate impact on present worth per dollar of harvest returns for each year error in estimating rotation length. (B) Approximate impact on present worth per dollar of annual costs for each year error in estimating rotation length.

about \$.10 for each dollar in annual costs because those costs will be borne for 1 less year (fig. 3B). In terms of the jack pine example, the total effect would be to increase the discounted value of net profits by (\$.006) x (\$200) + (\$.10) x (\$.40) = \$1.24 per acre.

Errors in Discount Rate

Finally, the partial derivative of present worth with respect to the discount rate can be written as:[4]

$$(9) \qquad \frac{\delta PW}{\delta i} = -\left[R\left(\frac{n}{(1 + i)^{n+1}}\right) - AC\left(\frac{1}{i^2} - \frac{1 + i + ni}{i^2\ (1 + i)^{n+1}}\right)\right].$$

The amounts of future costs and returns of a particular investment are assumed independent of *i*, the cost of using investment funds. If the cost of investment funds should increase from 6 to 7 percent, present worth would be decreased because discounted harvest returns would be less but

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[4] *This expression gives the average of the impact of decreasing the interest rate 1 percent and the somewhat smaller impact of increasing the rate a like amount. See section entitled "Impact Analysis by Computer."*

increased because discounted annual costs would be less (figs. 4A and 4B). For the jack pine example, present worth would decline to the point where this investment could only be undertaken at a loss.

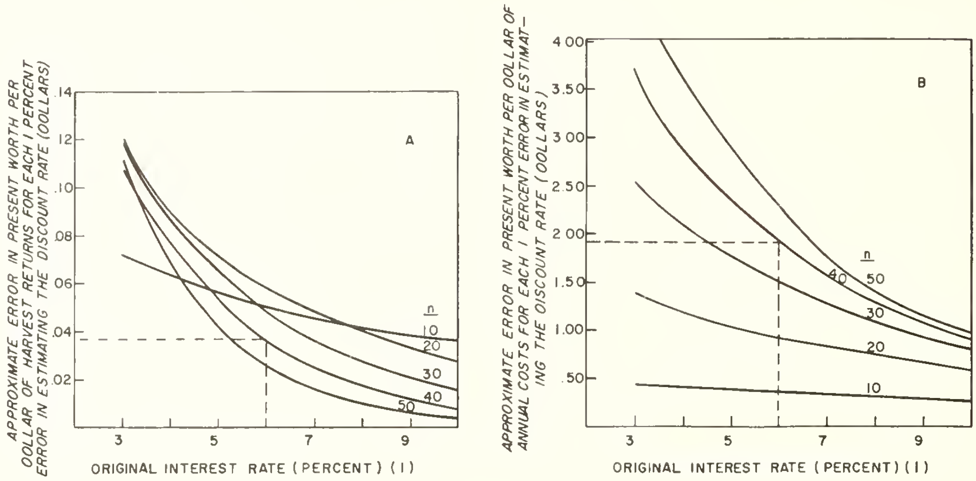


Figure 4.—(A) Approximate impact on present worth per dollar of harvest returns for each 1 percent error in estimating the discount rate. (B) Approximate impact on present worth per dollar of annual costs for each 1 percent error in estimating the discount rate.

Relative Importance of These Errors

We might summarize the findings of this jack pine example by noting that present worth will be altered by $1.00 if any of the following estimating errors are made:

	Error in absolute terms	*Error in relative terms (Percent)*
Establishment costs	$ 1.00	10.0
Annual costs	$ .07	17.5
Harvest returns	$10.00	5.0
Rotation length	1 year	2.5
Discount rate	.2 percent	3.0

This summary suggests that choosing an appropriate discount rate is important. If that rate is set too high, perhaps in an attempt to allow for uncertainties (Flora 1964), a desirable investment opportunity may be lost. On the other hand, using an artificially low rate (2 or 3 percent) will lead to investments that cannot be justified using a present worth criterion. The critical role of the discount rate suggests that its determination deserves expert attention: a rule-of-thumb approach is clearly inappropriate.

A measure of the relative importance of correctly estimating costs and returns can be derived by dividing the partial derivative of present worth with respect to a cost by that with respect to harvest returns. When these ratios for annual costs are plotted, for example, we see that at 6 percent over 40 years a dollar error in estimating annual costs will have about 150 times the effect of a dollar error in estimating harvest returns (fig. 5). Being off $.07 a year is equivalent to about a $10.00 error in estimating harvest returns, as illustrated by the jack pine problem. In general, mistakes in forcasting harvest returns are relatively important for short rotations while annual costs are the dominating factor over longer periods. All errors, except those concerning establishment costs (which are not discounted), are most critical when interest rates are low.

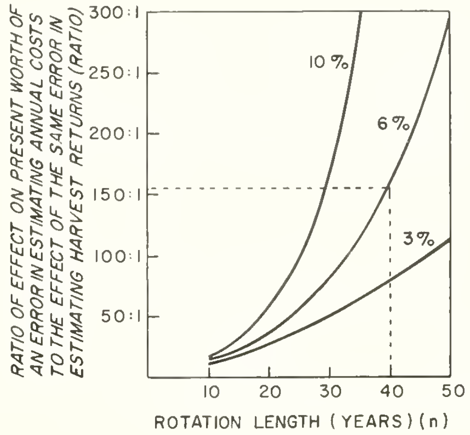


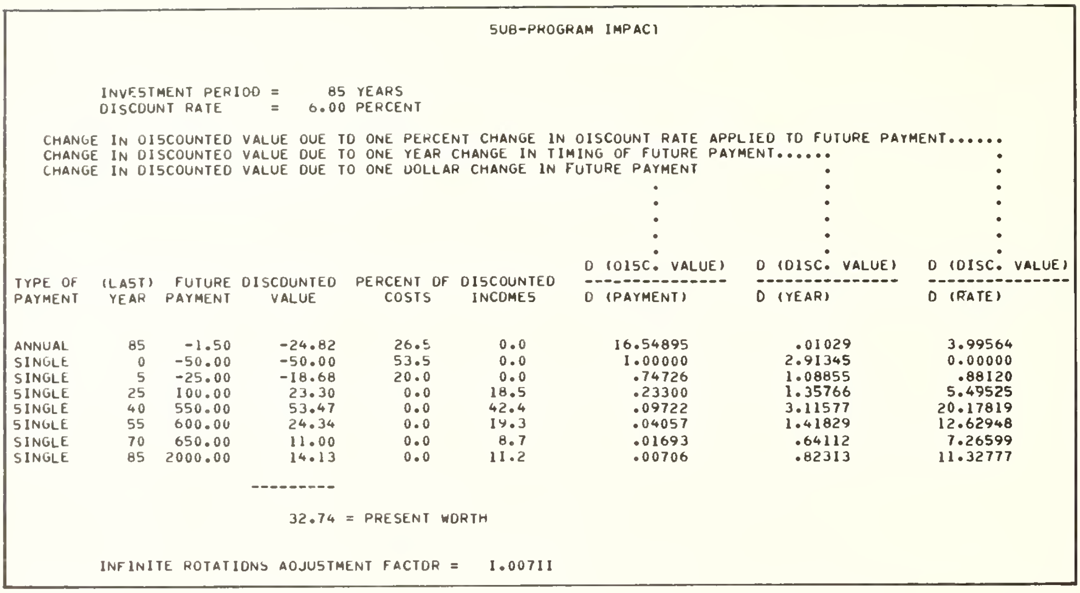
Figure 5.—The relative impacts on present worth of equal errors in estimating annual costs and harvest returns.

Impact Analysis By Computer

A computer routine titled IMPACT (see the Appendix for programing details) enables us to verify that these general conclusions remain valid for complex investments in which costs and receipts occur at irregular intervals. As an illustration consider a proposed investment in Douglas-fir management defined by the following anticipated costs and returns:

Year	*Cost or return per acre*	
Annually	–$ 1.50	Administration
0	– 50.00	Planting
5	– 25.00	Brush control
25	100.00	Thinning
40	550.00	Thinning
55	600.00	Thinning
70	650.00	Thinning
85	2,000.00	Clear cut

The computer printout quantifies the effects of errors in estimating each of these costs and returns (fig. 6).



```
                                                  SUB-PROGRAM IMPAC1


         INVESTMENT PERIOD =     85 YEARS
         DISCOUNT RATE     =   6.00 PERCENT

   CHANGE IN OISCOUNTED VALUE OUE TD ONE PERCENT CHANGE IN OISCOUNT RATE APPLIED TD FUTURE PAYMENT......
   CHANGE IN DISCOUNTEO VALUE DUE TO ONE YEAR CHANGE IN TIMING OF FUTURE PAYMENT......                  .
   CHANGE IN DISCOUNTED VALUE DUE TO ONE DOLLAR CHANGE IN FUTURE PAYMENT              .                 .
                                                                   .                  .                 .
                                                                   .                  .                 .
                                                                   .                  .                 .
                                                                   .                  .                 .
                                                            D (OISC. VALUE)    D (DISC. VALUE)   D (DISC. VALUE)
TYPE OF  (LAST)  FUTURE DISCOUNTED  PERCENT OF DISCOUNTED   ---------------    ---------------   ---------------
PAYMENT   YEAR  PAYMENT    VALUE       COSTS    INCOMES     D (PAYMENT)        D (YEAR)          D (RATE)


ANNUAL      85    -1.50     -24.82      26.5     0.0           16.54895            .01029          3.99564
SINGLE       0   -50.00     -50.00      53.5     0.0            1.00000           2.91345          0.00000
SINGLE       5   -25.00     -18.68      20.0     0.0             .74726           1.08855           .88120
SINGLE      25   100.00      23.30       0.0    18.5             .23300           1.35766          5.49525
SINGLE      40   550.00      53.47       0.0    42.4             .09722           3.11577         20.17819
SINGLE      55   600.00      24.34       0.0    19.3             .04057           1.41829         12.62948
SINGLE      70   650.00      11.00       0.0     8.7             .01693            .64112          7.26599
SINGLE      85  2000.00      14.13       0.0    11.2             .00706            .82313         11.32777

                         ---------

                             32.74 = PRESENT WORTH


         INFINITE ROTATIONS AOJUSTMENT FACTOR =   1.00711
```

Figure 6.—Analyzing the importance of estimation errors in the Douglas-fir problem by the IMPACT computer routine.

The size of each income and cost (a minus sign), whether it is a one-time or annual payment, and the year when it occurs (for annuities, the last year it occurs) are printed in the first three columns. The present values of the payments, which are calculated by using the discount rate and investment period printed at the top of the page, are listed and summed in the fourth column.

The percentage contributions of each of the individual discounted incomes and costs to total discounted incomes and costs are given in the next two columns. In the current example, planting costs (age 0) make up over half of total discounted costs, while the major item in discounted incomes is the thinning at age 40. This single thinning accounts for 42 percent of discounted incomes, compared to only 11 percent contributed by a much larger volume harvested at age 85. Further study shows that the first three thinnings account for 80 percent of all discounted incomes. Thus, in this preliminary analysis, one can quickly identify the important items for further study.

In the seventh column are listed the first three types of partial derivatives we have considered: those taken with respect to individual annual ($\delta PW / \delta AC$) and one-time or single ($\delta PW / \delta EC$ and $\delta PW / \delta R$) payments. For example, the present value of a \$2.50 annual payment (for 85 years at 6 percent) would be \$16.55 greater than the listed present value of \$24.82, which is for a \$1.50 annuity. An increase in the receipts expected at age 25 from \$100 to \$120 would increase present worth by (\$20 x .233 =) \$4.66. These partials (which have been tabulated by

Lundgren,[5] Marty and Neebe (1966) and others as discounting "multipliers") are multiplied by the corresponding future payments to get the column of discounted values.

The approximate effect of varying the investment period for each payment is quantified in the next-to-last column. If the $1.50 annuity were paid for 84 rather than 85 years, its present value would be about 1 cent less. That these entries are approximations is apparent when considering again the receipts expected in the 25th year. Receiving the $100 24 years from now would increase the present value by $1.39; waiting for 26 years would decrease it by $1.31. The column entry averages these two changes. The sum of this column of entries, when the directions of individual income and cost effects are considered, is comparable to $\delta PW / \delta n$, the total effect of delaying all investment elements 1 year (e.g., as when there is a 1-year regeneration lag).

The magnitude of error introduced by these approximations can be illustrated by summing the individual impacts, suggested by the partials, of delaying all payments 1 year in the Douglas-fir example. We assume annual administrative costs must be extended 1 year.

Year of payment		*Change in present worth (Dollars)*
Example	*+1*	
Annual		–0.01029
0	1	+2.91345
5	6	+1.08855
25	26	–1.35766
40	41	–3.11577
55	56	–1.41829
70	71	– .64112
85	86	– .82313
Total negative:		7.36626
Total positive:		4.00200

The net change in present worth estimated by this method is the algebraic sum of negative and positive changes, –$7.36 + $4.00 = –$3.36. The true net change, calculated by subtracting the present worth of the example ($32.74) from the present worth of the same investment with all payments delayed 1 year ($29.47), is –$3.27. Thus, in this example, the error in estimating the change in net worth from partials rather than calculating it directly is only $.09 (about 3 percent of the change in net worth).

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[5] *Allen L. Lundgren. Compound-discount interest rate multipliers for evaluating forest investments. (Unpublished tables on file at N. Cent. Forest Exp. Sta., St. Paul.)*

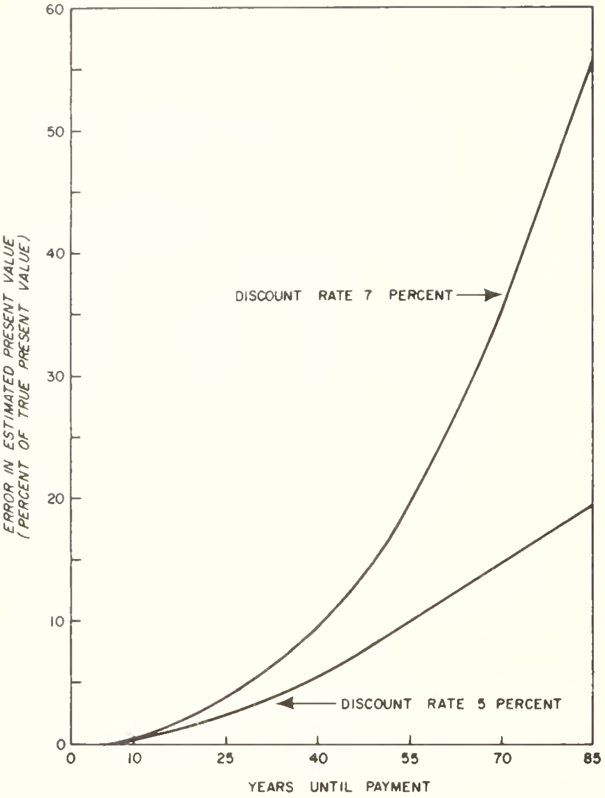

Figure 7.—The errors in present values introduced by using partials to estimate impacts of changing the discount rate from 6 percent.

Finally, the right-most entries express the sensitivity of the individual discounted values to a 1-percent change in the discount rate. The sum of these entries is analogous to the last graphed derivative, $\delta PW / \delta i$. Again, because the impact on present worth is greater for decreasing the discount rate than for increasing it a like amount, these partials are approximations. In the present case, though, the estimates are so poor that they might well be unacceptable. If, in the Douglas-fir problem, we were to rely on partials to determine present worth in substituting discount rates of 5 percent and 7 percent for the assumed 6 percent, the errors would be large. Present worth would be estimated as $84.77 in the former case, but it would actually be $102.53; if increasing the rate to 7 percent, –$19.29 would be estimated when –$7.45 is correct. The partials will always overestimate the impacts of increasing the rate and underestimate the impacts of decreasing the rate (fig. 7).

Obviously, the IMPACT routine is not adequate to tell everything about the sensitivity of present worth to the parameters defining an investment. It does, however, provide a fast and accurate first suggestion of which data items are likely to be most critical.

Sensitivity Of The Internal Rate Of Return

The rate earned by an investment when discounted costs and returns are equal is called the internal rate of return. For the earlier jack pine problem, *i* would be that rate if:

$$\frac{R}{(1 + i)^n} = AC\left[\frac{(1 + i)^n - 1}{i\ (1 + i)^n}\right] + EC. \qquad (10)$$

Rankings on the basis of this measure provide a means of choosing between investments. Again, though, the possibility of forecasting errors must be considered in evaluating the reliability of such rankings.

Starting from the earlier assumptions of $10.00 for establishment costs and $.40 a year in annual costs, internal rates of return can be calculated for several possible jack pine rotation lengths (table 1). The internal-rate-of-return criterion of desirability suggests that a 30-year rotation is most desirable, given our assumptions. However, if estimated prices are too high or yields too low, the ranking might be radically changed (fig. 8). Although a 30-year rotation would be preferred over the range of errors considered here, the shorter rotation varies all the way from being least desirable to second-most desirable. If errors were to vary by years because pulpwood prices change unexpectedly, any desirability ranking of the rotation lengths would be possible.

Table 1.—*Illustrative rates of return for alternative rotation lengths in growing jack pine*

Rotation	Estimated yield	Estimated price per cord	Estimated harvest returns	Internal rate of return
Years	Cords	Dollars	Dollars	Percent
20	9	6.00	50	6.4
30	20	6.50	130	7.5
40	28	7.00	200	6.6
50	34	7.50	250	5.6

In the case of the present worth criterion, errors in estimating final harvests were found to be most critical for short rotations. This is also true for the internal rate of return (fig. 8). The impact of an error is less for the longer rotations (the slopes of the curves decrease).

In considering how conservative estimates of returns should be to compensate for possible underestimates of costs, it is helpful to multiply both sides of the definitive equation by $(1 + i)^n$. This leads to:

$$R = AC\left[\frac{(1 + i)^n - 1}{i}\right] + EC(1 + i)^n. \qquad (11)$$

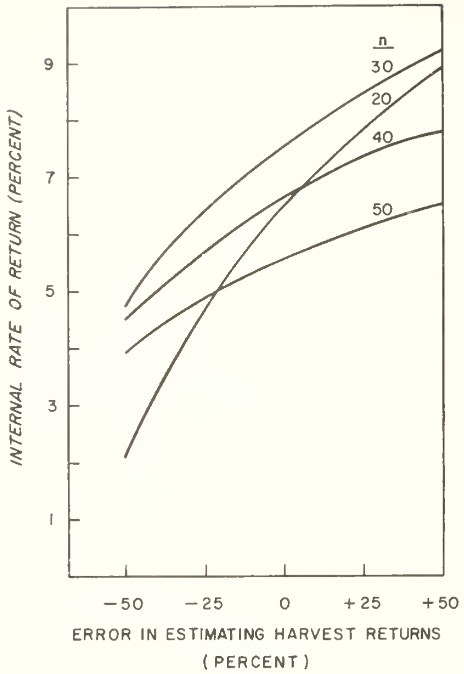

Figure 8.—The sensitivity of internal rate of return to errors in estimating harvest returns for the jack pine problem.

By again taking partial derivatives, we can determine the trade-offs of returns for costs that will just maintain the equality and that will leave the internal rate of return unchanged. For the current example where $i = 7.5$ percent in the optimal 30-year rotation:

$$\frac{\delta R}{\delta AC} = \frac{(1 + i)^n - 1}{i} = 103.40 \quad (12)$$

$$\frac{\delta R}{\delta EC} = (1 + i)^n = 8.75.$$

That is, each $1.00 underestimate of annual costs or establishment costs must be balanced, respectively, by underestimates of $103.40 and $8.75 of harvest returns if the internal rate of return is to be maintained. Substituting these values back into the preceding equation permits the calculation of the required returns for any levels of costs as:

$$R = 103.40\ (AC) + 8.75\ (EC). \quad (13)$$

A Final Word

This paper has explored how an arithmetic technique, the taking of partial derivatives, can be used to evaluate the "goodness" of data. In particular, the question of how much a calculated present worth will vary, given unit changes in a set of defining costs and returns, has been examined. We know what the impact will be if there is an estimating error — the companion question, which is not considered here, is what is the chance that there will be an estimating error. Given answers to both of these questions, a rational plan of data collection and analysis can be designed.

A basic premise is that data are collected to answer questions that guide actions. Ultimately, a decisionmaker must use his judgment to select a few of the many possible timber management opportunities. A sensitivity analysis such as this one can help to insure it will be an informed judgment.

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Appendix

The IMPACT Computer Routine[6]

The IMPACT routine was written in FORTRAN II in the form of a "subroutine." It was designed to be used with some standard investment analysis computer program. All of the following values, for each of as many as 100 annuities and/or single (nonrecurring) payments which define an investment, can be calculated:

1. The discounted value.
2. The percent contribution to total discounted costs or incomes.
3. The effect on the discounted value of (a) a $1.00 change in the future payment, (b) a 1-year change in the time of the future payment, and (c) a 1-percent change in the discount rate.

A multiplier is also calculated that can be used to adjust any discounted value if it is judged appropriate to assume a future infinite series of rotations.

Originally, the routine was written for Control Data Corporation 6400 and 6600 computers, but there should be little difficulty in adapting the routine to other machines.

Linking the IMPACT Routine To an Investment Analysis Program

The IMPACT routine can be called by any standard or "canned" investment analysis program (written in FORTRAN) if a subroutine call statement comparable to the following is supplied as linkage:

Subroutine IMPACT (PAY, NA, NS, INTIME, RATE). The arguments are defined below:

Variable	*Meaning*
RATE	The discount rate.
INTIME	The number of years until the last payment (the investment period or length of rotation).
NS	The number of (nonrecurring) single payments.
NA	The number of (recurring) annual payments.

---

[6] *Request for IMPACT source decks or comments on programming details should be addressed to Director, North Central Forest Experiment Station, Folwell Avenue, St. Paul, Minnesota* 55101.

Variable	*Meaning*
PAY (I, J)	The input data array, dimensioned in the subroutine as PAY (100, 2), which includes:
I = 1, NA	The annuities
J = 1	Last year of occurrence
J = 2	Amount of payment
I = (NA + 1), (NA + NS)	The single payments
J = 1	Year of occurrence
J = 2	Amount of payment

Explicit instructions for linking the IMPACT routine to my previously published investment analysis programs, NCRETURN (Schweitzer *et al.* 1967) and NCSUBPR (Schweitzer 1968), are available upon request.

Details of Program Construction

From the array PAY(I, J) supplied by the main program, IMPACT calculates and prints the output array P(I, J) which is defined below. Asterisks indicate that formulas are also given.

P(I, J) I = 1, (NA + NS)	The output data array dimensioned as P(100, 8)
J = 1	Same as PAY(I, 1)
J = 2	Same as PAY(I, 2)
J = 3	*Discounted value of payment
J = 4, 5	If payment is a cost (<0): P(I, 4) = discounted value as a percent of total discounted costs P(I, 5) = zero If payment is an income (>0): P(I, 4) = zero P(I, 5) = discounted value as a percent of total discounted incomes
J = 6	* $\frac{\partial(\text{discounted value})}{\partial(\text{payment})}$
J = 7	* $\frac{\partial(\text{discounted value})}{\partial(\text{year of occurrence})}$
J = 8	* $\frac{\partial(\text{discounted value})}{\partial(\text{discount rate})}$

The following formulas, which are presented in the text, were used in calculating these values.

Annual payments. – Define *n* as the smaller of P (I, 1) and INTIME

$$\begin{aligned}
&\frac{\delta\ (\text{discounted value})}{\delta\ (\text{payment})} = \frac{(1 + \text{RATE})^n - 1}{\text{RATE}\ (1 + \text{RATE})^n} \\
&\text{discounted value} = \text{P}(\text{I},\ 2) \times \left[\frac{\delta\ (\text{discounted value})}{\delta\ (\text{payment})}\right] \\
(14)\quad &\frac{\delta\ (\text{discounted value})}{\delta\ (\text{year of occurrence})} = \text{P}(\text{I},\ 2) \times \left[\frac{1}{\text{RATE}} \times \frac{\ln\ (1 + \text{RATE})}{(1 + \text{RATE})^n}\right] \\
&\frac{\delta\ (\text{discounted value})}{\delta\ (\text{discount rate})} = \text{P}(\text{I},\ 2) \times \left[\frac{1+\text{RATE}+(n \times \text{RATE})}{\text{RATE}^2\ (1 + \text{RATE})^{n+1}} - \frac{1}{\text{RATE}^2}\right]
\end{aligned}$$

Single payments.–Define *n* as P(I, 1) if P(I, 1) $\leq$ INTIME; otherwise $n = 0$.

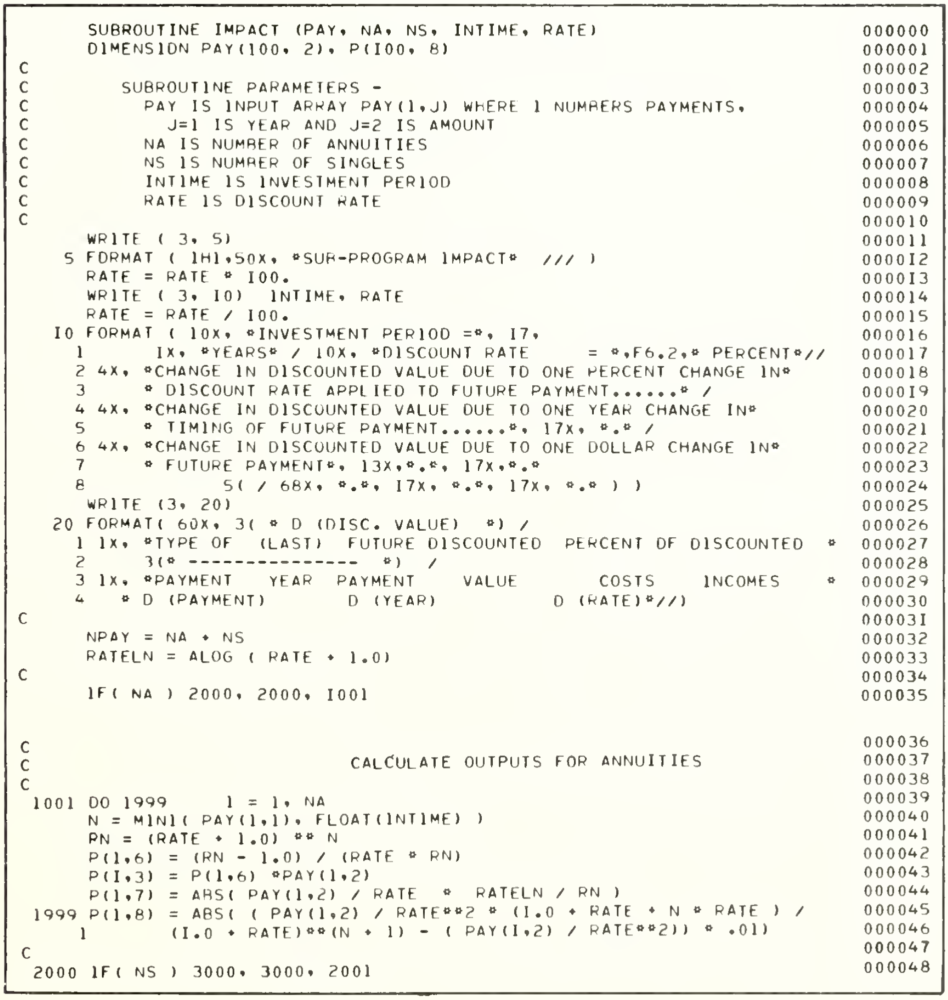
$$\begin{aligned}
&\frac{\delta\ (\text{discounted value})}{\delta\ (\text{payment})} = \frac{1}{(1 + \text{RATE})^n} \\
&\text{discounted value} = \text{P}(\text{I},\ 2) \times \left[\frac{\delta\ (\text{discounted value})}{\delta\ (\text{payment})}\right] \\
(15)\quad &\frac{\delta\ (\text{discounted value})}{\delta\ (\text{year of occurrence})} = \text{P}(\text{I},\ 2) \times \left[\frac{\ln\ (1 + \text{RATE})}{(1 + \text{RATE})^n}\right] \\
&\frac{\delta\ (\text{discounted value})}{\delta\ (\text{discount rate})} = \text{P}(\text{I},\ 2) \times \left[\frac{n}{(1 + \text{RATE})^{n+1}}\right]
\end{aligned}$$

Other FORTRAN variables used in writing the IMPACT routine are defined below:

Variable	*Meaning*
ADJ	Infinite rotations adjustment factor, calculated as: $\left[\frac{(1 + \text{RATE})^{\text{INTIME}}}{(1 + \text{RATE})^{\text{INTIME}} -1}\right]$
DC	Discounted costs
DI	Discounted incomes
N	n (as defined in the preceding formulas)
NAP1	NA + 1
NPAY	NA + NS
PW	Present worth
RATELN	ln(1 + RATE)
RN	$(1 + \text{RATE})^n$

Required library functions	
ABS	Take absolute value
ALOG	Take natural logarithm
FLOAT	Float a fixed variable
IFIX	Fix a floating variable
MIN1	Take the smaller value

A listing of the IMPACT routine source deck is presented as figure 9.

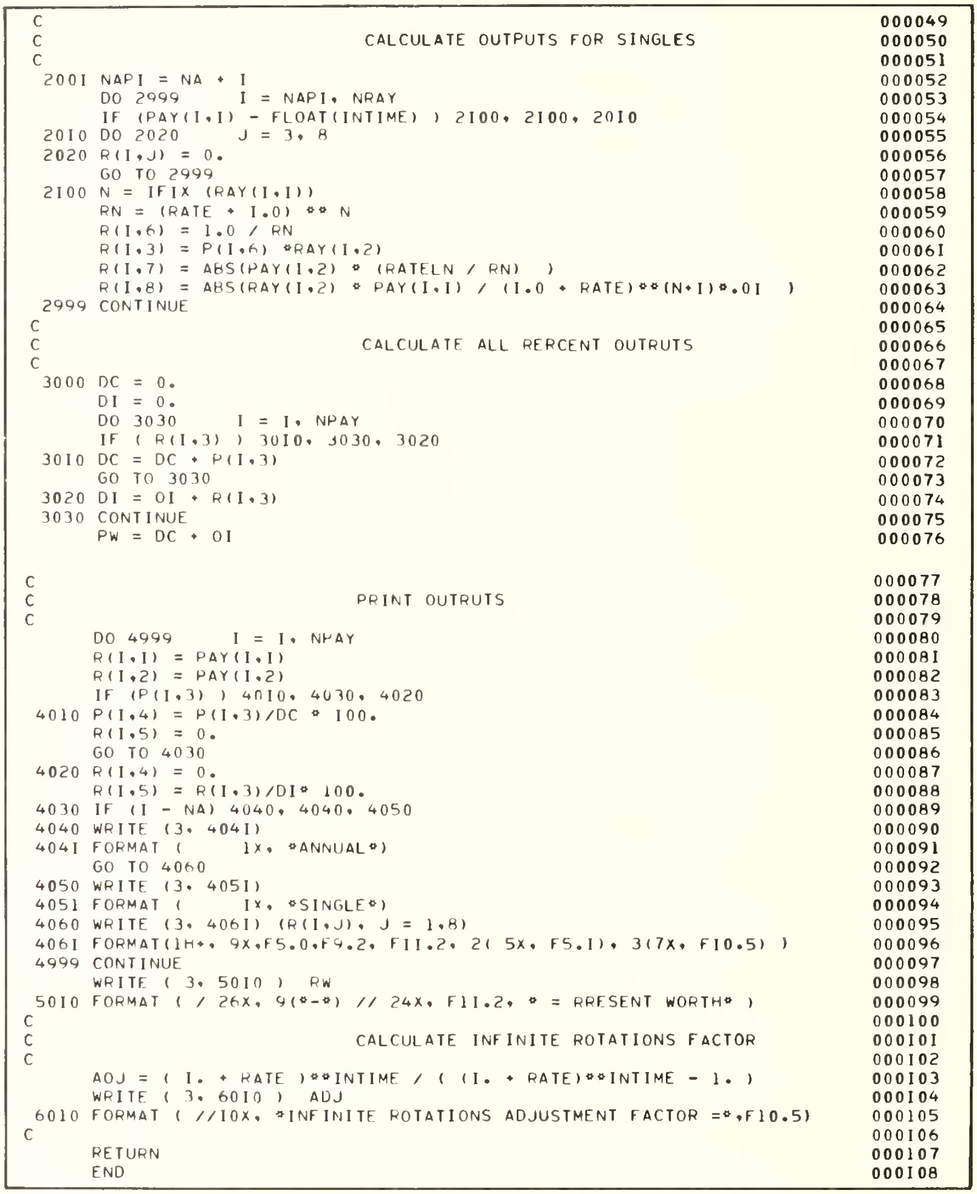


```
      SUBROUTINE IMPACT (PAY, NA, NS, INTIME, RATE)
      DIMENSION PAY(100, 2), P(I00, 8)
C
C        SUBROUTINE PARAMETERS -
C          PAY IS INPUT ARRAY PAY(I,J) WHERE I NUMBERS PAYMENTS,
C            J=1 IS YEAR AND J=2 IS AMOUNT
C          NA IS NUMBER OF ANNUITIES
C          NS IS NUMBER OF SINGLES
C          INTIME IS INVESTMENT PERIOD
C          RATE IS DISCOUNT RATE
C
      WRITE ( 3, 5)
    5 FORMAT ( 1H1,50X, *SUB-PROGRAM IMPACT*  /// )
      RATE = RATE * I00.
      WRITE ( 3, I0)  INTIME, RATE
      RATE = RATE / I00.
   I0 FORMAT ( 10X, *INVESTMENT PERIOD =*, I7,
     1       IX, *YEARS* / 10X, *DISCOUNT RATE     = *,F6.2,* PERCENT*//
     2 4X, *CHANGE IN DISCOUNTED VALUE DUE TO ONE PERCENT CHANGE IN*
     3     * DISCOUNT RATE APPLIED TO FUTURE PAYMENT......* /
     4 4X, *CHANGE IN DISCOUNTED VALUE DUE TO ONE YEAR CHANGE IN*
     5     * TIMING OF FUTURE PAYMENT......*, 17X, *.* /
     6 4X, *CHANGE IN DISCOUNTED VALUE DUE TO ONE DOLLAR CHANGE IN*
     7     * FUTURE PAYMENT*, 13X,*.*, 17X,*.*
     8            5( / 68X, *.*, I7X, *.*, 17X, *.* ) )
      WRITE (3, 20)
   20 FORMAT( 60X, 3( * D (DISC. VALUE)  *) /
     1 1X, *TYPE OF  (LAST)  FUTURE DISCOUNTED  PERCENT OF DISCOUNTED  *
     2      3(* ---------------   *)  /
     3 1X, *PAYMENT   YEAR  PAYMENT    VALUE       COSTS    INCOMES    *
     4   * D (PAYMENT)       D (YEAR)          D (RATE)*//)
C
      NPAY = NA + NS
      RATELN = ALOG ( RATE + 1.0)
C
      IF( NA ) 2000, 2000, I001


C
C                                CALCULATE OUTPUTS FOR ANNUITIES
C
 1001 DO 1999     I = 1, NA
      N = MIN1( PAY(I,1), FLOAT(INTIME) )
      RN = (RATE + 1.0) ** N
      P(I,6) = (RN - 1.0) / (RATE * RN)
      P(I,3) = P(I,6) *PAY(I,2)
      P(I,7) = ABS( PAY(I,2) / RATE  *  RATELN / RN )
 1999 P(I,8) = ABS( ( PAY(I,2) / RATE**2 * (I.0 + RATE + N * RATE ) /
     1        (I.0 + RATE)**(N + 1) - ( PAY(I,2) / RATE**2)) * .01)
C
 2000 IF( NS ) 3000, 3000, 2001
```

Figure 9.—Listing of IMPACT routine source deck.

Figure 9 Continued



```
C
C                            CALCULATE OUTPUTS FOR SINGLES
C
 2001 NAPI = NA + 1
      DO 2999      I = NAPI, NRAY
      IF (PAY(I,1) - FLOAT(INTIME) ) 2100, 2100, 2010
 2010 DO 2020      J = 3, 8
 2020 R(I,J) = 0.
      GO TO 2999
 2100 N = IFIX (RAY(I,1))
      RN = (RATE + 1.0) ** N
      R(I,6) = 1.0 / RN
      R(I,3) = P(I,6) *RAY(I,2)
      R(I,7) = ABS(PAY(I,2) * (RATELN / RN)  )
      R(I,8) = ABS(RAY(I,2) * PAY(I,1) / (1.0 + RATE)**(N+1)*.01  )
 2999 CONTINUE
C
C                            CALCULATE ALL RERCENT OUTRUTS
C
 3000 DC = 0.
      DI = 0.
      DO 3030      I = 1, NPAY
      IF ( R(I,3) ) 3010, 3030, 3020
 3010 DC = DC + P(I,3)
      GO TO 3030
 3020 DI = OI + R(I,3)
 3030 CONTINUE
      PW = DC + OI


C
C                            PRINT OUTRUTS
C
      DO 4999      I = 1, NPAY
      R(I,1) = PAY(I,1)
      R(I,2) = PAY(I,2)
      IF (P(I,3) ) 4010, 4030, 4020
 4010 P(I,4) = P(I,3)/DC * 100.
      R(I,5) = 0.
      GO TO 4030
 4020 R(I,4) = 0.
      R(I,5) = R(I,3)/DI* 100.
 4030 IF (I - NA) 4040, 4040, 4050
 4040 WRITE (3, 4041)
 4041 FORMAT (      1X, *ANNUAL*)
      GO TO 4060
 4050 WRITE (3, 4051)
 4051 FORMAT (      1X, *SINGLE*)
 4060 WRITE (3, 4061) (R(I,J), J = 1,8)
 4061 FORMAT(1H+, 9X,F5.0,F9.2, F11.2, 2( 5X, F5.1), 3(7X, F10.5) )
 4999 CONTINUE
      WRITE ( 3, 5010 )  RW
 5010 FORMAT ( / 26X, 9(*-*) // 24X, F11.2, * = RRESENT WORTH* )
C
C                            CALCULATE INFINITE ROTATIONS FACTOR
C
      AOJ = ( 1. + RATE )**INTIME / ( (1. + RATE)**INTIME - 1. )
      WRITE ( 3, 6010 )  ADJ
 6010 FORMAT ( //10X, *INFINITE ROTATIONS ADJUSTMENT FACTOR =*,F10.5)
C
      RETURN
      END
```

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- Managing and protecting the 187-million acre National Forest System.

The Forest Service does this by encouraging use of the new knowledge that research scientists develop; by setting an example in managing, under sustained yield, the National Forests and Grasslands for multiple use purposes; and by cooperating with all States and with private citizens in their efforts to achieve better management, protection, and use of forest resources.

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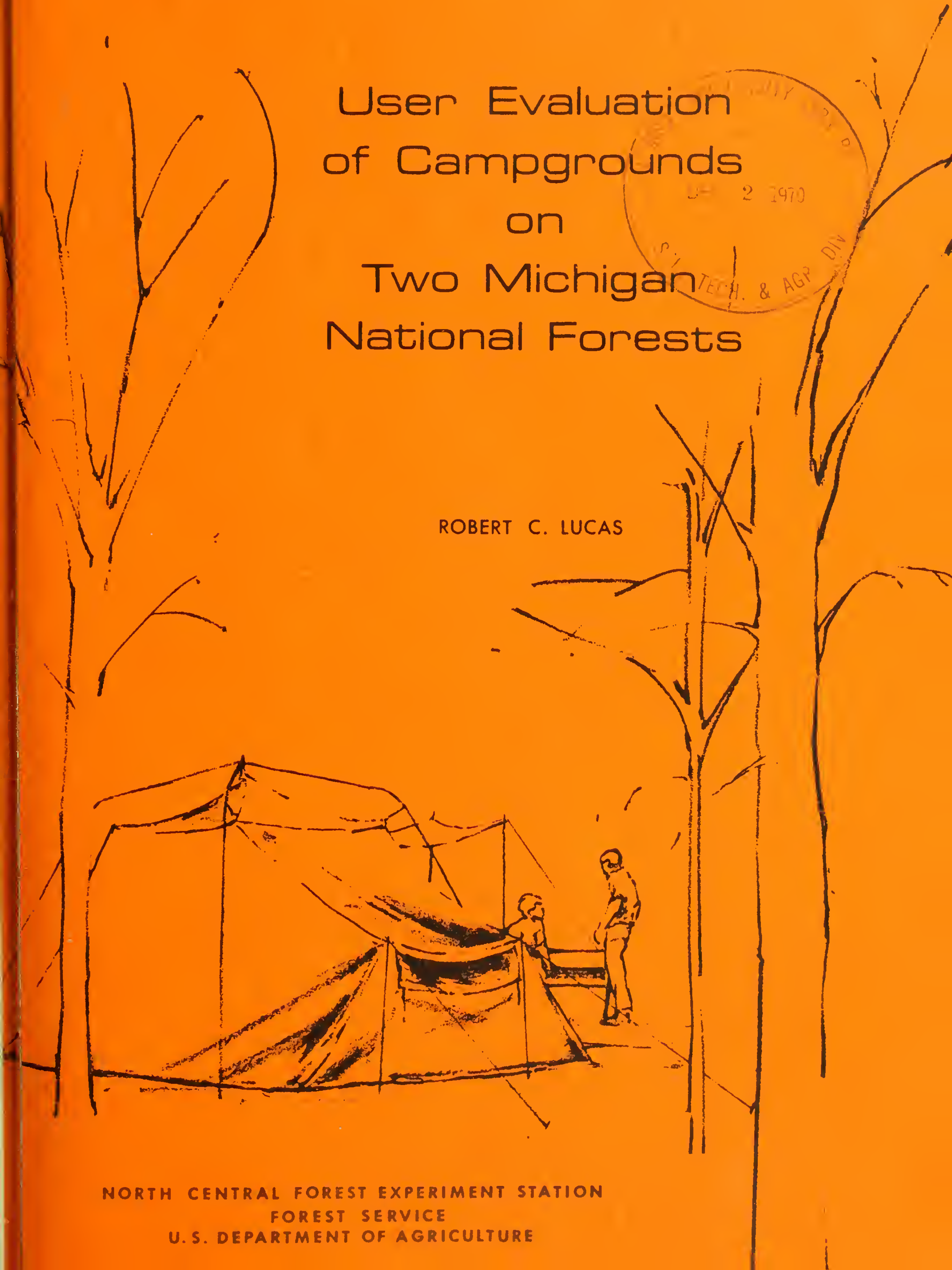
A FOREST SERVICE
CH PAPER NC-44

User Evaluation of Campgrounds on Two Michigan National Forests

ROBERT C. LUCAS



NORTH CENTRAL FOREST EXPERIMENT STATION
FOREST SERVICE
U.S. DEPARTMENT OF AGRICULTURE

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User Evaluation of Campgrounds on Two Michigan National Forests

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The Problem

Recreational areas vary widely in amount of use; some places are crowded, while others are only lightly visited. For example, the most popular Minnesota State Park campground received 14 times as much use per unit as the least popular in 1961 (Minnesota Outdoor Recreation Resources Commission 1965). Some Superior National Forest campgrounds in Minnesota received more than six times as many group visits per unit as others in 1961 (Lucas 1964), and more than five times as many in 1967.[1] Better selection of sites for recreational development, based on a better understanding of the reasons for uneven visitor distribution, could improve public enjoyment and operating efficiency.

Objective

The main objective of this study was to determine how variation in recreational use among campgrounds is related to characteristics of the campground sites and to people's ideas about them. Variation among sites within campgrounds is not included. Although use of campgrounds, picnic areas, beaches, and access points to lakes and streams on the Huron and Manistee National Forests in Michigan's Lower Peninsula was investigated, the study emphasized campground use; therefore, this paper will make only incidental references to other recreation uses.

Study Approach

The key to understanding the role of factors related to recreational use distribution lies in understanding people's perception of their environment. We assume people choose recreation sites from among those they know about on the basis of how desirable they think the places are for their purposes, and their view of the time, effort, and cost of visiting them. People vary in their knowledge, purposes, standards of desirability, and willingness to make the effort to use particular locations.[2] Some people study maps and plan trips all winter; others jump in the car with a vague destination and pull into the first place they see. We assume the pattern of total use is a composite of decisions arrived at in many different ways.

Therefore, I will first explain how observed differences in campground use relate to characteristics of existing sites.[3] Most of the factors that we would expect to influence use distribution fall into one of three

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[1] *Lime, David W. A spatial analysis of auto-camping in the Superior National Forest of Minnesota: models of campground selection behavior. (Ph.D. thesis on file at University of Pittsburgh.)*

[2] *The fact that half the Huron-Manistee campers in 1962 were visiting these forests for the first time suggests a greater than normal role for chance in location choices. This low level of prior experience may not be unusual, however; one-third of the campers on the Huron National Forest were newcomers in 1966 (Krejcarek, Don E. An analysis of family campers' socioeconomic characteristics, preferences, and attitudes toward fees on the Huron National Forest. Unpubl. M.S. thesis, Mich. State Univ.)*

[3] *This general approach is also used in a recent study of use of New York State Parks in the Adirondack region (Shafer and Thompson 1968).*

categories — resource characteristics at the site, the facilities there, and its relative location or accessibility. No fees were charged at any of the sites at the time of the study, and regulations were identical except for two locations where campers were turned away when the campground was full.

Second, I will analyze visitors' attitudes concerning the resources at the site, the type and quality of facilities, crowding and user conflicts, general satisfaction, and sources of information about the area. Resource quality ratings made as part of the National Forest Recreation Survey or NFRS (USDA Forest Service 1959) will be compared to visitors' ratings of the same resources.

The Study Area

The Huron and Manistee National Forests are located where the urbanized, industrialized Midwest ends and the northwoods begin (fig. 1). More than 90 percent of Michigan's people live south of the two National Forests; thus these Forests are in the front lines facing the northward flow of recreationists.

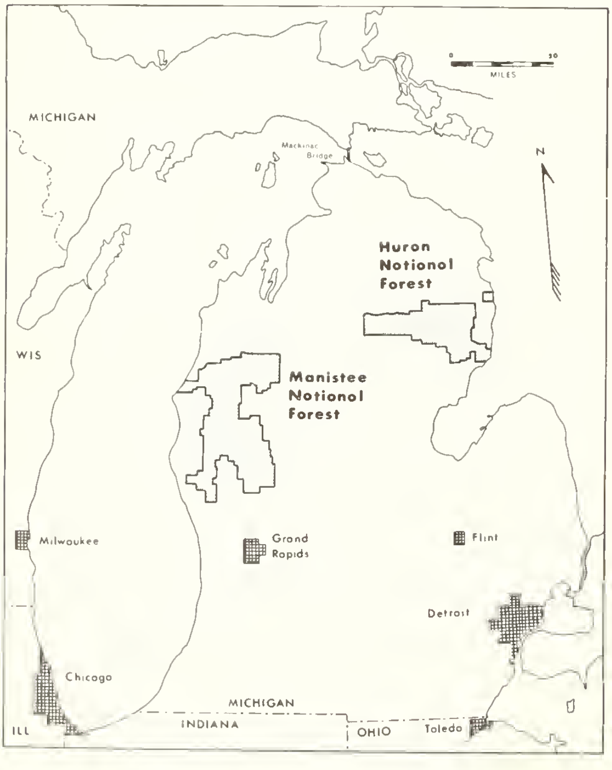

Figure 1. — The location of the Huron and Manistee National Forests.

The two Forests have similar recreational resources. Both are on sandy glacial deposits, sometimes rolling or hilly, but primarily a plain (University of Michigan 1967). Small lakes are common. Streams are numerous and many support trout (the Coho salmon fishery developed after this study). Second-growth forests of mixed deciduous and coniferous trees cover most of the area. Both Forests had substantial recreational facilities. There were 22 campgrounds, many with picnic areas, and eight separate picnic areas in 1962 when the fieldwork for the study was carried out. A dense network of roads crisscrossing both Forests made access generally easy, but hiking trails were limited. Official Forest Service recreation-use estimates show that the Huron-Manistee was among the most visited National Forests in 1962.

State Parks are fairly abundant near the Forests, especially along the Lake Michigan shore. There are several State Forests in the region, but they had little recreational development. Summer home and small resort development was extensive, but private campgrounds were few and small.

Study Methods

The Sample

The study was conducted from April 28 through September 14, 1962. This included all of the trout-fishing season. The camping season was divided into four 5-week periods.[4] Each campground was checked seven times during a 5-week period, each time on a different day of the week. Thus, each campground was checked a total of 28 times during the season. All checks were made between 1:00 p.m. and 9:00 p.m. Occupied sites were tallied, and one-fourth (as nearly as possible, but at least one) of them were selected as samples using random numbers. If there were one through five groups in the campground, one group was interviewed; if six through nine groups, two were interviewed, and so on. Occupied but unattended sample sites were revisited before leaving the area, and again later in the day if possible, before a substitute was randomly chosen. None of the campers

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[4] *For a more detailed explanation of sample date selection see King, David A. Sampling and length-of-stay bias adjustment; for a fuller treatment of sampling see Lucas, Robert C. The distribution of recreational use on the Huron-Manistee National Forests. (Unpublished reports on file at N. Cent. Forest Exp. Sta., St. Paul, Minn.)*

refused to be interviewed. This procedure produced 597 interviews, which means that 13 percent of the estimated number of groups visiting the area were interviewed.

The group spokesman, generally the head of the household, answered for the group. Whether interviewing the spokesman comes closer to revealing the group consensus (which we assume determines the group's decisions) than interviewing each individual is not known.

The interview data were affected by length-of-stay bias like all other on-site recreation surveys. Groups staying a longer time were overrepresented relative to those staying more briefly. This effect was removed by a computer program that weighted interviews (Lucas and Schweitzer 1965).

Use Estimates

Estimates of numbers of visitors and man-days of use at each location were made from the use tallies. Three sample campgrounds were checked 4 days each, every hour from 11:00 a.m. to 9:00 p.m., and ratios of overnight use to hourly use were used to adjust the observed use totals. For example, a campground with 18 sites occupied at 5:00 p.m. on a Friday would be estimated to have had 19.6 overnight groups. These adjusted estimates were then expanded for each location.

The minimum acceptable precision for recreation-use estimates given in the Forest Service Manual is ± 25 percent at the 67-percent confidence level. Estimates for 17 of 22 campgrounds met this standard. The five that did not received light use — less than half as much use per unit as the average for all campgrounds.

Resource and Location Variables

The resource variables were provided by the National Forest staff from the 1959-1960 inventory data (NFRS), supplemented when necessary.

Each location was classified in the NFRS as to location relative to water (USDA Forest Service 1959). There were six categories:

1. Accesible to a lake or reservoir (10 acres or more).
2. Accessible to a pond (under 10 acres).
3. Accessible to a river navigable for boats and canoes.
4. Accessible to a river navigable only by canoes.
5. Accessible to a small stream (nonnavigable).
6. Not accessible to a body of water.

There were no campgrounds in categories 2 and 6.

The NFRS ratings of the beach, fishing, boating water, canoeing water, and occupancy site quality (outstanding, good, fair, unsatisfactory, absent) were treated as equally spaced points on a scale, as they were in the inventory, and were assigned numbers 1, 3, 5, 7, and 9.

Relative location was measured in terms of road miles over normal travel routes:

1. Shortest distance to the Great Lakes.
2. Distance to the nearest State highway.
3. Distance to the nearest paved highway.
4. Distance to the nearest campground (whether National Forest, State Park, or other).
5. Average distance to the three nearest National Forest campgrounds (a measure of clustering).
6. Distance to Flint or Grand Rapids, Michigan, whichever was closest. This was a measure of relative distance from the main population of potential visitors, which was concentrated heavily in southern Michigan (King 1965, p. 3-5).

Recreational Use: Amount and Distribution

Campground use accounted for half of the total estimated man-days of recreation on the two Forests. Camper length of stay averaged 4.6 days. About 18 percent of all camping was at undeveloped spots, either within existing campgrounds but not at established units, or at lakes or streams having no developed campground. Only developed campgrounds were included in the locational analysis.

The Huron campgrounds fall in two clusters — four closely spaced sites in the western half of the Forest, and five more widely spaced sites in the east (fig. 2). The western cluster was somewhat more uniform in total man-days of use. The Manistee had about one-fourth more camping than the Huron, and had both the most- and the least-used campgrounds (fig. 3). All but three of the 13 Manistee campgrounds were located in the northern half of the Forest. The campgrounds with the greatest use were also in the north, with Sand Lake the center of the cluster and by far the most used.

Because the campgrounds varied in size, capacity had to be taken into account. Therefore figures 2 and 3 show use in terms of groups-nights per unit. (A campground with every developed unit occupied every night would equal 100 percent.) This is probably the unit of measurement most needed for resource planning.

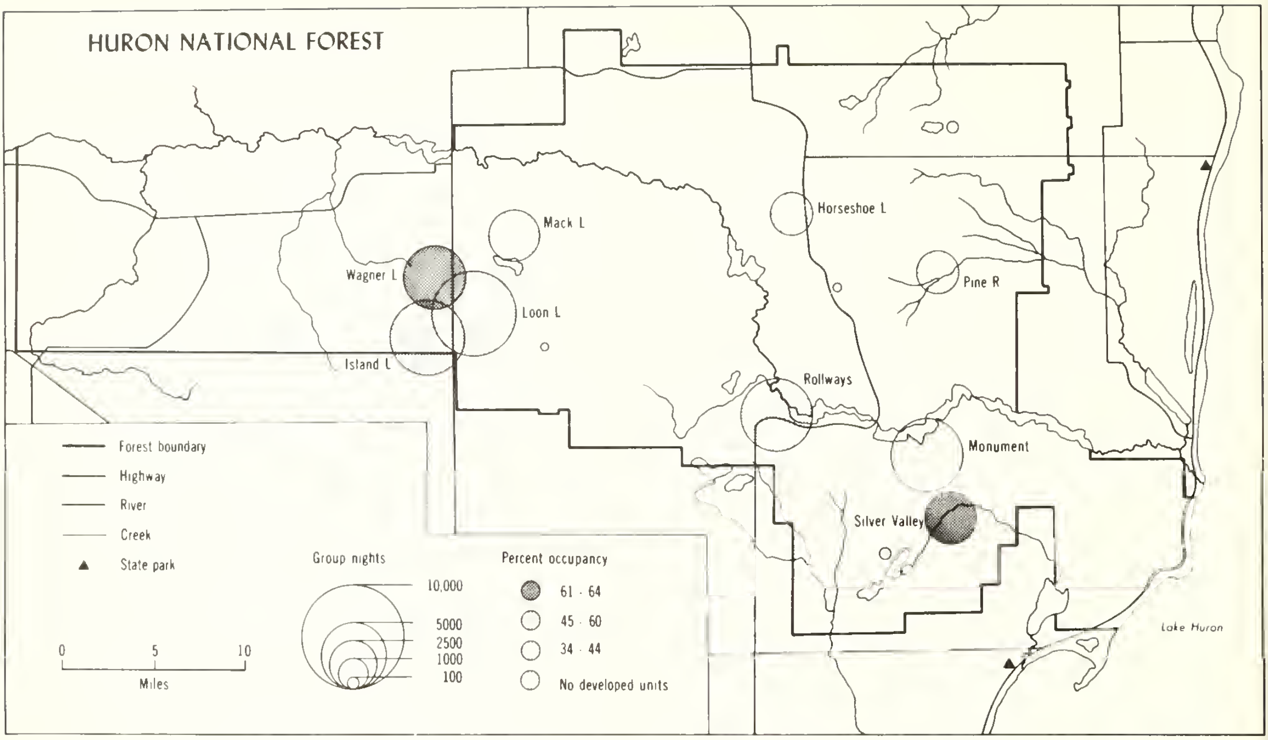

Figure 2. — Camping use, Huron National Forest, April 28-September 14, 1962.

Again, the Manistee had both more use and more variable use. Occupancy ranged from 40 to 97 percent on the Manistee compared with 34 to 64 percent on the Huron. The map shows no obvious pattern of occupancy rates. Heavily used campgrounds were found both on lakes and on streams, both along highways and in out-of-the-way places, and in the north and south.

Variables Related to Use Distribution

After examining each independent variable through simple correlations or class means, and on scatter diagrams, multiple regression was used to determine the variables useful in combination for estimating use, and how well they would estimate use.

Analysis of Variables Singly

Only one readily measured campground variable — creek location — appeared useful for estimating appeal when factors were considered singly (table 1). Use of campgrounds located near creeks averaged only 39 percent of capacity, compared to 58 to 71 percent for those near lakes or rivers.

The correlation of 0.40 (table 1) shown for the NFRS canoeing rating is somewhat misleading. There was little difference in use among outstanding, good, and fair canoeing locations; the big difference was between the places with canoeing opportunities and those without — the creek locations again.

Visitor rating of the fishing was strongly related to use — but this is difficult to apply directly in planning for new developments. The overall rating by visitors also was moderately associated with use per unit.

A few other variables were associated with use in the direction suggested by location theory (Haggett 1966) or common recreation planning assumptions, but not to any important extent: stream width, the length of beach at the site,[5] fishing quality, an im-

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[5] *If only the locations with beaches are considered, the relationship is stronger. Beach quality as measured for NFRS is weakly related to use, but less than beach length. Length and quality are closely correlated (r = 0.79). Most of the difference on the quality rating is between places with beaches and those without, and simple presence of a beach is almost as good an estimator as quality.*

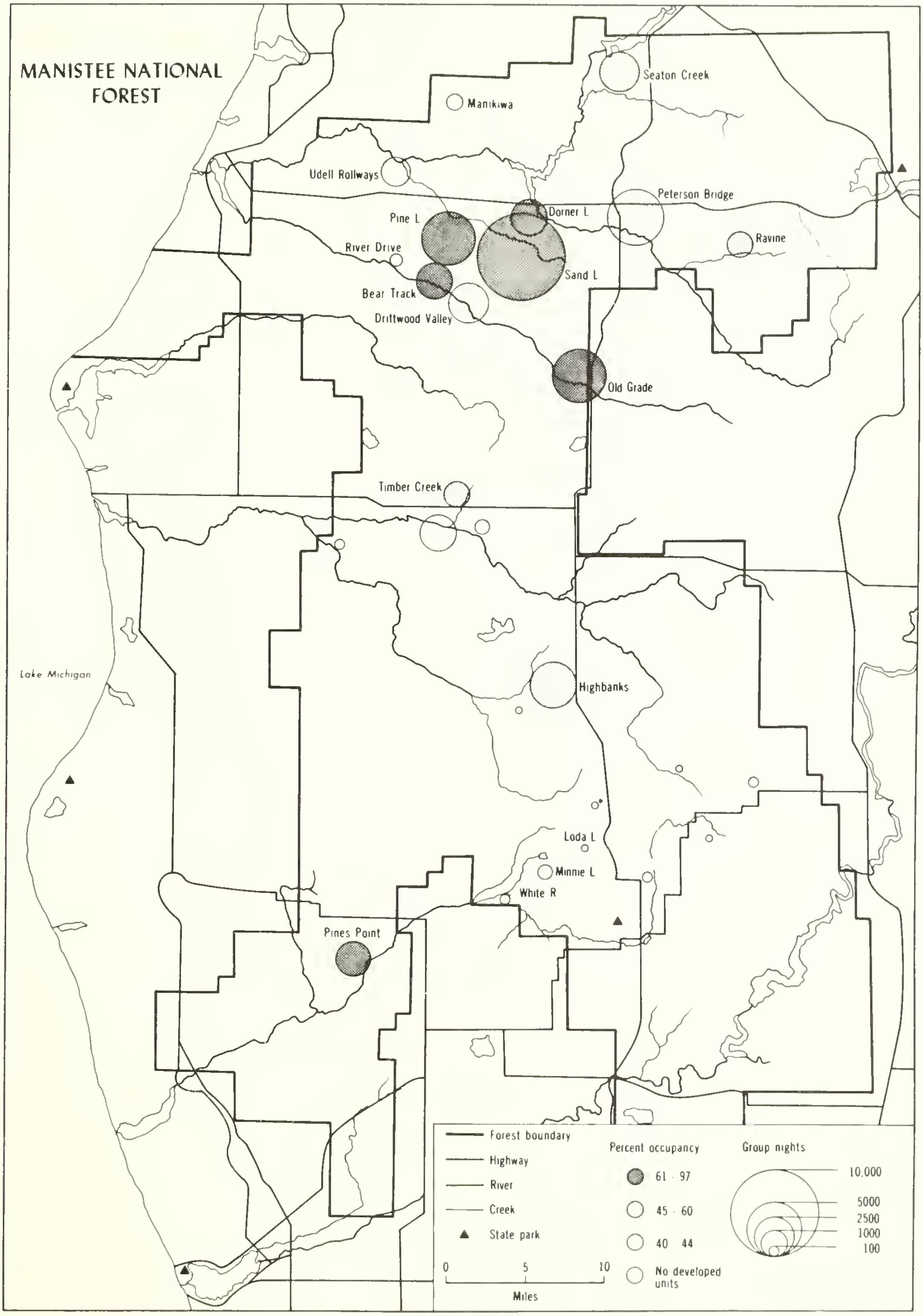

Figure 3. — Camping use, Manistee National Forest, April 28-September 14, 1962.

Table 1. — *Association of campground characteristics with group nights of camping per developed unit (22 locations)*

Characteristic	Simple correlation[1]		Average use as a percent of capacity
Physical resource features:			
Lake	--		58
Large river (boat-navigable)	--		71
Small river (canoeable)	--		66
Creek (unnavigable)	--		39
Size of lake, acres	-0.16		--
Width of stream, tenths of chains	.15		--
Size of lake or width of stream, relative to the mean for lakes or streams	-.16		--
Yards of beach	.32		--
Presence of beach	--		62
Absence of beach	--		54
Resource quality rating:[2]	Visitors'	NFRS	
Beach	.07	.24	--
Fishing	* .62	.31	--
Boating	.15	.05	--
Canoeing	.27	* .40	--
Occupancy site	.19	-.12	--
Resource composite at site	* .40	--	--
Development:			
Presence of developed boat access	--		65
Absence of developed boat access	--		52
Number of campground units	-.19		--
Huron	--		48
Manistee	--		63
Relative location:			
Distance to nearest paved road	.18		--
same, inverse (1/Distance + 1.0)[3]	-.21		--
Distance to State or Federal highway	.23		--
same, inverse (1/Distance + 1.0)	-.25		--
Distance to Flint or Grand Rapids	-.03		--
same, inverse	.07		--
same, inverse of distance squared	.09		--
Distance to Great Lakes	-.26		--
same, inverse	.18		--
Distance to nearest campground	-.23		--
same, inverse	.13		--
Average distance to three nearest Forest Service campgrounds	-.04		--
same, inverse	.21		--

[1] Pearson product-moment correlations are reported for campground characteristics measured on interval scales or on ordinal scales for which an assumption of equal intervals seemed acceptable. Average use as a percent of capacity for all campgrounds sharing a given characteristic is reported for characteristics in the form of nominal data.

[2] The higher the quality, the smaller the coded value. "Outstanding" = 1, "Good" = 3, etc., and "None" = 9. However, for ease in interpretation, signs have been reversed, so a positive correlation indicates a positive association of use and quality.

[3] The 1.0 was added because some distances were zero, which would lead to an inverse equal to infinity.

* = Significantly different from zero at 0.10 level.

proved boat access, proximity to Lake Michigan or Lake Huron, and campground clustering.

Some of the relations between independent variables and use were unexpected: lake size was negatively related to use although the range in size of lakes was small, and several small lakes had good beaches that were lacking at the large lakes; boating quality as estimated for NFRS was negatively related to use; campground size was weakly negatively related to use (except for one large, popular campground the negative correlation would have been stronger); distance to paved roads or to main highways was negatively related to use (the more accessible places were used less); and distance from the population concentrations to the south was positively related to use. Although the two Forests seem similar, the Manistee appears more attractive to campers.

Analysis of Variables in Combination

Most of the variables contributed little in accounting for use, and all but four were finally dropped from the multiple regression analysis. As a group, these four independent variables — campground size, yards of beach, type of water location, and the inverse of distance from the Great Lakes — accounted for 69 percent of the variation in use per unit[6] in the following equation:

$$Y_1 = 2.85 - 0.25X_{13} + 0.36X_{15} + 5.90X_{37} + 3.61X_{38} + 4.30X_{40} + 68.51X_{41}$$

R^2 = 0.69. Standard error of estimate of Y = 1.87 (mean of Y_1 = 7.95, standard deviation = 2.85), where

Y_1 = Estimated group nights per campground unit, in tens
X_{13} = Number of campground units
X_{15} = Yards of beach, in tens
X_{37} = Dummy variable, 1 = canoeable river location
X_{38} = Dummy variable, 1 = boat-navigable river location
X_{40} = Dummy variable, 1 = lake location (creek locations would be 0 for all three of these variables)
X_{41} = The inverse of road distance to the nearest Great Lake in miles (1/D)

The overall relationship is fairly strong, more so than for the variables singly, with an F-value larger than required for significance at the .01 level. The average discrepancy between observed and "predicted" use per unit for each campground was 17 percent. The difference exceeded 30 percent for three campgrounds (use was overpredicted) and the largest error was 53 percent.

The larger the campground, the less use per unit it received, on the average. This agrees with findings in part of the Superior National Forest (Lucas 1964) and with results from several Colorado National Forests.[7] This suggests that larger campgrounds are not necessarily more attractive because of their greater size, or because they generate more word-of-mouth advertising. It is possible, however, that the larger campgrounds are somewhat newer, and are not as well known yet.

The type of waterbody was moderately related to use. Greatest use was indicated for canoeable rivers, followed closely by lakes, then by larger, boat-navigable rivers, with creeks far behind. These relative weights were consistent no matter what other variables were included in the equations.

The amount of beach had a strong positive relation to use. The presence or absence of a beach was less strongly related than yards of beach.

Accessibility to Great Lakes shoreline was not very important. For example, a campground 20 miles from one of the Great Lakes would be predicted to have about 2 percent more use than one 40 miles away. (Distance to the Great Lakes varied from 8 to 46 miles.) I hypothesized that campgrounds closer to the Great Lakes would have greater use, but there is doubt if a causal relationship exists. There is no way of telling from the other data collected what the role of the Great Lakes shoreline might be. I do not know if many visitors were driving the scenic routes close to the Lakes, or combining stays at State Parks on the Great Lakes with camping on the National Forests, or perhaps overflowing from State Parks.

The equation containing only campground size and length of beach accounts for 25 percent of the variation in use per unit. If type of water location is added to campground size and beach length, 48 percent of the variation in use per unit is accounted for. If water locations are classified only as "creeks" and "other," R^2 is 0.45.

The "net effects" (Cooley and Lohnes 1962) or "coefficients of separate determination" (Mills 1955) indicated the following contributions by each variable to reducing the variance in use per unit:

Variable	*Contribution to R^2*	*Percent contribution*
Type of waterbody	0.29	42.0
Yards of beach	.20	29.0
Number of CG units	.10	14.5
Inverse of Great Lakes distance	.10	14.5
Total	.69	100.0

Some of the omitted variables were weakly related in the expected direction, but a few seemed backwards in their relation to use — at least at first glance. The presence of an established boat access was associated with greater use, but in combination with the variables included in the final equation, it did not improve use estimates. Most of the NFRS quality

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[6] *The same variables related to total group nights for the 140-day season accounted for 90 percent of the variation, with number of units and yards of beach dominant in this case (these two variables alone had an R^2 of 0.87). The average discrepancy between observed and predicted total use for each campground was 15 percent.*

[7] *Personal communication with Wendell Beardsley, Intermountain Forest and Range Experiment Station, Logan, Utah.*

ratings were slightly associated positively with use. Fishing quality came the closest to making a contribution. This does not mean that resource quality is irrelevant in explaining or predicting campground use. In fact, there was an improvement in estimates when the visitors' average composite rating of each site's resources was added to the final equation. Visitors' ratings of fishing also were associated with use, but all these visitor ratings were omitted because they would not be applicable to planning for potential sites.

The effect of the distance variables based on remoteness from paved roads or State highways was weak, and actually the opposite of what I expected based on general location theories. Distance from the area's main source of visitors to the south also seemed reversed. The more distant campgrounds were used *more,* not less. (The effect was small, however.) It is doubtful if the range of 70 miles in distance between the closest and farthest places was perceived as very important by most visitors. Greater distance to the campgrounds also means "up north," because the visitors almost all live to the south (King 1965). It is possible that the appeal of the northwoods is stronger than the friction of distance within the two National Forests. The proportion of residents in each county who visit the Forests does decline with increasing distance from the study area (King 1965), but it appears that the variation in remoteness within the Forests lies below some threshold of perceived importance in terms of cost or effort. Within the Forests, distance seems to act like a lure rather than as a cost or friction as it does in most human activity. Between home and the Forest boundary, distance assumes its normal role of a cost or deterrent. The decision to make the trip to the general area apparently is made in a different frame of reference than the choice of a specific destination within the Forests. This finding of a two-level distance effect is consistent with results of an earlier study of the Superior National Forest (Lucas 1964).

The campground clustering variable added nothing to the equation's predicting ability.

Much of the difference in use between the Huron and the Manistee was apparently due to differences in campground resource features between the two Forests, judging from the small effect of a dummy variable for Forest.

Finally, the difference between observed and predicted use as a percent of observed use was mapped for any indications of overlooked effects or variables. At four campgrounds discrepancies were 25 percent or more, all overpredicted. All four of these overpredicted locations had below-average use. They have no apparent common characteristic of location, development, or resources to suggest why they are less popular than would be expected.

In summary, campground use could be fairly wel predicted on the basis of a combination of physica resource features — type of water body and amoun of beach — and size of development. Quality ratings as measured, were not important, except perhaps fo fishing. Distance from population concentrations o main roads seemed unimportant.

Campground use is clearly not a simple functio of a few dominant characteristics. One feature apparently can offset the lack of another in a complex an variable way — for example, beaches seem to be a important attraction, but there are popular campgrounds without beaches. There seems to be no simpl shortcut to forecasting the drawing power of campground sites. Useful estimates probably will need t be based on environmental features considered i combination.

It might be added that what makes a popula campground does not necessarily make a popular picnic area. Total group picnic visits and total grou nights camping, in places with opportunities for both had a correlation coefficient of 0.44, but picnic visit and camping, with both on a use *per unit* basis, wer negatively associated (—0.42). (Both correlation were significant at the 0.05 level.) The variables related to picnicking use were also different than fo campground use. For example, beaches were not a important factor in picnic area use. However, sit quality, paved road access, and capacity were all significantly related to picnic use per unit with an R in multiple regression of 0.53. A convenient, adequat place appears to be all that most people are after fo such a brief, undemanding type of recreation a picnicking.

Visitor Attitudes

Some visitor attitudes are directly related to th use-distribution analysis, while others are important i relation to future use patterns and to visitor satisfaction.

General Satisfaction

The vast majority of visitors to the Huron an Manistee National Forests liked what they found ver much. This high level of satisfaction may not alway be recognized by administrators because the dissatisfied person is likely to complain, while the overwhelm

Table 2. — *Campers' satisfaction related to years of National Forest camping experience*
(In percent)

Years of experience	N (weighted for length of stay bias)	Answer to question: Do you think your group will visit this location again? Yes	No	Maybe	If answer was "no" or "maybe," the group: Was not satisfied with the site	Liked the site, but felt it too hard to reach	Liked the site, but preferred to visit new areas	Other
0	259	77	3	20	2	4	11	5
1	61	83	--	17	--	--	13	4
2-4	114	88	1	11	3	--	5	4
5-9	51	94	--	6	2	--	--	4
10+	99	98	--	2	--	2	1	1
All campers	584	84	1	15	2	2	8	4

ing majority quietly enjoy the area. Recreation surveys thus can be useful in putting complaints in a more balanced, objective picture. All the sample groups were asked if they thought their group would visit that particular location again. Only 1 percent said no, and most of these were seeking new places to see rather than dissatisfied.

Camper satisfaction was directly related to years of experience in National Forest camping (table 2). This was to be expected because the person who preferred a different type of area would be unlikely to keep camping on the National Forests. Some of the newcomers were exploring and testing, and would not be back (although about three-fourths of the first-timers thought they would return).

In future research, the attitudes of these "dropouts," and perhaps all first-year visitors, could well be separated. Any specific type of recreational area will be rejected or disliked by some people because areas and people both vary. It seems misleading to give equal weight to evaluations by people who are seeking a different type of area or experience. By analogy, a Chinese restaurant would do well to ignore the opinion about the food expressed by someone who ate there by mistake while seeking an Italian restaurant.

Visitors were also asked if there was anything about the location that they especially liked or disliked. Campers liked the lack of crowding and "just everything" (table 3). The only fairly common dislikes were related to the type, condition, or absence of facilities. Picnickers were even more satisfied than campers, 78 percent voicing no dislikes.

Variation in camper experience showed little relation to likes or dislikes. Tent campers and trailer campers had similar likes, but trailer campers had somewhat more dislikes — 42 percent compared with 31 percent for tent campers. Trailer campers complained more about facilities, and 5 of 176 trailer groups complained about *lack* of crowding, compared with only 1 of 302 tent groups.

Table 3. — *Campers' likes and dislikes*
(In percent)[1]

Answer to question: Is there anything about this place you particularly like or dislike?	Likes	Dislikes
Nothing	1	66
Everything	30	*
Scenery	12	*
Lack of crowding	33	1
Beach	9	2
Fishing	4	4
Facilities	7	17
Cleanliness or dirtiness[2]	4	4
Remote, hard to reach	1	--
Crowded	--	6

[1] Percents total more than 100 because some people gave more than one answer. N(weighted) = 593.

[2] Likes apply to cleanliness, dislikes apply to dirtiness.

* = Less than 0.5 percent.

Resources

The amount of recreational use was not closely associated with resource quality as measured in the NFRS inventory, but was associated with visitors' ratings; thus it is is apparent that NFRS resource ratings and visitors' ratings differ. Some of the differences are substantial (tables 4-8). Two-thirds of

Table 4. — *Visitors' rating of beach quality compared with NFRS rating*

NFRS rating[1]	Locations[2]	Visitors' rating of resource[3] Higher	Same	Lower
	Number	Percent	Percent	Percent
Outstanding	0	--	--	--
Good	4	38	48	15
Fair	6	81	18	1
Unsatisfactory	11	99	1	--
All ratings	21	68	26	6

[1] N(weighted) = 232.

[2] Number of locations at which interviews were obtained (tables 4-8).

[3] Percentages are based on group responses, corrected for length of stay bias, and only for visitors who said they had actually used the resource (tables 4-8).

Table 5. — *Visitors' rating of fishing quality compared with NFRS rating*

NFRS rating1/	Locations	Visitors' rating of resource		
		Higher	Same	Lower
	Number	Percent	Percent	Percent
Outstanding	21	--	16	84
Good	17	11	35	54
Fair	7	39	38	23
Unsatisfactory	0	--	--	--
All ratings	45	7	26	67

1/ N(weighted) = 355.

Table 6. — *Visitors' rating of boating water quality compared with NFRS rating*

NFRS rating1/	Locations	Visitors' rating of resource		
		Higher	Same	Lower
	Number	Percent	Percent	Percent
Outstanding	4	--	24	76
Good	16	29	54	17
Fair	12	84	1	15
Unsatisfactory	0	--	--	--
All ratings	32	28	48	24

1/ N(weighted) = 141.

Table 7. — *Visitors' rating of canoeing water quality compared with NFRS rating*

NFRS rating1/	Locations	Visitors' rating of resource		
		Higher	Same	Lower
	Number	Percent	Percent	Percent
Outstanding	2	--	72	27
Good	9	32	24	44
Fair	0	--	--	--
Unsatisfactory	0	--	--	--
All ratings	11	15	50	35

1/ N(weighted) = 42.

Table 8. — *Visitors' rating of site quality compared with NFRS rating*

NFRS rating1/	Locations	Visitors' rating of resource		
		Higher	Same	Lower
	Number	Percent	Percent	Percent
Outstanding	0	--	--	--
Good	39	64	33	3
Fair	9	92	8	0
Unsatisfactory	0	--	--	--
All ratings	48	67	31	2

1/ N(weighted) = 833.

the visitors gave beaches (table 4) and site quality (table 8) a higher rating than did NFRS, and two-thirds gave fishing (table 5) a lower rating.

If the places considered better by the planner are also thought to be better by the potential user, it matters little what adjective is chosen, at least if the planner is only trying to compare locations with regard to one resource. He still could make misleading evaluations of resource combinations, however. A more serious problem is presented by the visitors' reversal of NFRS ratings. This is the case with boating water ratings (table 6). Seventy-six percent of the visitors gave a lower rating to boating sites rated as outstanding by NFRS, and 84 percent gave a higher rating to sites rated only fair by NFRS.

To get a further view of the relative importance of the different resource elements, the visitors' overall site ratings were correlated with their ratings of a number of resource elements (table 9). The overall rating question was presented as referring to "the whole area all together — the surroundings, fishing water, water for boating, and so on — except the facilities." The strongest association was with the visitors' ratings of the general surroundings, or essentially "scenery." Fishing was also important. Beach quality was positively associated with overall quality, but weakly. Boating quality — as judged by visitors who said they had boated — had a slight negative association with overall site quality, for which there is no apparent explanation.

Table 9. — *Correlations of visitors' average site resource rating*[1] *with their average overall, composite site rating*

Type of resource	Number of locations	Correlation coefficient
Beach (places with beaches)	21	0.26
Fishing (all developed sites)	30	* .51
Boating (all developed sites)	30	-.24
Canoeing (all streams)	45	* .29
General site environment ("scenery") (all developed sites)	30	* .60

1/ Based only upon responses of visitors who used the resource element.

* = Significantly different from zero at 0.05 level.

Facilities and Layout

At least three-fourths of the campers were satisfied with every type of facility checked, and tent sites, tables, and roads scored over 90 percent approval. (Picnickers were even more satisfied with facilities.) Toilets, boat launching areas, signs, and fireplaces drew the most complaints from campers.

Table 10. — *Campers' use of facilities and reasons for nonuse*
(In percent)

Type of facility[1]	Use and evaluation of facility				
	Used	Not used		Not available	
		No interest	Unaccept.	Desired	Not desired
Water supply	97	3	--	--	--
Toilets	96	4	--	--	--
Tent or trailer site	92	6	*	2	--
Fireplace	71	20	*	7	2
Firewood	90	8	--	2	*
Table	85	8	--	7	*
Boat launching area[2]	27	28	*	14	32
Signs and information	99	1	--	*	*
Rentals (boats, etc.)	3	9	--	22	66
Campground roads	99	1	--	--	--
Hiking trails	40	47	--	9	4

[1] N(weighted) = 594.

[2] Data on boat launching areas are reported only for lakes and large, boat-navigable rivers. For lakes and large rivers, N(weighted) = 468.

* = Less than 0.5 percent.

Hardly any campers said that they had not used some facility because it was unsatisfactory (table 10). However, substantial numbers of campers failed to use some facility because of their lack of interest in it. About half of the campers were not interested in available hiking trails, but about two-thirds of the people at campgrounds without trails said they would like them. Except for boat ramps and rentals, missing facilities were generally desired.

More experienced campers expressed somewhat more satisfaction with toilets, fireplaces, firewood, and signs and information than those with fewer years of camping.

In general, visitors were also highly satisfied with the number of individual family units, their spacing, screening vegetation, and amount of use. Over two-thirds of the camping groups said they liked the size of the campground they were using (table 11). Thirty percent said the campground was too small, and only 2 percent said they felt it was too large. The people who felt more sites were needed may have been reacting more to difficulties in finding a spot than to the small campground environment itself. Tent campers seemed to be the type most in favor of small campgrounds. The variation in attitude among campers with different amounts of experience was small.

It appears that the campers who preferred large campgrounds usually wound up in the large campgrounds, and vice-versa. The larger the campground, the more campers who wanted it larger still (table 11). The very small campgrounds (3 to 6 units) had the most satisfied customers, and the customers with the least enthusiasm for expansion. The campers in small campgrounds (7 to 11 units) were the only ones who expressed some feeling that the campgrounds were already too big.

Spacing between campground units, which averaged about 100 feet, received 90 percent approval. Only 6 percent of the groups said the spacing was too wide, while 4 percent said it was too close. Again, tent campers (the largest group) preferred more privacy than trailer campers. One trailer camper out of seven felt too much room was left between camping spots.

Over 90 percent of the campers approved of the screening vegetation between units. Only 1 percent thought the vegetation was too dense, but 7 percent felt there was too little. Again, the trailer campers seemed to want less privacy. Attitude toward screening vegetation was not related to camping experience. Screening seems more important than spacing —

Table 11. — *Campers' opinions of the number of units in the campground, by shelter type and campground size*

Camping group characteristic	N(weighted)	Opinion of number of units		
		Too few	About right	Too many
		Percent	Percent	Percent
Shelter type:				
Tent	323	27	69	4
House trailer	185	34	64	1
Tent trailer	54	20	80	--
Pickup camper	7	46	54	--
Station wagon	7	54	46	--
Other	17	38	62	--
Campground size:				
Very small (8) (3-6 units)	97	22	76	2
Small (8) (7-11 units)	195	25	70	5
Medium (6) (16-22 units)	286	35	64	1
All camping groups	594	30	68	2

Table 12.—*Campers' opinions on number of units, spacing, and number of other campers by forest*
(In percent)

Forest	N(weighted)	NUMBER OF UNITS			
		Too few	About right	Too many	No opinion
Huron	257	35	65	1	--
Manistee	337	26	71	3	--
Forest	N(weighted)	SPACING BETWEEN UNITS			
		Too close	About right	Too far	No opinion
Huron	257	2	89	8	1
Manistee	337	6	91	3	--
Forest	N(weighted)	NUMBER OF OTHER CAMPERS			
		Too many	About right	O.K. with more	No opinion
Huron	250	3	73	24	--
Manistee	334	12	75	13	--

about twice as many campers wanted more screening as wanted more distance between units. The obvious physical relation between distance and screening effect may not have been recognized clearly by the visitors. Actually, as spacing gets tighter, screening probably drops at an increasing rate as trampling from one unit overlaps that from the next.

If the large number of experienced campers who were new to National Forest camping were "graduating" from State Parks, it does not appear that they were applying State Park standards to the National Forest campgrounds. Most Michigan State Park campgrounds are much larger, have more closely spaced units, and have less screening. But the campers who were new to the National Forests tended to favor small campgrounds with widely spaced units more than the oldtimers. The future effect of increasing camping experience is difficult to predict from a single study. The newcomers may shift toward the views of more experienced people as they grow older, or their views could represent a shift in taste. Future research focused on attitudes will have to cope with this problem, and repeated surveys over time seem necessary.

The Manistee campers were less inclined to favor campground enlargement and clearly preferred wide spacing of units (table 12). This may be related to the big-city origin of many of the Huron campers, which draws more from the Detroit metropolitan area. If differences like this exist between two such similar Forests, it emphasizes again the need for caution in applying study results from one area directly to another.

Recreational Use

There was no significant amount of complaining about too many people on beaches, in boats, canoes, or on fishing streams. In fact, a majority said more canoeing would have been acceptable to them.

There was some negative reaction to the number of campers (last section of table 12), especially on the Manistee, where 12 percent of the groups said the campground was too full. Part of this was because the Manistee had several campgrounds that were more fully occupied than did the Huron (figs. 2 and 3). However, Manistee campers objected to crowding of *every* type much more than Huron campers, which is consistent with the difference between the visitors' to the two Forests in terms of attitudes on campground size and spacing.

Water skiing seems to cause friction some places, but on the Huron only 5 percent of visitor groups complained about water skier numbers at campgrounds where it took place and 28 percent said they could tolerate more. The Manistee campers were a bit touchier (9 percent complained), but even here the situation does not seem bad, especially when one recalls the refuge from water skiing available at the campgrounds on small lakes and streams.

Type of shelter and years of experience in National Forest camping were both unrelated to crowding attitudes.

Sources of Information

Campers found out about the campground they were using primarily by talking with friends and acquaintances (43 percent). The same result has appeared in many other outdoor recreation studies. The free Forest Service map-brochure (4 percent), stories by outdoor writers in newspapers or magazines (1 percent), and tourist information booths (1 percent) were no match for "a guy at the plant," "my neighbor," and so on. However, some of the press releases and maps may still be crucial as the *original* source of new information that is then dispersed through the person-to-person network. The second most common reason, however, was "drove by and dropped in" (19 percent), a sort of random search, and this is probably where most of the new knowledge comes from. Third most important were road maps (10 percent). This stresses the importance of getting all State highway and oil company maps to show public recreation sites.

Management Implications

The most obvious implication of the study is that the recreation resource management on the Huron and Manistee National Forests is doing a good job of satisfying the public. It is hard to imagine any program receiving much more complete approval than the recreational management of these areas. There are opportunities for improvement, of course, and some change and much growth will be necessary, but there seems to be no need for major shifts in design of areas or facilities, or in their operation.

Capacity and use, however, are not well balanced. A national motel chain would be concerned if some of its motels were almost full every night while others had two-thirds of their rooms empty. Too much capital would be tied up in poor producers, and too many potential customers would be turned away from the full motels. An analogous situation exists in the Huron-Manistee campgrounds. It should be added, however, that the variation in campground occupancy for these Forests was less than that reported for any of the other areas studied, perhaps due to the great accessibility and heavy use pressure in Lower Michigan.

The question of desirable campground size is not completely analogous to the motel situation, however, because it must be answered in terms of somewhat different management objectives. The larger campgrounds received less use per unit than small ones. This means that smaller campgrounds produced more recreation per unit, and maybe even more per dollar of input, depending upon the economies of scale of campground construction and operation. (One study of this question in three Colorado National Forests indicated no relationship between campground size and construction costs, and only a slight tendency for operation and maintenance costs per unit to decline as campground size increased (Beardsley 1967).) But, unlike the motel, the objective is not to get maximum use per campground unit. Too much use hastens physical deterioration, reduces freedom of choice by visitors, and also raises costs of maintenance, according to the study by Beardsley (1967). The goal is some optimum level of use, with only moderate variation from campground to campground.

It appears that the occupancy rate may be lowered somewhat by enlarging campgrounds; supply does not necessarily create its own demand in treadmill fashion. However, the problem still is one of assessing a location's attractiveness, and matching the size of the development to it.

Because of variation in peoples' desires and in areas' potentialities, diversity in campground size seems both necessary and desirable. Unless small campgrounds can be shown to be substantially more costly per man-day of use, it would appear to be a mistake to eliminate them. On the other hand, there seems to be a distinct desire for, and acceptance of larger campgrounds by most campers.

Diversity is also supported by the differences in attitudes that showed up between users of the two superficially similar Forests. Areas that look much alike may still attract rather different people. A standard pattern of development does not seem appropriate here, and even less so nationwide.

The visitors indicated no serious problem of overuse or use conflicts. If the capacity of the campgrounds keeps pace so that most campers can find a place, and if diversity of size and setting is maintained within the system, camper satisfaction with the number and types of other users should remain high.

Picnic area locations should apparently be chosen independently of campground locations. The range of possible locations for "successful" picnic areas seems much wider than for campgrounds. A reasonably attractive spot near a highway seems sufficient for most picnic areas.

A better flow of information between land managers and the public about recreational areas and opportunities seems desirable. Many people apparently wound up in the type of place they preferred, but largely without help from official Forest Service maps and brochures. More effort here could pay real dividends — the extensive person-to-person communication network can greatly multiply the transmission of knowledge distributed by the Forest Service. Better knowledge of available alternatives could produce more uniform and efficient use, and increase public satisfaction. Information helpful in choosing a campground might include number of family units, kind of fish, size of lake, presence or absence of water skiing, navigability of streams, miles of hiking trails, and even type of water supply and toilets (some of this information is already provided). This information would require more frequent revising of maps, but it could be worth it. It would be desirable to have Forest Service recreation opportunities fully and accurately reported on highway maps; these maps were used by far more Huron-Manistee campers than were the Forest Service maps. The large number of campers who found campgrounds by just driving and looking emphasizes the importance of adequate signing.

Finally, the study results show that the expense and effort of detailed resource quality measurement seem a doubtful investment at this time. This is especially true of summary quality ratings. We do not yet know enough to measure recreation resource quality in terms meaningful to use potential. The resource data that best accounted for recreational use were usually straightforward physical resource measurements, such as yards of beach, rather than quasi-objective quality ratings. Directly measured, raw data have several advantages: First, there is little doubt about comparability. Lake size, shoreline material, slope, and tree species, for example, are fairly objective measures. Second, direct, physical data are flexible and adaptable; as knowledge about the significance of various resource elements becomes available, the data can be interpreted or scaled. This is also true for some future type of recreation as unforeseen now as water skiing was 30 years ago or as snowmobiling was 15 years ago. This adaptability is maximized by keeping data in the original units — for example, "a 114-acre lake," not "a lake between 100 and 250 acres." Such data lend themselves to later classification or combination without remeasurement. Much of this information can be recorded best on maps or map overlays, which can indicate relative location; for example, maps can show whether a sand beach is in front of a stand of big pine or across the lake in a way that even the most complex tables never can.

Future Research

Further study of the relation of recreational use to recreation area characteristics appears worthwhile. Even a modest improvement in evaluating the drawing power of recreation sites nationwide could increase efficiency enough to save millions of dollars each year for the National Forests alone. For many reasons, campground use should have top priority for future study: it is the major use at developed sites, the major investment (at least for the Forest Service), and a fixed investment.

An unanswered question relating to use measurement is the stability of use patterns from year to year. Total use may fluctuate in response to weather, but I would expect that the relative use distribution would remain about the same. (Of course, gradual changes are to be expected as roads change and new areas are built.) Past experience suggests stability, and data for Superior National Forest campgrounds for 1961 and 1967 show similar rankings in use.

Another use question, not included in this study, concerns visitor distribution *within* campgrounds. This has been studied somewhat[8] (Love 1964), and work is now in progress at the North Central Forest Experiment Station. The pulling power of the campground site as a whole and the attractiveness of units within the campground are obviously interrelated.

Campground use should be measured separately for each type of shelter, such as tent, trailer, tent trailer, motor home, and pickup camper. Some of these types of campers probably differ significantly in their evaluations of locations. For example, trailer campers stayed much closer to paved roads than other campers in the Colorado study (Beardsley 1967). It might be possible to predict each type of use better than the total use. This could also be useful in campground design because there is some variation in requirements for the various types of campers.

The most obvious location factor that appears important and was inadequately measured in this study is scenic attractiveness. Subjective expert ratings, perhaps by a panel of landscape architects, more objective measurement of elements thought to be scenically important, and interviews in depth to probe reactions to scenery should be tried and compared.

It is likely that important location factors have been overlooked. A thorough study of the location choice process is needed. We know much of the information for such choices comes from friends. What qualities do these friends notice and report? How accurate is their information? What do they distort, and how? How conscious is the location

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[8] *LaPage, Wilbur F. A study of campsite selection among visitors to a small, Forest Service campground. (Unpublished report on file at Northeast. Forest Exp. Sta., Upper Darby, Pa.)*

choice? How often are places looked over and passed up? Why? How aware are people of alternatives? How are distance and travel viewed — as a cost, or as part of the fun? Study of the choice process might uncover new variables, redefine existing ones, or confirm interpretations based on relating use patterns to location characteristics.

An analytical approach, such as discriminant analysis, which could simply classify possible development sites as "below average," "average," or "above average" might be preferable to numerical use predictions. This could conform better to the realities of the complex, multifactor relationships, and avoid an impression of more precision than is warranted. Such a general classification would still be a substantial improvement in present predicting abilities and could be very helpful in resource planning.

Several key attitudes require more analysis. A recreation program that ignores quality is certain to be a failure, and efforts to better measure quality should have top priority. This will require imaginative research design, drawing upon psychology and sociology. Again, even a small improvement in knowledge could pay handsome returns in increased recreational output for the American public.

Another important attitude that still is not well understood concerns campground size. The main problem is a confusion of utilitarian and esthetic viewpoints. A camper wants to find a spot to camp when he pulls in, and he wants there to be enough units to provide one for him (and probably some to spare). He may also have some ideal size range in mind, assuming getting a space would not be a problem. But he may answer overly simplified questions about campground size from either viewpoint. The average size of public campgrounds is growing rapidly, but with little knowledge of what this implies in terms of use, quality, or economic efficiency.

Finally, recreation research needs to do much more time-series or trend analysis.[9] How are participation and attitudes changing over time, for individuals and for the public generally?

These questions must be answered if recreation resource management is ever going to aspire to do more than struggle to catch up.

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[9] *The major effort of this sort has been by Wilbur LaPage of the USDA Forest Service, Northeast. Forest Exp. Sta.*

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SD.A. Forest Service
search Paper NC-45

System Identification Principles in Studies of Forest Dynamics

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CONTENTS

SYSTEM IDENTIFICATION PRINCIPLES IN STUDIES OF FOREST DYNAMICS

ROLFE A. LEARY

This paper is directed at the following problem: Given observational data on a dynamic system, what are the equations governing its behavior? This is known as a system identification or inverse problem, and in its broadest sense involves both unknown algebraic form and numerical constants in the governing equations. Clearly, knowledge of the exact, or even approximate, equations governing a system makes effective control a realistic possibility. The ability to control a dynamic forest system is of concern in the many instances where man has developed preferences for certain system states and is prepared to take action to ensure their existence.

Methods of dealing with both types of unknowns are discussed. The first part concerns concepts that are helpful in rationalizing the algebraic form of the governing equation right-hand sides. The second part deals with parameter estimation. The only type of governing equations considered are linear and nonlinear first-order ordinary differential equations.

The methods of regression analysis, so widely used in modeling forest systems, are not used in this study. In a traditional regression approach one is attempting to solve two problems simultaneously. First is that of modeling the causal mechanisms of observed system behavior. Second is the problem of making inferences about the population from which the observed systems were selected. Regression methods are particularly inadequate for the first problem. Thus, the above problems are treated separately, attention being given to the first.

The approach used here is to transform the problem from one of having several observations through time on a forest system in some inconvenient tabulated form to one of having a single point located in state space corresponding to the system state at each observation. This is essentially a problem of trajectory identification. It can, in many cases, be solved by considering it as a multipoint boundary value problem of differential equations and applying the computational procedure called quasilinearization (Bellman and Kalaba 1965, Lee 1968). The approach used here in state space is entirely analogous to that used in physical space for such problems as orbit determination (Bellman 1962).

It is assumed that the reader is interested in developing mathematical models of forest-system dynamics. The level of mathematics is variable. As a rule, the lowest level of mathematics that conveys correct meaning is used. Most of the concepts and techniques discussed are not new; they have been gleaned from numerical analysis, numerical methods, differential equations, mathematical analysis, control theory, and general systems theory. In anticipation that some readers may not be familiar with these subjects and how they relate to the problem of modeling dynamic forest systems, each topic is discussed verbally and related to example models. No attempt is made at an exhaustive treatment. Instead, the emphasis is on synthesizing the concepts into a comprehensive, workable scheme.

In the following discussion the assumed goal is to develop governing equations for the standing crop in northern hardwood forests. Plot data from the Argonne Experimental Forest in northeastern Wisconsin are used.

GENERAL CONCEPT OF A SYSTEM

The past decade or two has witnessed the evolution of a holistic or systems approach to the study of complex entities. During this time the forester's ability to talk and write about systems has in some cases outdistanced his ability to quantify or model them. The concepts and techniques that follow may help to fill the gap in modeling dynamic forest systems.

The success of the approach hinges on successfully combining the following: (1) the researcher's knowledge of the biological processes involved in the system he is studying, (2) his ability to quantify these processes, and (3) the ability of the digital computer to do the calculations necessary for successful solution of the inverse problem.

Initially, a proper definition of the entities involved is required. The following definition suits the purposes well (Hall and Fagen 1956). "A system is a set of objects together with relationships between the objects and the attributes." Objects are the parts or components of a system and attributes are properties of objects.

This study is concerned with modeling the primary production of forest systems. Although only the vegetative portion of the ecosystem is modeled, the theories and methods discussed are applicable to ecosystem production in general. Due to the limited historical data available, the examples given deal only with the component capable of producing merchantable timber.

Thus, what is being modeled is a subsystem of the total vegetation system. Where does one draw the line or boundary for his system? It is helpful to place system boundaries at natural breaks in the hierarchy of systems. Thus, the system may be restricted to all vegetation of species capable of producing merchantable timber in the northern hardwood forest. But, this would be an unnatural boundary. Hence, our system includes all woody and herbaceous vegetation in the northern hardwood type even though our examples, for reasons stated previously, consider only timber species.

CHARACTERIZING SYSTEMS

A meaningful discussion of alternative methods of characterizing systems presupposes a well-defined end objective. Usually the interest is in gaining insight into the system's inner workings and developing a predictive capability, via a mathematical model. A logical beginning would be to characterize a system by its components. In the examples from later sections, where the vegetative portion of the ecosystem is treated as a system, components may be defined on the basis of taxonomic criteria, physical size or function (in the distribution and accumulation of energy), or by some combination. For example, an individual plant, species, diameter or height class, or species x diameter class may be considered as a system component. Clearly, the number of possible ways of defining system components is enormous. For the purposes of this study, where our interest is in system productivity, a combination of physical size and function was deemed most desirable.

Once the components for the system have been defined, a question arises as to what attribute should be used to characterize each component. A most elemental approach would be to use presence or absence of qualifying individuals as the component attribute. A next logical refinement would be the frequency of occurrence of qualifying individuals. For ecological succession studies this level of refinement might be suitable. In productivity studies, it is still too elemental for reasons that will be evident in the section on dynamic models. Having gone from the binary (1-present: 0-absent) level to the discrete (positive integers), any further refinement necessitates passing to the continuous. This method of describing system components is common; basal area of qualifying individuals is an often-used attribute. In later examples, sum of diameters of trees in each component is used.

One could, of course, go further and compute volume from diameter (d.b.h.) and height and use this as the attribute. The advisability of the last step is questionable, however. As a general rule, component attributes should be the most elemental measures of individuals in the component consistent with the type of model used.

To summarize briefly, then, a system is comprised of objects or components and each component may be characterized by various attributes.

At this point it is convenient to use a slightly different approach, and consider the system components as coordinate axes of an abstract space — called state space. The component attributes specify the system location with respect to each coordinate axis. All system components plus their attributes specify the system location in state space, and are called state variables.

In later examples, species group (based on shade tolerance) and size (diameter class) were used as the basis for delineating the system components. The size attribute selected was sum of diameters. Thus, sums of tree diameters at breast height in each species group x diameter class are the state variables. Of course, the units in which each is expressed need not be the same. That is, the first m state variables may represent standing crop of m browse species and be expressed in pounds per acre, the $m + 1 \ldots n$ variables may represent standing crop in timber species and be expressed in inches of diameter for all qualifying individuals. The variety of expressions are, obviously, limited only by the user's needs. It is not at all difficult to conceive of system components numbering into the hundreds. Theoretically, the more state variables employed, the more refined the characterization of the system will be. However, in practical applications, one must strike a compromise between increasing realism and decreasing tractability as the number of variables increases.

The state variable approach to system characterization is well established in engineering and control theory (De Russo *et al.* 1965). There is much to be gained in terms of conciseness of expression and ease of mathematical analysis of forest systems by its adoption in our work. The combination of a state variable formulation of forest systems with the versatility of systems of nonlinear differential equations makes a powerful predictive scheme. The remainder of this paper, then, uses a state variable approach to forest system modeling.

DYNAMIC SYSTEMS

An underlying objective of this paper is to develop a capability of predicting system development through time; hence, the concern is with dynamic systems. But first it is instructive to consider a static system.

Take for example a system with state variables $Y1$ and $Y2$. The system state is given, then, by the ordered pair $(Y1, Y2)$. The only type of systems of interest to us are interactive ones, that is, where the variable $Y1$ affects and is affected by $Y2$. Thus, a static interactive linear model is given by the system of simultaneous equations

$$\begin{aligned} a_{11}Y1 + a_{12}Y2 &= b_1 \\ a_{21}Y1 + a_{22}Y2 &= b_2. \end{aligned} \qquad (1)$$

The system state is given by the values of $Y1$ and $Y2$ that satisfy these equalities.

The primary concern here is the differential equation formulation of system development through time, and the discussion that follows deals with it exclusively. As a means of introducing this formulation, consider a system characterized by a single state variable; i.e., the scalar case. The equation that specifies the instantaneous rate of change, with respect to time, in system state takes the form

$$dY/dt = f(Y,t) \tag{2}$$

where f is a suitably chosen function,
Y is the dependent variable, and
t is time, the independent variable.

The actual state, as opposed to rate of change of state, is obtained by solving equation (2) for Y.

What are the possible forms the function f may take? In general, they may be as follows:

(a) $$dY/dt = 0. \tag{3}$$

This seemingly uninteresting form, in the context of a system of equations, is very useful, and is used extensively in the later sections on parameter estimation.

(b) $$dY/dt = f(t). \tag{4}$$

This form, which states that rate of change in Y is dependent only on time, is used extensively in engineering and other physical sciences, but in the context of open systems of living organisms it is of limited usefulness.

(c) $$dY/dt = g(Y). \tag{5}$$

This equation states that the rate of change of Y is a function solely of the amount present of Y, and is the form of most biological growth functions.

(d) $$dY/dt = h(Y,t). \tag{6}$$

In the case of equation (6), rate of change in Y is related to both amount present of Y and time. No further reference is made to this form.

In this study functions f and g are points of beginning and are not, in the case of f, obtained by differentiating some other function F with respect to time. This approach allows use of differential equations of the form (4) and (5) regardless of whether their solutions may be expressed in closed form. The approach is one of selecting a function of the form f or g that has the desired qualitative properties (see section on "Some Qualitative Properties of System Equations"), and describing the system on that basis, without regard to the closed form expression for F or G if such form does, in fact, exist. G is understood to be the closed form solution of equation (5).

Some most used forms of g and the names commonly associated with them are:

$g = a\ Y^2 + b\ Y$ (7) logistics, autocatalytic

$g = a\ Y\ \ln(\frac{A}{Y})$ (8) Gompertz

$g = a\ Y^c + bY$ (9) von Bertalanffy, Chapman-Richards.

Equation (9) is a special case of what is called the Bernoulli equation; i.e., a nonlinear equation of the form

$$dY/dt + Q(t)\ Y^n = R(t)\ Y. \tag{10}$$

The logistics equation has Q(t) = –a, R(t) = b, and n = 2.

The Bernoulli equation is one of the few nonlinear equations that may be solved in closed form. This is accomplished for the von Bertalanffy function by first substituting

$$Z = Y^{1-c}$$

in (9), which gives

$$\begin{aligned} dZ/dt &= (1-c)\ Y^{-c}\ (a\ Y^c - b\ Y) \\ &= (1-c)\ (a - b\ Y^{1-c}) \\ &= (1-c)\ (a - b\ Z). \end{aligned} \tag{11}$$

Equation (11) is linear in the variable Z and may easily be solved (see page 14). Once the solution in Z is obtained, it is expressed in terms of Y; i.e.,

$$Y = Z^{1/(1-c)}.$$

To this point, the discussion has been of a system described by a single equation. This approach is, of course, not new. The following observations and contrasts of examples from the literature may be made. In many cases the investigators dealt exclusively with the function G — i.e., the solution of the differential equation — and only incidentally mention that it was the solution of a particular differential equation. Naturally, this meant they were dealing only with equations having solutions expressible in closed form. In a more recent approach[1] the differential equation was fit to observational data, and the solution was obtained analytically.

Let us consider the case where Y is a vector (or an ordered n-tuple of numbers). Only through this treatment can the full benefit of the state variable approach be reaped.

Our model may have the following form:

$$\begin{aligned} dY1/dt &= f_1(Y1,Y2,Y3, \ldots ,Yn,t,a,b,c, \ldots.) \\ dY2/dt &= f_2(Y1,Y2,Y3, \ldots ,Yn,t,a,b,c, \ldots.) \\ &\vdots \\ dYn/dt &= f_n(Y1,Y2,Y3, \ldots ,Yn,t,a,b,c, \ldots.) \end{aligned} \tag{12}$$

where Yi, i = 1,2, ... ,n are the dependent variables,
t is time, and
a,b,c, ... are parameters of state such as soil and climate (Lotka 1928).

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[1] *Moser, J. W., Jr. Growth and yield models for uneven-aged forest stands. (Unpublished Ph.D. thesis on file at Purdue Univ., Lafayette.)*

Because *a,b,c* . . . are usually time invariant, they may be dropped, leaving

$$dYi/dt = f_i(Y1,Y2,Y3, \ldots ,Yn,t), \; i = 1,2, \ldots ,n. \qquad (13)$$

If equation (13) is modified further by eliminating *t* from the right-hand side,

$$dYi/dt = f_i(Y1,Y2,Y3, \ldots ,Yn), \; i = 1,2, \ldots ,n. \qquad (14)$$

This is called an autonomous system of equations. All the examples that follow are in terms of autonomous systems of equations.

Equation (14) states that the rate of change of state variable $Y1$ may be a function of the amount present — in our case standing crop — in $Y1,Y2,Y3, \ldots, Yn$, and likewise for the other state variables $Y2,Y3,Y4, \ldots Yn$. It follows from this formulation that system development, as reflected by changing state variables, is influenced by system position in state space. In the context of the later examples, a model such as (14) indicates the diameter growth of individuals in a specific diameter x species group class may be influenced by the amount present in each (or none) of the other classes, depending upon the form of the f_i, $i = 1,2,3, \ldots n$.

Returning to the question concerning the origin of the functions f_i, the system of equations (14) is a point of beginning. This being the case, it is clear that the researcher must rationalize the form of the right-hand sides (r.h.s.) of (14) on the basis of his knowledge of the processes involved. Two general principles in this regard are: (1) the r.h.s. of (14) should possess qualitative properties, individually and as a set, that do not violate biological principles, and (2) the r.h.s. of (14) should reflect the causal pathways between interacting parts of the forest system.

The process of rationalizing the form of the r.h.s. of (14) may conveniently be broken down into two phases: (1) specifying the state variables that should be included in the r.h.s. of each equation in (14), and (2) specifying the algebraic form of the relationships between the state variables selected in phase 1.

An example of phase 1 from Darlington Woods in Indiana follows:[2] The problem is rationalizing the r.h.s. of an interactive dynamic model that has timber size classes as system components.

The biological basis for the r.h.s. is that of competition for the ecosystem resources of space and light (solar radiation). It is based on the following two premises: (1) the amount of light (relative to full sunlight) received by a forest system (stand) component affects its growth and development, and (2) the amount of solar radiation received by a forest stand component is primarily a function of its position in the stand.

If a stand is uneven-aged with resultant diversity in tree size, the trees in the lower strata receive radiation of a different intensity and quality than trees in the overstory. If a stand is even-aged, a larger percentage of the trees receive nearly the same intensity and quality radiation because they tend to be more nearly the same height.

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[2] *Leary, R. A. A multidimensional model of even-aged forest growth. (Unpublished Ph.D. thesis on file at Purdue Univ., Lafayette.)*

It is constructive to consider this problem in the manner of Hutchinson (1957) in terms of the relationships between ecological niches of potential competitors. An ecological niche is here defined as all of the ecosystem resource levels that a species is currently capable of using. Hutchinson (1957) distinguishes two types of niches. The fundamental niche of a particular forest component is all of the ecosystem resource levels that a component is capable of using at the present time in the absence of competition. The fundamental niche of a particular forest component may thus be shown as a region or set of values in space, the co-ordinate axes of which correspond to the various resources. If only two ecosystem resources are considered, the fundamental niche may be shown as a region in the x - y plane.

The concept of a realized niche arises when competition for ecosystem resources exists. The realized niche of a competitor may be defined as the set of ecosystem resource levels it is capable of using and has available for use free from competition. It may also be defined as the fundamental niche minus the intersection subset.

If N_1 is the fundamental niche of forest component $Y1$, and N_2 is the fundamental niche of forest component $Y2$, the realized niche of component $Y1$ is

$N_1 - \{N_1 \cap N_2\} = RN_1$. ($\{N_1 \cap N_2\}$ is the set of all ecosystem resource levels (a,b) that are simultaneously in both sets N_1 and N_2.)

Likewise, the realized niche of component $Y2$ is

$$N_2 - \{N_1 \cap N_2\} = RN_2.$$

Of special interest is $\{N_1 \cap N_2\} = N_2$. In such cases, $RN_2 = \square$, where $\square$ is the null (empty) set. A similar condition could, of course, exist for N_1; i.e., $RN_1 = \square$.

The two ecosystem resources that were used as a basis for the model may be shown graphically as a region in two-dimensional space (Miller 1967) (fig. 1). Let us assume the diagonally hatched area represents the set of light and space resource levels that stand component $Y1$ is presently capable of using in the absence of competition. This area therefore represents the fundamental niche of $Y1$.

If the horizontally hatched area is the set of space and light resource levels that stand component $Y2$ is capable of using in the absence of competition, it is the fundamental niche of component $Y2$.

It is clear from figure 1 that the fundamental niche of the lower stratum component ($Y2$) is a proper subset of the fundamental niche of the larger component ($Y1$). Miller (1967) describes two possible outcomes of competition between components $Y1$ and $Y2$ where $N_2 \subset N_1$. The first is where competition proceeds in favor of $Y1$ and given adequate time, only $Y1$ survives. The second is where $Y2$ survives in all regions of the x - y plane corresponding to $N_1 \subset N_2$, and both components survive.

The hypothesis underlying the proposed model is that the growth and development of a stand component is influenced by the amount of the contained component present, and the amount of the containing component(s) present in the stand.

Thus, for stands comprised of n components, if Y_i indicates the amount of the i^{th} stand component and if N_i indicates the fundamental niche of the i^{th} component, and if

$N_1 \subset N_2$ and $N_1 \subset N_3$ and $N_1 \subset N_4$ and ... $N_1 \subset N_n$, then
$dY1/dt = f_1(Y1,Y2,Y3, \ldots ,Yn)$.

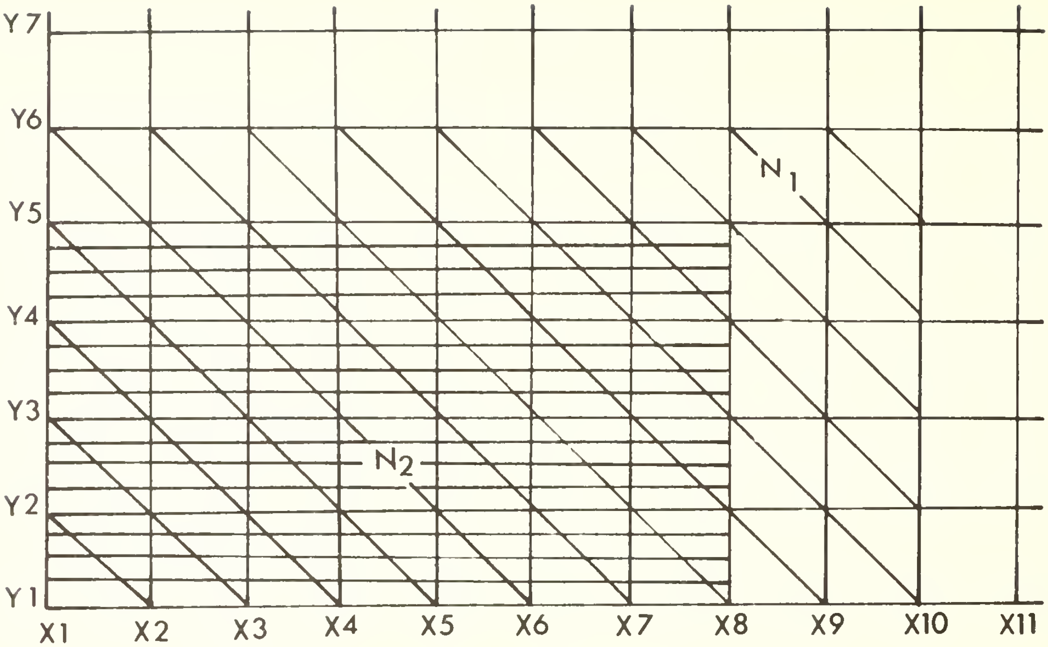


Figure 1. — Fundamental niches N_1 and N_2 of stand components $Y1$ and $Y2$ relative to ecosystem resources of space (Y) and light (X).

Likewise, if

$$N_2 \subset N_3 \text{ and } N_2 \subset N_4 \text{ and } N_2 \subset N_5 \text{ and } \ldots N_2 \subset N_n \text{, then}$$
$$dY2/dt = f_2(Y2,Y3, \ldots ,Yn).$$

If we continue in this manner and consider a model for a pure stand divided into five height classes (components),

$$\begin{aligned} dY1/dt &= f_1(Y1,Y2,Y3,Y4,Y5) \\ dY2/dt &= f_2(\quad Y2,Y3,Y4,Y5) \\ dY3/dt &= f_3(\qquad Y3,Y4,Y5) \\ dY4/dt &= f_4(\qquad\quad Y4,Y5) \\ dY5/dt &= f_5(\qquad\qquad Y5) \end{aligned} \tag{15}$$

where Yi is the standing crop in the i^{th} height class,
$Y1$ is the shortest component, and
t is time, in years.

Implicit in the above argument is that the understory does not affect overstory development. This is, of course, not universally true.

As an example of phase 2 applied to a reduced system, consider a simple linear additive relationship between components, such as

$$\begin{aligned} dY1/dt &= a_{11}Y1 + a_{12}Y2 + a_{13}Y3 \\ dY2/dt &= \qquad\qquad a_{22}Y2 + a_{23}Y3 \\ dY3/dt &= \qquad\qquad\qquad\qquad a_{33}Y3 \end{aligned} \tag{16}$$

or nonlinear multiplicative relationships such as

$$\begin{aligned} dY1/dt &= (a_{11}Y1^{a_{12}} + a_{13}Y1)\, e^{a_{14}Y2 + a_{15}Y3} \\ dY2/dt &= (a_{21}Y2^{a_{22}} + a_{23}Y2)\, e^{a_{24}Y3} \\ dY3/dt &= (a_{31}Y3^{a_{32}} + a_{33}Y3). \end{aligned} \qquad (17)$$

One must often compromise between a desire for the realism that would favor equation (17), and the often limited availability of historical data that would favor equation (16). Involved is the question concerning the evolution of r.h.s. from first approximations based on limited data to more refined r.h.s. as more information is gathered concerning the processes involved. It has been suggested that there is a need for a dynamical yield (standing crop) function revised periodically to incorporate results from new practices (Moser and Hall 1969). This may involve updating parameters as well as substituting more realistic r.h.s.

SOME QUALITATIVE PROPERTIES OF SYSTEM EQUATIONS

Earlier is was stated that " . . . the researcher must rationalize the form of the r.h.s. . . . on the basis of his knowledge of the processes involved." It is not sufficient to simply hypothesize a governing equation form and then test it with observational data. A valuable and potentially enlightening first step is to examine the behavior of governing equation solutions to eliminate from further consideration all equations that violate irrefutable biological "laws." Because the goal in rationalizing the governing equations is to identify the causal pathways of system component interaction, nonviolation of these biological "laws" is a necessary, but not sufficient, condition for attaining this goal.

It is instructive to ask what, in terms of a system model, is measured. Certainly parameters are not measured directly nor is instantaneous change. Rather, the standing crop (system state) or some attribute thereof is measured. Because the solution may not be expressable in closed form, the following discussion is concerned with inferring qualitative properties of governing equation solutions from the r.h.s. of the governing equation. One-dimensional models are discussed first.

The properties the solution should possess are often apparent. For solutions describing undisturbed standing-crop development, such as standing crop in survivor trees, these properties might be required: (1) nonnegativity for nonnegative values of the independent variable, (2) upper boundedness for nonnegative values of the independent variable, and (3) monotonic nondecreasing for nonnegative values of the independent variable. Clearly, properties (2) and (3) imply an upper asymptote. If property (1) was extended to include all values of the independent variable, then (1), (2), and (3) imply an upper and lower asymptote.

If the discussion is extended to include standing crop development in general, not just survivor standing crop, there is no universal agreement on the suitability of an upper asymptote. MacKinney *et al.* (1937) state, " . . . the yield (standing crop) curve is limited between zero . . . at inception and a finite maximum . . . at that advanced age before the stand commences to break up." Implicit is that the amount of standing crop does not remain at the maximum level indefinitely.

If it does not remain at the maximum (asymptotic) level indefinitely it may decrease monotonically or through oscillations. A decreasing standing crop is not considered in this study.

Let us discuss solutions of one-dimensional models, types given by equations (4) and (5), in terms of properties (1), (2), and (3). As an example of the form in equation (5) consider equation (9), and for form in equation (4) consider a polynomial approximation to a current annual increment curve such as equation (18),

$$dY/dt = at^2 + bt + c \qquad (18)$$
$$a<0,\ b>0,\ c>0.$$

Consider the requirement that the solution be nonnegative. In the case of equation (9) ; i.e.,

$$dY/dt = a\ Y^c + b\ Y$$
$$a<0,\ c>0,\ b>0,$$

it is clear that given a positive initial condition, standing crop cannot be negative. If Y ever becomes zero, it cannot deviate from zero. Equation (18) contains no such restriction, however, and standing crop may be negative.

Consider next the property of upper boundedness. How is this property of the solution inferred from equations (9) and (18)? Clearly, boundedness is a property of the solution of (9) if at any point

$$a\ Y^c + b\ Y = 0 \qquad (19)$$

The upper bound corresponds to the value of Y at which equation (19) is true. In equation (18), standing crop at time t_1 corresponds to the area under the polynomial curve between $t = 0$ and $t = t_1$. Upper boundedness is a property of the solution if $a < 0$. This will generally be the case; hence, equations of the form (18) possess upper boundedness.

In many cases, especially when the state variables represent physical dimensions of surviving trees, the solution should possess a monotonic increasing character, hence, property (3). How is this property reflected in governing equations such as (9) and (18)? Monotonicity is a property of the solution if the r.h.s. of the governing equation is greater than zero for all $t > 0$. In the case of equation (9), it cannot be otherwise. For equation (18), monotonicity is a property of the solution only if $a > 0$. If the coefficients *a, b,* and *c* have been determined from actual data, this will usually not be the case; hence, monotonicity is not a property of governing equations like equation (18).

These properties, nonnegativity, upper boundedness, and positive monotonicity, are but a few of many qualitative properties of solutions that may be inferred by considering the governing equations. Note that the solutions of governing equations (9) and (18) are expressible in closed form, hence their qualitative properties need not have been investigated via the governing equations. These examples were used since it is easy to verify the conclusions.

As an example of a governing equation, without a solution expressible in closed form, consider

$$dY/dt = a\ Y\ e^{bY} \qquad (20)$$
$$a>0,\ b<0.$$

Given a positive initial condition, $Y(0)$, the solution is clearly monotonic increasing. The upper bound occurs at that value of Y for which

$$a Y e^{bY} = 0.$$

There exists no value of Y for which this equality is true, hence there is no upper bound. If $Y(0) = 0$, it cannot deviate from zero, hence $Y(0) \geq 0$ insures that the solution of governing equation (20) is monotonic nondecreasing. Further examination would reveal other properties.

A governing equation solution may be characterized as an element of one or more of the following sets (fig. 2). It is apparent that the solution of equation (9) is an element of the set formed by the intersection of sets 1, 2, and 3 (region A in figure 2). Likewise, the solution of equation (20) is a member of sets 1 and 3 but not 2: i.e., region B.

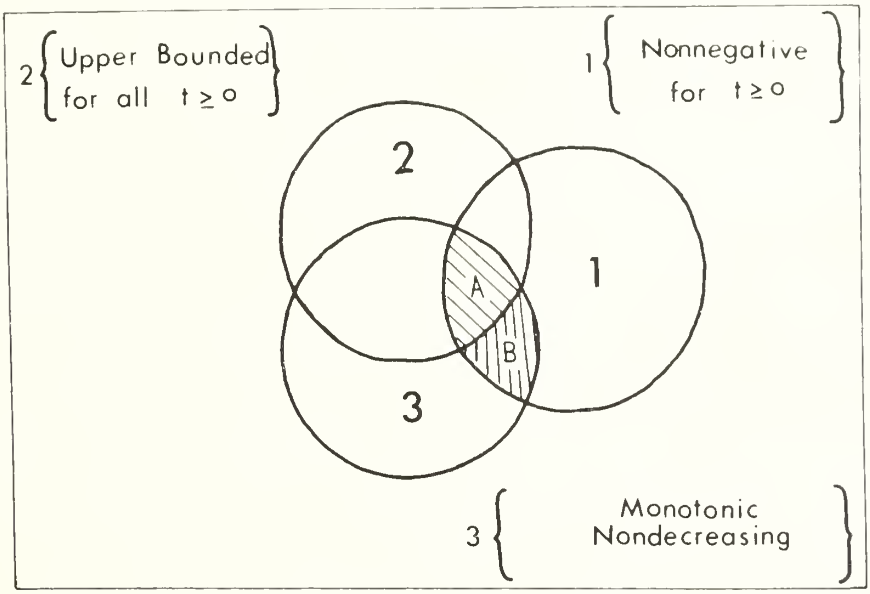


Figure 2. — Relationship of properties 1, 2, and 3 for solution of governing equation.

What conclusions may be drawn from the previous discussion? One is that governing equations, the solutions of which are contained in region A, are suitable only for describing forest system components such as standing crop in survivor trees. They are not, therefore, suitable for describing total standing crop because this may involve an initial increase but subsequent decline (MacKinney *et al.* 1937).

It may be said that figure 2 characterizes a function (solution) space, a space in which the elements (points) are functions (solutions of differential equations). This is in contrast with the more familiar value space where the elements correspond to values of functions. Also, in contrast with the value space, which is finite-dimensional, many function spaces of interest are infinite-dimensional (Rosen 1967). It is obvious there are infinitely many functions in region A that are solutions of the differential equation

$$dY/dt = -aY^{c} + bY. \qquad (21)$$

One need only let the parameters *a, b,* and *c* vary over the positive real numbers.

The quasilinearization method used later allows choosing a starting point in function (solution) space, and constructing a sequence of functions (solutions) that in many cases converges to the function desired.

Consider briefly the qualitative properties of systems of equations. Again the purpose is to examine the r.h.s. of the differential equations and on that basis infer properties of the solution.

Of all the "irrefutable" biological laws, perhaps the one most commonly violated in modeling forest systems is that of nonnegativity. This, of course, need not be the case as shown by the following nonlinear multiplicative system:

$$\begin{aligned} dY1/dt &= (a_{11}Y1^{a_{12}} + a_{13}Y1)\, e^{a_{14}Y2 + a_{15}Y3} \\ dY2/dt &= (a_{21}Y2^{a_{22}} + a_{23}Y2)\, e^{a_{24}Y3} \\ dY3/dt &= (a_{31}Y3^{a_{32}} + a_{33}Y3). \end{aligned} \qquad (22)$$

It is immediately apparent, assuming the coefficients are based on actual field data, that $Y1$, $Y2$, and $Y3$ cannot become negative. This is so because whenever $Y1$, $Y2$, or $Y3$ becomes zero, the corresponding derivative becomes zero, and furthermore cannot deviate from zero again.

The same cannot be said for the following linear additive system:

$$\begin{aligned} dY1/dt &= a_{11}Y1 + a_{12}Y2 + a_{13}Y3 \\ dY2/dt &= a_{21}Y1 + a_{22}Y2 + a_{23}Y3 \\ dY3/dt &= a_{32}Y2 + a_{33}Y3. \end{aligned} \qquad (23)$$

Clearly, even though $Y1$ may be zero, $dY1/dt$ is still influenced by $Y2$ and $Y3$. In later applications $Y2$ and $Y3$ have inhibiting influences on $Y1$, so the coefficients a_{12} and a_{13} are negative, thus giving a negative derivative and eventually a negative standing crop, $Y1$. Leary[2] compensated for this problem by eliminating from the system (23) any component that became negative, a procedure suggested by Lotka (1928).

The property of boundedness for all $t \geqq 0$ is a possible point of disagreement. In fact it is a very restrictive condition to impose on a solution. Although the statement may be made that everything worldly is finite, hence functions that describe standing crop should be bounded above, a question arises as to how one treats something like standing crop in surviving trees. By definition, surviving trees are living trees, and living trees do not remain static in size. Hence, it appears that by definition, functions that describe standing crop of surviving trees should be monotonic nondecreasing and unbounded above. An example of a nonlinear multiplicative model with possibly unbounded solutions is

$$\begin{aligned} dY1/dt &= a_{11}Y1\, e^{a_{12}(Y1 + Y2 + Y3)} \\ dY2/dt &= a_{21}Y2\, e^{a_{22}(Y2 + Y3)} \\ dY3/dt &= a_{31}Y3\, e^{a_{32}(Y3)} \end{aligned} \qquad (24)$$

Notice the above system is nonnegative and may be unbounded above, although well behaved for all t. Due to its relative simplicity, nonnegativity, and conformity to the requirements for functions representing survivor standing crop, this is the system used in later sections to demonstrate the quasilinearization method of solving the inverse problem.

EXAMINING SYSTEM DEVELOPMENT

Let us briefly discuss methods of examining system development i.e., of solving the governing equations. In what follows, the unknown model parameters are assumed known. The purpose of discussing these topics now is that many things common to methods of solution are used in the methods of parameter estimation discussed later.

Solution methods may be categorized as analytic, graphic, and numeric. The concepts and approaches contained in these methods are as follows:

(1) Analytic solutions: (a) the role of initial and boundary conditions in isolating a unique solution to a single equation or system of equations, (b) the solution of linear equations, and (c) the principle of superposition, which forms an integral part of the quasilinearization method.

(2) Graphic (geometric) solutions: the construction of direction fields is a useful approach for (a) exhibiting the approximate system development for a wide range of initial conditions in an easily understood manner, (b) understanding the boundary condition formulation of the inverse problem, and (c) exhibiting convergence of the quasilinearization method to the correct system parameters.

(3) Numeric solutions: the algorithms used in obtaining numerical solutions are an efficient means of examining the large systems of nonlinear equations encountered in modeling forest systems.

Several of the above areas are the subjects of books in themselves. The purpose is, therefore, to introduce the ideas involved through examples, give a brief discussion of applications, and refer the reader to authoritative sources for a more thorough discussion.

Analytic Solutions

It is perhaps well to distinguish between an analytic formulation of the problem and its analytic solution. An analytic formulation is used in all cases regardless of the solution method (analytic, graphic, or numeric). Conceivably, problems could be formulated verbally, which might be a good beginning for someone with little experience in this area, but they must be converted to analytic form for solution. As a simple example consider: "The growth rate of an organism is directly proportional to the amount of it that is present." Expressing this in analytic form gives

$$dY(t)/dt = a\,Y(t), \qquad (25)$$

where a is a constant of proportionality.

An analytic solution, in the context of the above example, is a function $Y(t)$ which, when differentiated with respect to time, results in equation (25). Such a function is easily obtained by direct integration of both sides of

$$dY(t)/aY(t) = dt;\ \text{i.e.,}$$

$$\int \frac{dY(t)}{aY(t)} = \int dt,\ \text{or}$$

$$\ln Y(t) = at + C_4,\ \text{where } C_4 = a(C_2 - C_1),\ \text{or}$$

$$Y(t) = e^{at}\, e^{C_4},\ \text{or}$$

$$Y(t) = C_5\, e^{at}, \qquad (26)$$

where $C_5 = e^{C_4}$ and is the constant of integration.

The role of C_5 in the complete solution (26) may be seen from graphs of $Y(t) = C_5 e^{at}$ for various values of C_5 (fig. 3). Clearly, there is a family of solutions. How does one specify a particular element of the family shown in figure 3 as the unique solution? This is accomplished by specifying a constraint on the solution $Y(t)$ at a value of t. The common case is where one specifies $Y(t) = b$ when $t = 0$. This is an initial condition requirement that $Y(t) = C_5 e^{at}$ must meet; i.e.,

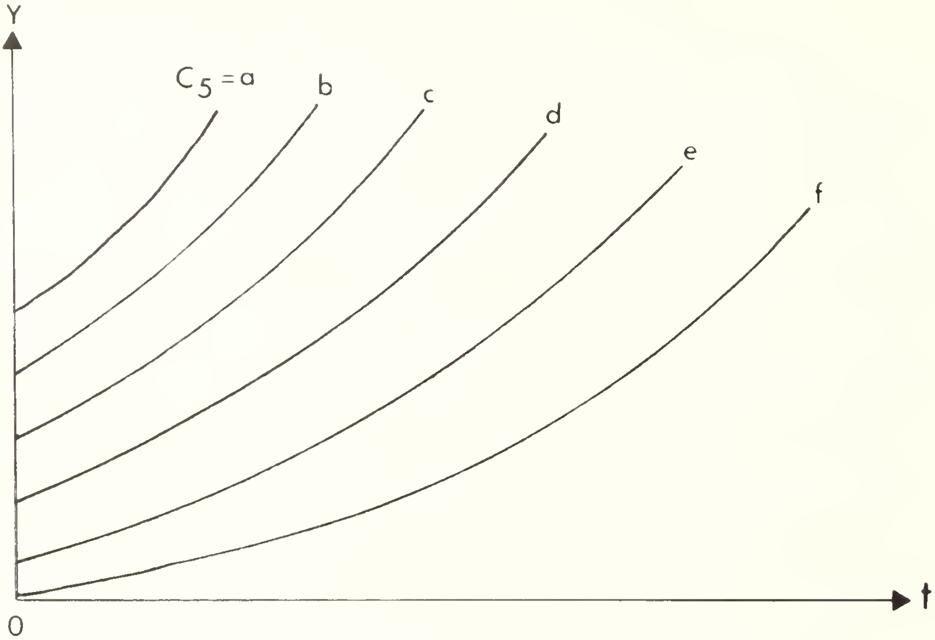


Figure 3. — Family of solutions of equation 25.

$$Y(0) = b = C_5 e^{a0}, \text{ or } b = C_5. \tag{27}$$

Thus, the unique solution of

$$dY(t)/dt = a Y(t), \text{ with}$$
$$Y(0) = b, \text{ is}$$
$$Y(t) = b e^{at}.$$

For a more involved example, consider the generalization of von Bertalanffy's equation (9), for which a solution for Z in terms of t is desired. Starting with equation (11) and multiplying through by $-b\, dt/(a - bZ)$ gives

$$-b\ dZ/(a - bZ) = -b(1-c)dt. \tag{28}$$

Integrating both sides gives

$$\ln(a - bZ) + C_1 = -b(1-c)t + C_2. \tag{29}$$

Solving for Z, we have

$$Z = \frac{a}{b} - \frac{C_3}{b} e^{-b(1-c)t},$$
$$C_3 = e^{C_2 - C_1}.$$

Thus

$$Y = \left[\frac{a}{b} - \frac{C_3}{b} e^{-b(1-c)t}\right]^{\frac{1}{1-c}} \tag{30}$$

If $Y_t = Y_0$ at $t = 0$,

$$Y_t = Y_0 = \left[\frac{a}{b} - \frac{C_3}{b} e^0\right]^{\frac{1}{1-c}}, \text{ or } Y_0^{1-c} = \left[\frac{a}{b} - \frac{C_3}{b}\right], \text{ and}$$

$$Y = \left[\frac{a}{b} - (\frac{a}{b} - Y_0^{1-c}) e^{-b(1-c)t}\right]^{\frac{1}{1-c}} \tag{31}$$

The above examples were chosen so that they would be simple, yet exhibit what is meant by analytic solutions and how they are obtained; i.e., using basic rules of integration, derived in elementary calculus. There are many special techniques for finding analytic solutions of differential equations, for instance, use of integrating factors, method of undetermined coefficients, method of variation of parameters, and others. They are mentioned here because these are the techniques that are taught in the standard courses in differential equations. But for our purposes they are of limited value.

The reader should clearly understand the role that initial conditions play in obtaining unique solutions, because later two-point and multipoint boundary value problems are discussed that presuppose an understanding of these ideas.

Solutions to homogeneous and nonhomogeneous linear differential equations are required to implement the quasilinearization method of solving multipoint boundary value problems. Hence, it is appropriate to say something about the form of their solution.

Let $L(D)$ be a linear differential operator (Rainville 1964, Kaplan 1964),

$$L(D) = a_n(t)D^n + a_{n-1}(t)D^{n-1} + \dots + a_1(t)D + a_0,$$

$$\text{where } D = d/dt, \; D^k = d^k/dt^k,$$

and consider the differential equations

$$\begin{array}{ll} L(D)Y = R(t) & (L) \\ L(D)Y = 0 & (LH). \end{array} \tag{32}$$

LH is called the homogeneous or complementary equation associated with *L*.

It is possible to write down the entire set of solutions of *L* in the following way: (1) Determine any one solution of *L*, called a particular solution, $Y = Y_p(t)$. (2) Determine the set of all solutions to *LH*. Let it be $Y = Y(t,c_1,c_2,c_3,\dots,c_n)$ where the c_i are arbitrary constants. (3) Then $Y = Y_p(t) + Y(t,c_1,c_2,\dots,c_n)$ is the complete set of solutions to *L*. The function $Y(t,c_1,c_2,\dots,c_n)$, called the complementary function, or Y_c, has a standard form also. It is $Y_c = c_1y_{h1}(t) + c_2Y_{h2}(t) + \dots + c_ny_{hn}(t)$, where $y_{h1}(t) \dots y_{hn}(t)$ are n linearly independent solutions of *LH*.

As an example consider:

$$\begin{array}{ll} Y'' + Y = t^2 & \text{(L)} \\ Y'' + Y = 0 & \text{(LH)} \end{array} \tag{33}$$

1. $Y = t^2-2$ is a solution of L

2. $y_{h1} = \cos\ t, y_{h2} = \sin\ t$ are linearly independent solutions of *LH,* therefore

$$Y = t^2 - 2 + c_1 \cos t + c_2 \sin t \tag{34}$$

is the complete solution of L. The arbitrary constants c_1 and c_2 may be specified by initial or boundary conditions.

Consider a system of linear first order equations similar to those of equation (16):

$$\begin{array}{l} dY1/dt = a_{11}Y1 + a_{12}Y2 + a_{13}Y3 \\ dY2/dt = a_{21}Y1 + a_{22}Y2 + a_{23}Y3 \\ dY3/dt = a_{31}Y1 + a_{32}Y2 + a_{33}Y3. \end{array} \tag{35}$$

It is known that the solution of linear constant coefficient differential equations such as (35) contain terms of the form e^{rt}. Let them be as follows:

$$\begin{array}{l} Y1 = A\ e^{rt} \\ Y2 = B\ e^{rt} \\ Y3 = C\ e^{rt}. \end{array}$$

Using these as trial solutions of (35) gives, upon substitution,

$$\begin{array}{l} rA\ e^{rt} = a_{11}A\ e^{rt} + a_{12}B\ e^{rt} + a_{13}C\ e^{rt} \\ rB\ e^{rt} = a_{21}A\ e^{rt} + a_{22}B\ e^{rt} + a_{23}C\ e^{rt} \\ rC\ e^{rt} = a_{31}A\ e^{rt} + a_{32}B\ e^{rt} + a_{33}C\ e^{rt}. \end{array} \tag{36}$$

Dividing through each equation by e^{rt} and transposing gives

$$\begin{array}{rrrl} (a_{11}-r)A + & a_{12}B & a_{13}C & = 0 \\ a_{21}A + & (a_{22}-r)B & a_{23}C & = 0 \\ a_{31}A + & a_{32}B & + (a_{33}-r)C & = 0. \end{array} \tag{37}$$

A homogeneous system of algebraic equations has nontrivial solutions if and only if the determinant of the coefficient matrix equals zero; i.e.,

$$\begin{vmatrix} a_{11}-r & a_{12} & a_{13} \\ a_{21} & a_{22}-r & a_{23} \\ a_{31} & a_{32} & a_{33}-r \end{vmatrix} = 0. \tag{38}$$

Expanding the determinant gives a polynomial in r. The three roots of this polynomial, called the characteristic polynomial, are known as the characteristic numbers or eigenvalues of the matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}. \qquad (39)$$

For simplicity in the following discussion, it is assumed that the r_i, $i = 1, 2, 3$ are real and distinct.

The eigenvalues of a matrix A are those scalar numbers r for which

$$Ax = rx, \qquad (40)$$

where x is a column vector,
A is a matrix, and
r is a scalar.

This may be put into a more familiar form as

$$(A - rI)x = 0, \qquad (41)$$

where I is the identity matrix and 0 is the zero column vector. This is a homogeneous matrix equation that has nontrivial solutions if and only if

$$\det(A - rI) = 0.$$

Clearly, this is the same form as equation (38). Associated with each eigenvalue r is a vector x for which equation (40) holds. This vector is called the associated eigenvector.

Returning to equation (35) a general solution may be written down directly. It is

$$\begin{matrix} Y1 \\ Y2 \\ Y3 \end{matrix} = k_1 \begin{bmatrix} G_{11} \\ G_{12} \\ G_{13} \end{bmatrix} e^{r_1 t} + k_2 \begin{bmatrix} G_{21} \\ G_{22} \\ G_{23} \end{bmatrix} e^{r_2 t} + k_3 \begin{bmatrix} G_{31} \\ G_{32} \\ G_{33} \end{bmatrix} e^{r_3 t} \qquad (42)$$

where r_1, r_2, and r_3 are the eigenvalues of matrix (39), and
G_{1i}, G_{2i}, and G_{3i} are the associated eigenvectors.

The constants k_i, $i = 1, 2, 3$ are determined by initial conditions on $Y1$, $Y2$, and $Y3$.

The relevance of this discussion may be seen as follows. It is clear from equation (40) that, for a specified value of r, say, $r = 2.5$, multiplying the vector x by the matrix A has the same effect as multiplying x by the scalar 2.5. Multiplying a vector by a positive scalar increases its length, but does not change its direction. Hence, eigenvalues serve as expanders or contractors of the eigenvectors in equation (42). The eigenvectors G_{1i}, G_{2i}, and G_{3i} are linearly independent (a fundamental result from linear algebra for distinct eigenvalues), and span the 3-space. Hence, they form a basis of this space (Perlis 1952). This means that any point in this space may be located by a linear combination of the basis vectors, G_{1i}, G_{2i}, G_{3i}. (The most familiar basis for 3-space is the orthogonal set

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}).$$

Consider again equation (42). At a particular time, say t_1, the vectors G_{ij} have a specified length and direction. Using basic rules for vector addition (i.e., the parallelogram law), the sum of three vectors is again a vector that locates a point in space, in this case, our state space. After an elapse of time, say of length Δt, this point will have moved to a different position in state space, the distance from that at time t being determined by

$$e^{r_1 \Delta t}, \; e^{r_2 \Delta t}, \text{ and } e^{r_3 \Delta t}.$$

Clearly, the eigenvalues r_1, r_2, and r_3 determine how much the system state changes in a specified time period Δt.

At this point the following question seems relevant: Does it not stand to reason that, everything else held constant, forest stands on good sites should progress faster through state space than stands on poor sites? If this is so, does it not follow that a model such as (35) fit to historical data from stands on good sites would have larger eigenvalues associated with its coefficient matrix than the same model fit to historical data from stands on poor sites?

Computing the eigenvalues for models comprised of many equations can be bothersome, because the equations will usually be integrated numerically. To secure the essential information desired concerning the size of the eigenvalues and to circumvent the problem of computing them, use is made of the following result from linear algebra (Faddeev and Faddeeva 1963). The trace of a matrix is identical to the sum of its eigenvalues. In terms of the above example,

$$\text{trace (A)} = a_{11} + a_{22} + a_{33} = r_1 + r_2 + r_3.$$

Thus, the trace of a coefficient matrix for a model such as (35) may well be a measure of the productivity of the site on which the stand is growing.

Graphic or Geometric Solutions

Perhaps in the strictest sense of the word "solution," graphic methods are not sufficiently precise. Nonetheless, they are useful in giving a picture of the solution's behavior over a wide range of initial conditions. Before discussing the basic method, recall that the solution of

$$dY(t)/dt = aY(t) \qquad (43)$$

is a locus of points in 2-space corresponding to different values of Y and t. This was seen in figure 3.

The method of tangents or direction fields gives a representation of this locus of points by the construction, at a suitable grid of points in Y - t space, of a short line segment with the appropriate slope, $dY/dt|_{Y,t}$. If the grid points are sufficiently close together, the locus of points corresponding to the true solution would possess a set of lines tangent to it. The logic in using the method of tangents is that one constructs the "tangent" field, and from it infers the locus of points that would be the solution for any given starting value (initial condition). Figure 4 shows a schematic direction field for equation (43). Obviously, the slope increases as Y increases, and time is not a factor in its determination.

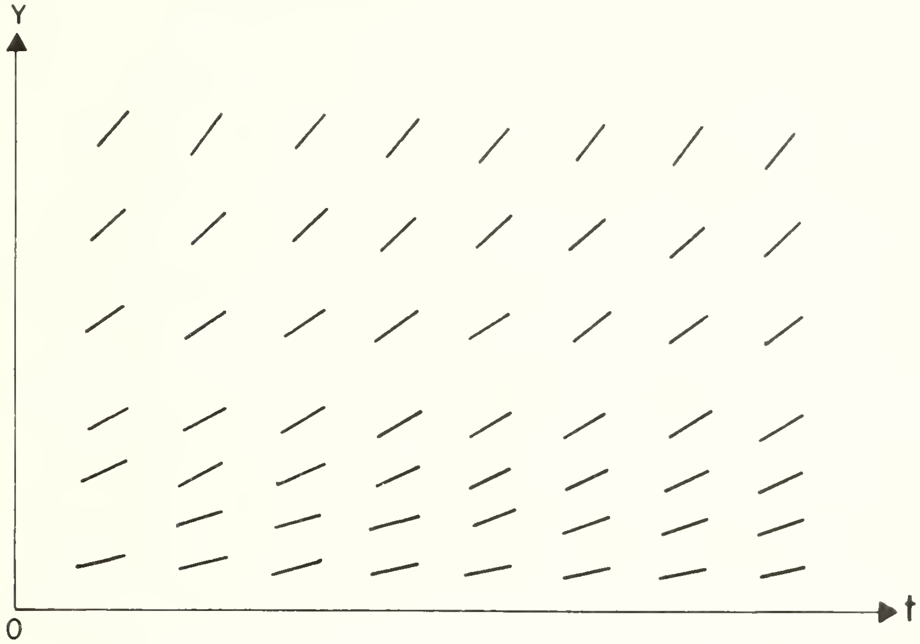


Figure 4. — Schematic direction field for equation (43).

Clearly, the form of the solution curves in figure 3 could be inferred from the direction field in figure 4.

Numeric Solutions

A numeric solution of a differential equation is a tabulated set of values with, in the case of equation (43), the values of Y and t at which it occurred. The table could, conceivably, consist of but two entries; i.e., Y_o, t_0 the beginning conditions, and Y_f, t_f, the final conditions. Rapid access to intermediate solution values and associated times, as well as initial and terminal conditions is a practical necessity for the methods of later sections, hence they are stored internally in the central memory of the computer.

The problem of developing algorithms, of which there are many, for solving differential equations numerically is indeed vast. The purpose at this point is to indicate that the single-step method of Runge-Kutta has been found satisfactory in the numerical solutions required in employing the quasilinearization method. Briefly, the Runge-Kutta (fourth order) method uses information about the solution and the slope of the solution at a single point (value of the independent variable) to project ahead a short distance, Δt, along the time axis, in the case of equation (43). The information concerning the solution and its slope at time t_1 is contained in the following constants:

$$m_1 = f(Y(t_1),\ t_1)\ \Delta t$$
$$m_2 = f(Y(t_1) + m_1/2,\ t_1 + \Delta t/2)\ \Delta t$$
$$m_3 = f(Y(t_1) + m_2/2,\ t_1 + \Delta t/2)\ \Delta t$$
$$m_4 = f(Y(t_1) + m_3,\ t_1 + \Delta t)\ \Delta t.$$

They are combined as follows to project the value of Y at t_2: $Y(t_2) = y(t_1) + 1/6\ (m_1 + 2m_2 + 2m_3 + m_4)$.

The above equations may be generalized directly to handle systems of differential equations (Conte 1965, Henrici 1964, Ralston 1965).

A fourth-order Runge-Kutta (RK) procedure was used in the examples of later sections. The routine, RKGS, from the Scientific Subroutine Package (IBM), contained a facility for doubling or halving the step size according to an error criterion specified by the user. This feature proved useful in detecting unstable systems, thus allowing the procedure to be terminated before excessive computing time was consumed.

On the surface, one might expect a multipoint method, such as a predictor-corrector, to be faster and cheaper, because it requires evaluation of the r.h.s. of the differential equation only twice per step, as opposed to four times for the RK method. However, predictor-corrector methods are not self-starting and it is common practice to use a fourth-order RK routine to compute the first four solution values, Y_{t_1}, Y_{t_2}, Y_{t_3}, Y_{t_4}, which the predictor-corrector then uses to project ahead to Y_{t_5}. As the routines are applied in later sections, four applications of the RK procedure traverses the entire integration interval, with the result that the predictor-corrector scheme is never implemented. Hence, a simple, self-starting, single-step, fourth-order RK method is used.

CHARACTERIZING THE SYSTEM STATE

Much time was spent examining how the system may be characterized in a manner that does not violate certain biological principles and how a system may be projected through time; i.e., by solving the governing equation. It was shown that at any point in time, t_k, the state of the system is given by the vector of state variables

$$[Y1(t_k)\ \ Y2(t_k)\ \ Y3(t_k)\ \ \ldots\ \ Yn(t_k)]^T.$$

This is a convenient means of expressing the system state; however, for our purposes it lacks sufficient conciseness for ease in mathematical analysis. A measure of the above vector that satisfies our needs is a vector norm.

Perhaps the most familiar vector norm is the Euclidean norm defined as (Faddeev and Faddeeva 1963),

$$||Y||_3 = [(Y1^2 + Y2^2 + Y3^2 + \ldots + Yn^2)]^{\frac{1}{2}}, \qquad (44)$$

or the distance of the point ($Y1\ Y2\ Y3 \ldots Yn$) from the origin of our state space. There is no obvious meaning to the Euclidean norm for our purposes, hence, attention is directed at what is called the second vector norm, defined as

$$||Y||_2 = |Y1| + |Y2| + |Y3| + \ldots + |Yn|, \qquad (45)$$

where $|Y1|$ is, in our case, the absolute value of the standing crop in component $Y1$. Notice that if our governing equations possess the property of nonnegativity, equation (45) may be simplified to

$$||Y||_2' = Y1 + Y2 + Y3 + \ldots + Yn,$$

which is, of course, total standing crop (assuming the units of Yi are conformable for addition).[2] Likewise, given two adjacent points in state space corresponding to times t_1 and t_2,

$$||Y(t_2)||_2' - ||Y(t_1)||_2'$$

gives growth in total standing crop.

A geometric picture of system (stand) development through time may be obtained if the sequence of state vectors

$$\begin{bmatrix} Y1(t_1) \\ Y2(t_1) \\ Y3(t_1) \\ \cdot \\ \cdot \\ \cdot \\ Yn(t_1) \end{bmatrix}, \begin{bmatrix} Y1(t_2) \\ Y2(t_2) \\ Y3(t_2) \\ \cdot \\ \cdot \\ \cdot \\ Yn(t_2) \end{bmatrix}, \ldots, \begin{bmatrix} Y1(t_n) \\ Y2(t_n) \\ Y3(t_n) \\ \cdot \\ \cdot \\ \cdot \\ Yn(t_n) \end{bmatrix}$$

is interpreted in a geometric manner.

One such manner is to present these vectors as bar charts (Knuchel 1953). Or one might convert the bar charts to a series of polygons by connecting the midpoints for each class. Superimposing several of these polygons on the same coordinate system shows that in the case of even-aged forest stands development is characterized by the propagation of a "wave" across the size (d.b.h.) classes (fig. 5).[2]

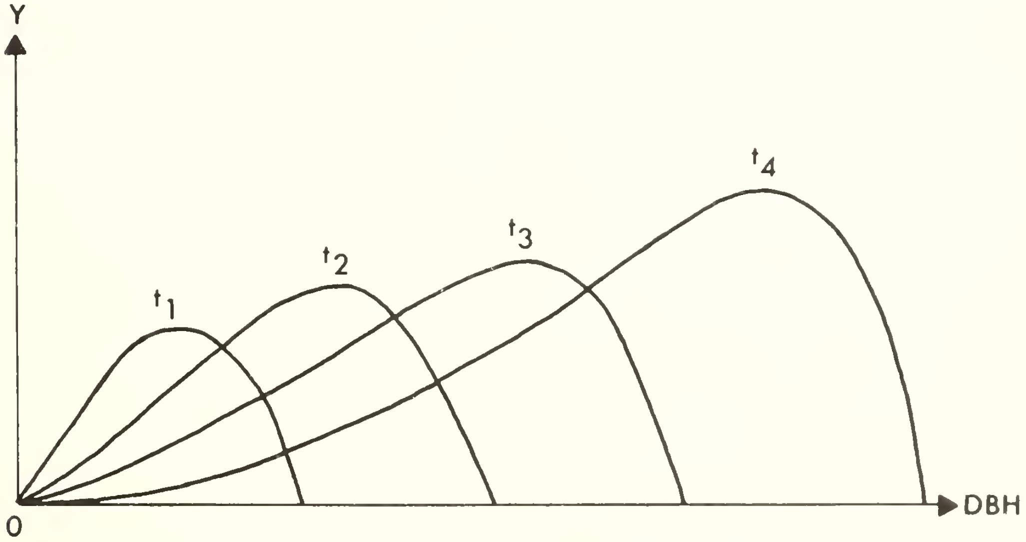


Figure 5. — Forest stand development in terms of wave propagation.

The interpretation employed in the remainder of this paper is that of the sequence of state vectors, each element of which locates a point in state space, tracing out a trajectory in state space. A simple example for three-dimensional space is shown in figure 6. Clearly, given an initial condition (point in state space), the question of solving the governing equation(s) is equivalent to determining the trajectory the system will follow through time.

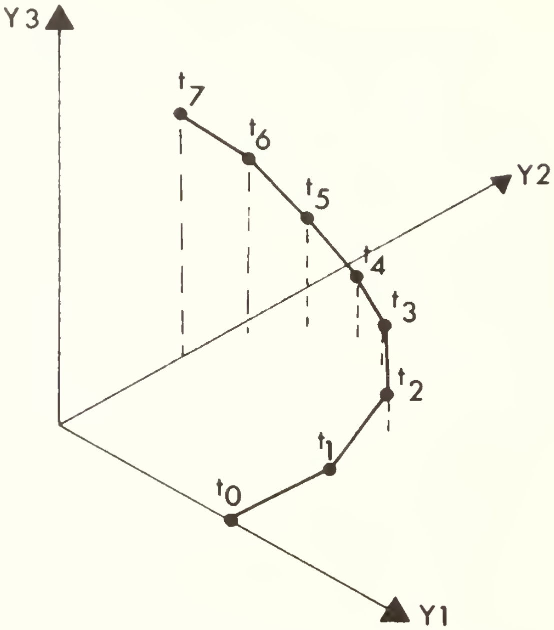


Figure 6. — Hypothetical forest stand development in terms of point transformations.

THE INVERSE PROBLEM

The problem central to constructing dynamic mathematical models of forest systems is the inverse of that just stated, that is, instead of asking "given certain governing equations for our system, what trajectory will the system follow?", one asks, "given observations on the system trajectory, what are the governing equations?" In its most basic form, the inverse problem may involve unknown forms of the governing equations. However, at this point the assumption is that the form of the governing equations is known, but numerical values of the parameters are not. Thus, our inverse problem may be rephrased as follows: "Given observations on the system trajectory, what are the numerical values for governing equation parameters that put the predicted system states in the desired conformity with the observed system states?"

What is the real world relevance of the inverse problem? Basically it rests on man's desire to exert control over his environment because of his preference for certain system states. In this case states take the form of stand structure, species composition, etc. As manipulators of forest systems, land managers may ask if leaving the system undisturbed will result in desired future states. Future states can be predicted by projecting the system into the future using past system states as a guide in determining governing equation parameters. If projected future states are not those most desired, a decision must be made as to how, given the resources available, a manager can alter the present state to insure the desired future states. This line of discussion leads naturally to the questions of multistage decision processes and optimal control, which are interesting but not part of our theme.

Boundary value problems are discussed in the next section, and we see that by using observations on the system trajectory as boundary conditions, the inverse problem may be solved. That is, the equations governing our system may be completely specified.

BOUNDARY CONDITION FORMULATION OF INVERSE PROBLEM

By specifying an initial condition for our governing equation, a particular element from a family of solutions may be isolated. See figure 7 for a schematic example using the generalized von Bertalanffy equation. For a further example, consider the following growth function similar to one suggested by Roston (1962):

$$dY/dt = c_1Y - c_2\int_0^t Y\ dt. \tag{46}$$

The growth of *Y* is positively related to the amount of *Y* present, but is inhibited by an accumulated proportion of *Y*. The inhibiting component may reflect an encroachment on available growing space (ecosystem resources) or an accumulation of toxic compounds that may be by-products of metabolism or environmental pollutants.

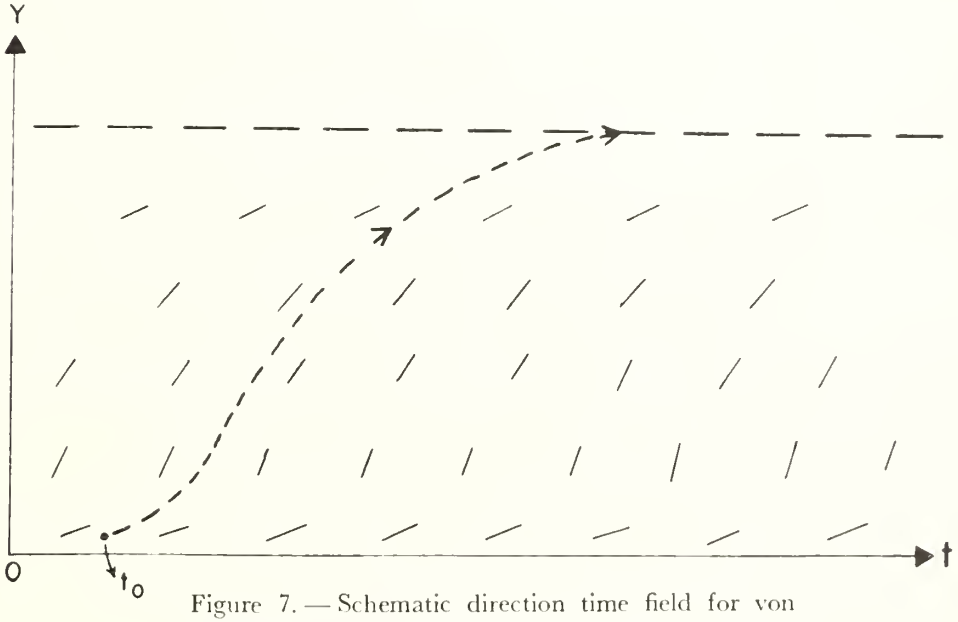


Figure 7. — Schematic direction time field for von Bertalanffy's equation showing role of initial condition.

Differentiation of equation (46) using Leibnitz's rule gives the linear homogeneous second-order differential equation

$$d^2Y/dt^2 - c_1dY/dt + c_2Y = 0. \tag{47}$$

Recall from the section on "Examining System Development" that such an equation has a solution comprised of a linear combination of homogeneous solutions of the form

$$Y = B_1 e^{r_1 t} + B_2 e^{r_2 t}, \tag{48}$$

where r_1 and r_2 are roots of the characteristic equation

$$z^2 - c_1 z + c_2 = 0.$$

In this case B_1 and B_2 are determined from initial conditions that involve Y and dY/dt; e.g.,

$$\begin{aligned} Y(0) &= a \\ dY/dt\Big|_{t=0} &= b. \end{aligned}$$

This is an initial value problem, because the constraints on Y and dY/dt are at time zero.

Consider now the following changes in constraints on the solution of equation (47). Instead of specifying both conditions at $t = 0$, one is specified at time $t = 0$ and the other at $t = t_f$. Furthermore, Y must meet both of these constraints instead of dY/dt meeting one as above. Thus, the problem is to solve

$$d^2Y/dt^2 - c_1 dY/dt + c_2 Y = 0, \tag{49}$$

with boundary conditions

$$\begin{aligned} Y(0) &= a \\ Y(t_f) &= b. \end{aligned} \tag{50}$$

Equations (49) and (50) constitute a linear two-point boundary value problem for which there are straightforward methods of solution.

However, many of the two-point boundary value problems that arise in the physical sciences, the calculus of variations, and control theory are nonlinear, and have no straightforward methods of solution. Some of these may, however, be solved by the quasilinearization method.

The solution of (47) may be placed in the framework of a system of differential equations by making the substitution, $Z = dY/dt$, and in place of equation (49) constructing the system of two first-order equations

$$\begin{aligned} dZ/dt &= c_1 Z - c_2 Y \\ dY/dt &= Z \end{aligned} \tag{51}$$

with the boundary conditions

$$\begin{aligned} Y(0) &= a \\ Y(t_f) &= b. \end{aligned} \tag{52}$$

Clearly, Y has two constraints and Z none. Successful numerical solution of (51) subject to (52) hinges on the ability to select a value for Z at $t = 0$ such that when $t = 0$, $Y = a$, and when $t = t_f$, $Y = b$.

If constraints on the solution exist at more than two points, it is referred to as a multipoint boundary value problem. As an example, consider the nonlinear first-order equation

$$dY/dt = aYe^{bY} \tag{53}$$

with boundary conditions

$$Y(t_1) = m_1, \quad Y(t_2) = m_2, \quad Y(t_3) = m_3. \tag{54}$$

In our applications, the constant parameters a and b in equation (53) are considered as unknowns and the following changes in the form of equation (53) are made:

(1) Consider a and b to be time-dependent variables; i.e., functions of time of the form

$$da/dt = 0$$
$$db/dt = 0, \text{ and}$$

(2) convert the single equation to a system of equations. Thus, a simple parameter estimation problem that will be solved in the following section is the multipoint boundary value problem

$$\begin{aligned} dY/dt &= aYe^{bY} \\ da/dt &= 0 \\ db/dt &= 0 \end{aligned} \tag{55}$$

with boundary conditions

$$Y(t_i) = m_i, \quad i = 1, 2, 3. \tag{56}$$

Here again, successful numerical solution rests on selecting values for $a(0)$ and $b(0)$ such that conditions (56) are met, where m_i are observed system states.

Likewise for systems of equations:

$$\begin{aligned} dY1/dt &= a\ Y1\ e^{b(Y1 + Y2 + Y3)} \\ dY2/dt &= c\ Y2\ e^{d(Y1 + Y3)} \\ dY3/dt &= f\ Y3\ e^{g(Y3)} \\ da/dt &= 0 \\ db/dt &= 0 \\ dc/dt &= 0 \\ dd/dt &= 0 \\ df/dt &= 0 \\ dg/dt &= 0 \end{aligned} \tag{57}$$

with boundary conditions

$$\begin{aligned} Y1(t_i) &= m_{1i}, \quad i = 1, 2, 3 \\ Y2(t_i) &= m_{2i}, \quad i = 1, 2, 3 \\ Y3(t_i) &= m_{3i}, \quad i = 1, 2, 3. \end{aligned} \tag{58}$$

Our approach will again be to select values for $a, b, c, d, f,$ and g at $t = 0$, such that conditions (58) are met.

It is highly unlikely that values for $a, b, c, d, f,$ and g at $t = 0$ could be selected such that $Y1\ (t_i) = m_{1,i}$, $Y2(t_i) = m_{2,i}$, $Y3(t_i) = m_{3,i}$ for all i. Hence, it is important that there is a means of making an initial estimate for $a, b, c, d, f,$ and g, and, furthermore, a means of improving these estimates until a criterion of goodness of fit between $Y1\,(t_i)$, $Y2(t_i)$, and $Y3(t_i)$, and $m_{1,i}$, $m_{2,i}$, $m_{3,i}$, respectively, is met.

The quasilinearization method considered in the next section provides us with such a procedure. First, however, it is advisable to review some basic concepts concerning iteration functions.

Use of an iteration function (I.F.) involves making an initial guess at the answer, substituting it into an I.F., and getting a new approximation to the answer. The new approximation is then substituted into the I.F. and another approximation is obtained. This process is repeated until the value substituted into the I.F. is the same (to however many digits accuracy one desires) as the value given by the I.F. as the new approximation.

An I.F. is any function of the form $x^{(k+1)} = F(x^{(k)}, x^{(k-1)}, x^{(k-2)}, \ldots)$, where the superscript indicates the independent variable at the iteration. One seeks a "fixed point" of the I.F., which is that value of x, say α, such that $\alpha = F(\alpha)$. Thus, when $x = \alpha$, the iteration has converged.

There are many types of I.F.'s, the primary differences among them being (a) their interval and rate of convergence, (b) the number of previous iterations that are used, and (c) the number of initial approximations that must be specified to "start" the process. One might say that an ideal method would be one that is globally convergent, of at least second order (i.e., converges quadratically), and needs only one initial approximation to get started.

In what follows, the Newton-Raphson method is discussed in relation to (b) and (c) above. Using Traub's (1964) classification of I.F.'s, the Newton-Raphson method is a one-point function requiring evaluation of the derivative. "One-point" means the I.F. is of the form $x^{(k+1)} = F(x^{(k)})$, that is, only the previous iterate is used in obtaining a new approximation. It is well to note that most I.F.'s use information about the shape of the curve of the underlying function to speed convergence.

The form of the Newton-Raphson I.F. may be developed in the following way. Consider the function f shown in figure 8. The value of x at which $f(x) = 0$ is sought. A first approximation is obtained by selecting a value for x, $x^{(1)}$, and evaluating f; i.e., $f(x^{(1)})$. Using the point-slope formula we have

$$Y - f(x^{(1)}) = f'(x^{(1)})(x - x^{(1)}). \tag{59}$$

Setting $Y = 0$ and solving for x gives

$$x = x^{(1)} - f(x^{(1)})/f'(x^{(1)}). \tag{60}$$

If x is closer to α than $x^{(1)}$ — i.e., $|x - \alpha| < |x^{(1)} - \alpha|$ — we take x as our new estimate of the root and repeat the process. In general this gives

$$x^{(n)} = x^{(n-1)} - f(x^{(n-1)})/f'(x^{(n-1)}), \tag{61}$$

which is the Newton-Raphson iteration function, and is abbreviated

$$x^{(k)} = F(x^{(k-1)}),$$

where F corresponds to the right-hand side of equation (61).

In the next section it will be seen that the quasilinearization method is abstractly equivalent to the Newton-Raphson method. A fundamental difference, however, is that in the discussion above, the concern is with approximations (to the root α) in *value* space, whereas with the quasilinearization method the concern is with Newton-Raphson type approximations in *function* space. The concern, then, is with convergence to a function rather than to the value (root) of a function.

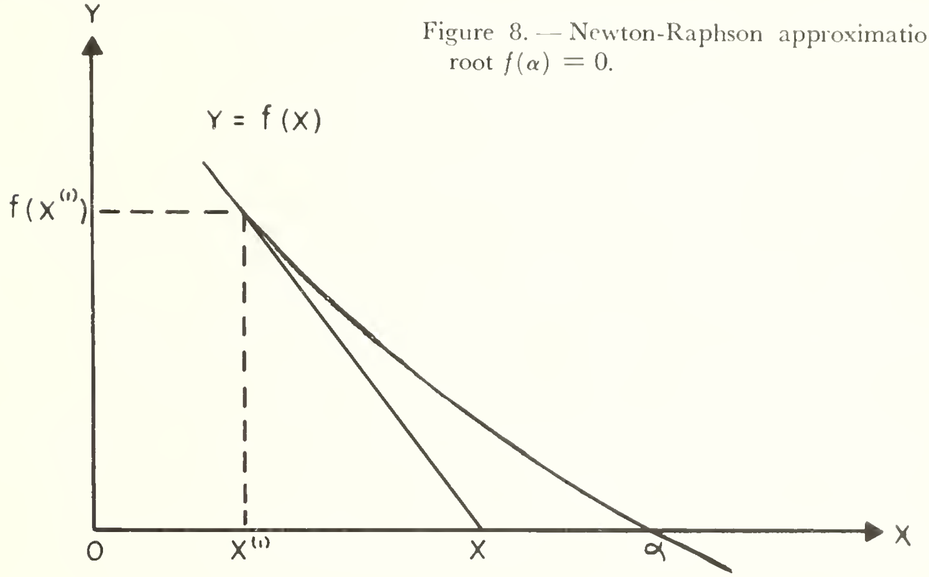


Figure 8. — Newton-Raphson approximation to the root $f(\alpha) = 0$.

SOLUTION OF THE INVERSE PROBLEM USING QUASILINEARIZATION

The approach in solving the following inverse problems is one of (a) converting nonlinear differential equations into approximately equivalent linear equations, and (b) solving the resulting linear boundary value problem using the method of complementary functions and standard numerical integration techniques. Why use this approach? It is because there are straightforward methods for solving linear boundary value problems with constant or variable coefficients. To see this, consider the following example of a linear nonhomogeneous second-order equation with variable coefficients.

$$d^2Y/dt^2 + P(t)\ dY/dt + Q(t)\ Y = R(t) \tag{62}$$

with boundary conditions

$$\begin{aligned} Y(0) &= a \\ Y(t_f) &= b. \end{aligned} \tag{63}$$

Linear equation theory dictates that the complete solution of (62) be of the form

$$Y(t) = y_p + c_1 y_{h1} + c_2 y_{h2}, \tag{64}$$

where y_p is the solution of the nonhomogeneous equation, and y_{h1} and y_{h2} are independent solutions of the homogeneous equation. How, then, do we form the particular and homogeneous solutions? First, one may obtain a particular solution by numerically integrating equation (62) subject to the initial conditions

$$Y(0) = a$$
$$dY/dt\Big|_{t=0} = 0.$$

Notice that the initial condition on Y corresponds to the first boundary condition in (63). Second one may obtain a homogeneous solution of equation (62) by numerically integrating

$$d^2Y/dt^2 + P(t)\ dY/dt + Q(t)\ Y = 0 \tag{65}$$

subject to any convenient initial conditions. As an example, one may use

$$\begin{aligned} Y(0) &= 1 \\ dY/dt\Big|_{t=0} &= 0. \end{aligned} \tag{66a}$$

Two linearly independent homogeneous solutions of (62) may be required, so

$$\begin{aligned} Y(0) &= 0 \\ dY/dt\Big|_{t=0} &= 1 \end{aligned} \tag{66b}$$

are used as initial conditions for the second.

The above are initial value problems. Because the first boundary condition has been satisfied by $y_p(t)$, our concern now is that the second boundary condition $y(t_f) = b$ is met. This is accomplished by forming the general solution at $t = 0$ and $t = t_f$, via superposition, which gives two linear algebraic equations in two unknowns, c_1 and c_2; i.e.,

$$\begin{aligned} Y(0) &= y_p(0) + c_1\ y_{h1}(0) + c_2\ y_{h2}(0) = a \\ Y(t_f) &= y_p(t_f) + c_1\ y_{h1}(t_f) + c_2\ y_{h2}(t_f) = b. \end{aligned} \tag{67}$$

What values should c_1 and c_2 take to insure that the boundary conditions (63) are met? By substituting the initial conditions into the first equation of (67)

$$a + c_1\ 1 + c_2\ 0 = a.$$

Clearly, c_1 must be zero. If $c_1 = 0$, then c_2 must equal

$$(b - y_p(t_f))/y_{h2}(t_f)$$

if the second boundary condition is to be satisfied.

Thus, this linear two-point boundary value problem is solved by computing one particular solution and two linearly independent homogeneous solutions. Then, a system of linear algebraic equations is solved for the integration constants so that the boundary conditions are satisfied. If we desire the complete solution, as we will later, we may use these integration constants in superposition.

If equation (62) is converted to a system of two first-order equations by making the substitution

$$dY/dt = Z, \tag{68}$$

the nonhomogeneous equation is

$$\begin{aligned} dY/dt &= Z \\ dZ/dt &= -P(t)\ Z - Q(t)\ Y + R(t) \end{aligned} \tag{69}$$

and the homogeneous equation is

$$\begin{aligned} dY/dt &= Z \\ dZ/dt &= -P(t)\ Z - Q(t)\ Y. \end{aligned} \tag{70}$$

The initial conditions for the solution of equation (69) (the particular solution) are

$$\begin{aligned} Y(0) &= a \\ Z(0) &= 0, \end{aligned} \tag{71}$$

and for the solution of equation (70) (the homogeneous solutions) are

$$\begin{matrix} Y(0) = 1 & & Y(0) = 0 \\ & \text{and} & \\ Z(0) = 0 & & Z(0) = 1. \end{matrix} \tag{72}$$

Because the boundary conditions involve Y and not Z, the integration constants c_1 and c_2 are again determined using equation (67). This approach is used in the following sections.

Thus, it is possible to solve linear boundary value problems, equations (62, 63), by using numerical integration techniques and methods of solving systems of linear algebraic equations. To see how it is possible to convert a nonlinear boundary value problem into a linearized form, consider the nonlinear second-order equation

$$d^2Y/dt^2 = f(Y(t),t) \tag{73}$$

with boundary conditions

$$\begin{aligned} Y(0) &= a \\ Y(t_f) &= b. \end{aligned}$$

Quasilinearization connotes replacing the nonlinear differential system by a "nearby" linear system. The nearby linear system is derived, in the case of equation (73), by expanding $f(Y,t)$ in a Taylor's series in Y about Y_0:

$$f(Y,t) = f(Y_0,t) + f_Y(Y_0,t)(Y-Y_0) + \frac{f_{YY}(Y_0,t)}{2!}(Y-Y_0)^2 + \ldots.$$

Retaining only the linear portion, the differential equation (73) takes the form

$$d^2Y/dt^2 = f(Y_0,t) + f_Y(Y_0,t)\ Y - f_Y(Y_0,t)\ Y_0, \tag{74}$$

where, if Y_0 is presumed a known function of t, the equation is linear in Y.

Starting with some initial guess function $Y_0(t)$, a sequence of functions is generated by means of the equation

$$\begin{aligned} d^2Y_{n+1}/dt^2 &= f_Y(Y_n(t),t)Y_{n+1} + f(Y_n(t),t) - \\ &\qquad f_Y(Y_n(t),t)Y_n(t) \end{aligned} \tag{75}$$

$$\text{or}\quad d^2Y_{n+1}/dt^2 = a(t)Y_{n+1} + b(t),$$

where $a(t)$ and $b(t)$ are known (previously computed and stored internally in the computer) functions of t.

Clearly, this recurrence relation is analogous to a one-point I.F. requiring evaluation of the derivative; i.e., the Newton-Raphson I.F.

How does one know when convergence of the above recurrence relation has occurred? Roughly speaking it is when the function $Y_n(t)$, the solution at the n^{th} iteration, deviates less than some prescribed amount from the function $Y_{n+1}(t)$, the current solution.

Most of this work involves systems of first-order equations, hence, the analogous recurrence relation for such systems is

$$\frac{d\vec{Y}_{n+1}}{dt} = f(\vec{Y}_n(t),t) + J(\vec{Y}_n(t),t)\ (\vec{Y}_{n+1} - \vec{Y}_n(t)), \tag{76}$$

where $\longrightarrow$ indicates column vectors and f is the vector function of r.h.s. of the governing equations, and J is the Jacobian matrix defined as

$$J(Y_n) = \begin{bmatrix} \frac{\partial f_1}{\partial y_{1,n}} & \frac{\partial f_1}{\partial y_{2,n}} & \frac{\partial f_1}{\partial y_{3,n}} & \cdots & \frac{\partial f_1}{\partial y_{m,n}} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \frac{\partial f_m}{\partial y_{1,n}} & \frac{\partial f_m}{\partial y_{2,n}} & \frac{\partial f_m}{\partial y_{3,n}} & \cdots & \frac{\partial f_m}{\partial y_{m,n}} \end{bmatrix}. \tag{77}$$

Again, $Y_n(t)$ is a computationally known vector function on the interval $0 \leq t \leq t_f$. Because equation (76) is a linear nonhomogeneous first-order equation, superposition may be used to form the complete solution as

$$Y(t) = y_p(t) + c_1 y_{h1}(t) + c_2 y_{h2}(t) + c_3 y_{h3}(t) + \ldots + c_m y_{hm}(t).$$

The details of the above procedure are perhaps best seen through an example. Assume the existence of three observations on a system trajectory,

$$\begin{aligned} Y(0) &= m_1 \\ Y(t_1) &= m_2 \\ Y(t_f) &= m_3, \text{ and} \end{aligned} \tag{78}$$

the hypothesized governing equation

$$dY/dt = aYe^{bY}. \tag{79}$$

To completely determine the equation governing the system, numerical values for a and b must be determined such that conditions (78) are met. Converting a and b to functions of time and constructing a system of equations gives

$$\begin{aligned} dY/dt &= aYe^{bY} &&= f_1 \\ da/dt &= 0 &&= f_2 \\ db/dt &= 0 &&= f_3. \end{aligned} \tag{80}$$

In terms of equation (76), we have

$$\begin{matrix} \frac{dY_{n+1}}{dt} = \\ \frac{da_{n+1}}{dt} = \\ \frac{db_{n+1}}{dt} = \end{matrix} \begin{bmatrix} a_n Y_n(t) e^{b_n Y_n(t)} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial Y_n} & \frac{\partial f_1}{\partial a_n} & \frac{\partial f_1}{\partial b_n} \\ \frac{\partial f_2}{\partial Y_n} & \frac{\partial f_2}{\partial a_n} & \frac{\partial f_2}{\partial b_n} \\ \frac{\partial f_3}{\partial Y_n} & \frac{\partial f_3}{\partial a_n} & \frac{\partial f_3}{\partial b_n} \end{bmatrix} \begin{bmatrix} Y_{n+1} - Y_n \\ a_{n+1} - a_n \\ b_{n+1} - b_n \end{bmatrix}. \tag{81}$$

Equation (81) reduces to

$$\frac{dY_{n+1}}{dt} = a_n Y_n e^{b_n Y_n} + \frac{\partial f_1}{\partial Y_n}(Y_{n+1}-Y_n) + \frac{\partial f_1}{\partial a_n}(a_{n+1}-a_n) + \frac{\partial f_1}{\partial b_n}(b_{n+1}-b_n) \qquad (82)$$

$$\frac{da_{n+1}}{dt} = 0$$

$$\frac{db_{n+1}}{dt} = 0$$

with boundary conditions $Y(0) = m_1$, $Y(t_1) = m_2$, and $Y(t_f) = m_3$. This, then, is a multipoint boundary value problem formulation of our inverse problem. Notice that equation (82) is a linear nonhomogeneous differential equation, and as such has a solution of the form

$$\begin{matrix} Y(t) = y_p(t) \\ a(t) = a_p(t) \\ b(t) = b_p(t) \end{matrix} + c_1\begin{bmatrix} y_{h1}(t) \\ a_{h1}(t) \\ b_{h1}(t) \end{bmatrix} + c_2\begin{bmatrix} y_{h2}(t) \\ a_{h2}(t) \\ b_{h2}(t) \end{bmatrix} + c_3\begin{bmatrix} y_{h3}(t) \\ a_{h3}(t) \\ b_{h3}(t) \end{bmatrix}, \qquad (83)$$

where $[y_p\ a_p\ b_p]^T$ is the solution of the nonhomogeneous equation (82) and

$$\begin{bmatrix} y_{h1}(t) \\ a_{h1}(t) \\ b_{h1}(t) \end{bmatrix}, \quad \begin{bmatrix} y_{h2}(t) \\ a_{h2}(t) \\ b_{h2}(t) \end{bmatrix}, \text{ and } \begin{bmatrix} y_{h3}(t) \\ a_{h3}(t) \\ b_{h3}(t) \end{bmatrix}$$

are independent solutions of the homogeneous form of (82); i.e.,

$$dY_{n+1}/dt = \frac{\partial f_1}{\partial Y_n} Y_{n+1} + \frac{\partial f_1}{\partial a_n} a_{n+1} + \frac{\partial f_1}{\partial b_n} b_{n+1} \qquad (84)$$

$$da_{n+1}/dt = 0$$

$$db_{n+1}/dt = 0.$$

The logic in selecting initial conditions for particular and homogeneous solutions is entirely analogous to that used in equations (66a) and (66b). For the particular solution the first boundary condition is used as the initial condition on Y and $a(0) = b(0) = 0$; i.e.,

$$\begin{bmatrix} m_1 \\ 0 \\ 0 \end{bmatrix}.$$

To obtain three independent homogeneous solutions the convenient initial conditions

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

may be used. These, again, are initial value problems that may be handled in a direct manner using numerical integration techniques.

To insure that the second and third boundary conditions are met, we form the general solution, equation (83). Because the boundary conditions involve Y, not a or b, we consider the solution for Y at the three times 0, t_1 and t_f; i.e.,

$$\begin{aligned} Y(0) &= y_p(0) + c_1 y_{h1}(0) + c_2 y_{h2}(0) + c_3 y_{h3}(0) = m_1 \\ Y(t_1) &= y_p(t_1) + c_1 y_{h1}(t_1) + c_2 y_{h2}(t_1) + c_3 y_{h3}(t_1) = m_2 \\ Y(t_f) &= y_p(t_f) + c_1 y_{h1}(t_f) + c_2 y_{h2}(t_f) + c_3 y_{h3}(t_f) = m_3. \end{aligned}$$

This is similar to equation (67). Proceeding in an analogous manner here we have for the first equation

$$m_1 + c_1\ 0 + c_2\ 0 + c_3\ 1 = m_1,$$

which dictates that c_3 be zero. The result is two equations in two unknowns,

$$\begin{aligned} y_p(t_1) + c_1 y_{h1}(t_1) + c_2 y_{h2}(t_1) &= m_2 \text{ and} \\ y_p(t_f) + c_1 y_{h1}(t_f) + c_2 y_{h2}(t_f) &= m_3, \end{aligned} \qquad (85)$$

which may easily be solved for c_1 and c_2. Only two homogeneous solutions are required; i.e., so initial conditions $[0\ 1\ 0]^T$ and $[0\ \ 0\ \ 1]^T$ are used.

Once c_1, c_2, and c_3 are known, they are substituted into equation (83) and the solution vector function

$$\begin{bmatrix} Y_{n+1}(t) \\ a_{n+1}(t) \\ b_{n+1}(t) \end{bmatrix}$$

is formed. After a check for convergence, by making a comparison of

$$\begin{bmatrix} a_{n+1}(0) \\ b_{n+1}(0) \end{bmatrix} \text{ vs } \begin{bmatrix} a_n(0) \\ b_n(0) \end{bmatrix},$$

one may, if need be, return to equations (81) and (82) and consider the vector function just determined as

$$\begin{bmatrix} Y_n(t) \\ a_n(t) \\ b_n(t) \end{bmatrix}.$$

In cases where many observations are available on a system trajectory, such as

$$Y(t_i) = m_i, \quad i = 1, 2, 3, \ldots, n,$$

we have an overdetermined system corresponding to equation (85). In such cases each boundary condition may not be met, so some measure of goodness of fit is used between

$$\begin{aligned} &Y(t_i) \text{ and } m_i;\ \text{i.e.,} \\ &Y(t_i) \cong m_i \text{ for all } i. \end{aligned} \qquad (86)$$

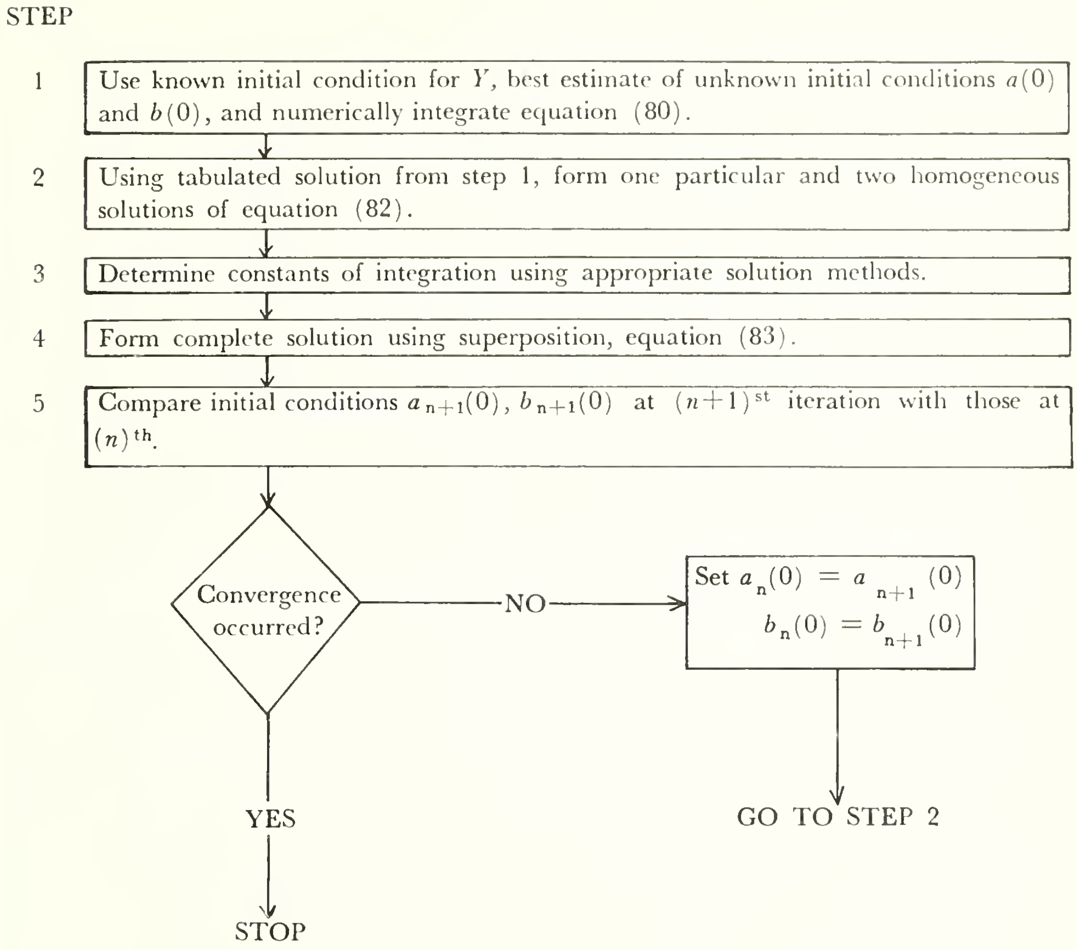
The criterion used in later sections is that of least squares. Thus, integration constants c_1 and c_2 are determined such that they minimize

$$S = \sum_i (Y(t_i) - m_i)^2. \qquad (87)$$

There is some indication that other measures of goodness of fit may be more appropriate than least squares, for instance, minimax (Bellman 1964).

The following flow-chart conveys a more concise picture of the steps involved:

OPERATION



Example 1

For a first example consider the following observational data on one system component in a northern hardwood stand located on the Argonne Experimental Forest in Wisconsin. The component is defined as all sugar maple (*Acer saccharum*) trees that were living in 1968 and were present in the 4- to 8-inch diameter class in 1947. Summarizing the historical records for this component on a plot gives

Year	*Sum of tree diameters* *(Inches)*
1947	60.6
1949	63.9
1950	65.4
1951	66.9
1954	72.6
1958	78.6
1962	85.9
1967	92.5
1968	93.4

These form the boundary conditions that guide the solution of equation (80). Because only two homogeneous solutions are required, the system is overdetermined. To guide our solution, the least squares criterion for goodness of fit is used. In other words, we want to determine c_1 and c_2 in (85) in a manner that minimizes

$$S = \sum_i (Y(t_i) - m_i)^2, \text{ where}$$

$$S = \sum_i (y_p(t_i) + c_1 y_{h1}(t_i) + c_2 y_{h2}(t_i) - m_i)^2. \qquad (88)$$

This may be accomplished by solving the simultaneous equations

$$\frac{\partial S}{\partial c_1} = 0 \quad \text{and} \quad \frac{\partial S}{\partial c_2} = 0$$

for c_1 and c_2.

Using a computer program written by the author, this nonlinear multipoint boundary value problem is solved by following the steps outlined in the flow-chart.

The initial parameter estimates were

$$a(0) = .05$$
$$b(0) = -.01.$$

The estimates of parameter values at each iteration were

Iteration	*a*	*b*
1	0.075469	−0.017809
2	.090660	− .019302
3	.091015	− .019167
4	.091041	− .019172
5	.091040	− .019172

The equation governing this system component is, therefore,

$$dY/dt = .0910\ Y\ e^{-.019172\ Y}. \qquad (89)$$

Integrating this equation with known initial conditions on Y, $Y(0) = 60.6$, gives the following values:

Year	*Observed*	*Predicted*	*Difference*
1947	60.6	60.6	—
1949	63.9	64.0	−0.1
1950	65.4	65.7	− .3
1951	66.9	67.4	− .5
1954	72.6	72.4	.2
1958	78.6	78.8	− .2
1962	85.9	85.1	.8
1967	92.5	92.4	.1
1968	93.4	93.8	− .4

A similar trial using the same source and type of data, but for the 8- to 12-inch diameter class of sugar maple, gave the following results

Iteration	*a*	*b*
Initial estimate	0.050000	−0.010000
1	.075405	− .020263
2	.134159	− .030693
3	.161696	− .030475
4	.161992	− .030541
5	.161949	− .030537
6	.161951	− .030537

Thus, the equation governing this system component is

$$dY/dt = .1619\ Y\ e^{-.030537\ Y} \tag{90}$$

Integrating this equation with known initial conditions, $Y(0) = 63.2$, gives

Year	*Observed*	*Predicted*	*Difference*
1947	63.2	63.2	—
1949	66.1	66.1	0.0
1950	67.0	67.5	– .5
1951	68.2	68.8	– .6
1954	72.8	72.8	.0
1958	77.9	77.7	.2
1962	83.0	82.2	.8
1967	87.4	87.3	.1
1968	87.8	88.3	– .5

Thus, it is possible to use observed system states as boundary conditions, and by solving the associated multipoint boundary value problem, to determine equation parameters that produce close agreement between predicted and observed states. Because the system components are well behaved, it follows logically that a projection for a limited time beyond 1968 may result in acceptable estimates of future component values.

The above examples, although showing close agreement between predicted and observed states, are inadequate in that they do not treat the forest as an interactive system. Hence, consideration is given to a more inclusive model that reflects simplified assumptions concerning interactions among three system components.

Example 2

The concern here is with finding numerical values for the parameters that best relate the predicted system state, as determined by

$$\begin{aligned} dY1/dt &= a\ Y1\ e^{b(Y1 + Y2 + Y3)} \\ dY2/dt &= c\ Y2\ e^{d(Y2 + Y3)} \\ dY3/dt &= f\ Y3\ e^{g(Y3)}, \end{aligned} \tag{91}$$

to the observed state given below. In this case, six homogeneous solutions are required, one for each unknown initial condition (parameter) (see equation (57)). Given nine observations in time, an overdetermined system is again present, so the least squares criterion for goodness of fit is used. In other words, we minimize

$$S = \sum_i ((Y1(t_i)-m_{1,i})^2 + (Y2(t_i)-m_{2,i})^2 + (Y3(t_i)-m_{3,i})^2) \tag{92}$$

where $m_{1,i}$ is the i^{th} observation on the first state variable, and

$$Y1(t) = y_{p1} + c_1 y_{h1} + c_2 y_{h2} + c_3 y_{h3} + c_4 y_{h4} + c_5 y_{h5} + c_6 y_{h6}.$$

The normal equations are derived in a straightforward manner by taking the partials of (92) with respect to c_1, c_2, c_3, c_4, c_5, and c_6, and equating them to zero.

Using the data in table 1 as our boundary conditions gives the following results:

Iteration	*Parameter*					
	a	*b*	*c*	*d*	*f*	g
Initial estimate	0.09100	– 0.00100	0.16190	– 0.01000	0.18370	– 0.08650
1	.03219	– .00056	.11380	– .01335	.11488	– .07443
2	.06551	– .00765	.14142	– .01948	.15608	– .08283
5	.12152	– .00958	.18381	– .02285	.15132	– .08029
7	.12145	– .00957	.18385	– .02285	.15132	– .08029

Table 1. — *Observed stand states for all sugar maple on a plot on the Argonne Experimental Forest*

Year	D.b.h. class (inches)		
	4 to 8	8 to 12	12 to 16
	(Sum of diameters in inches)		
1947	60.6	63.2	27.1
1949	63.9	66.1	28.2
1950	65.4	67.0	28.4
1951	66.9	68.2	28.9
1954	72.6	72.8	30.2
1958	78.6	77.9	31.6
1962	85.9	83.0	33.3
1967	92.5	87.4	34.8
1968	93.4	87.8	35.1

Hence, the complete governing equations are

$$\begin{aligned} dY1/dt &= .12145\ Y1\ e^{-.00957(Y1 + Y2 + Y3)} \\ dY2/dt &= .18385\ Y2\ e^{-.02285(Y2 + Y3)} \\ dY3/dt &= .15132\ Y3\ e^{-.08029(Y3)} \end{aligned} \qquad (93)$$

Using these equations and the known initial conditions

$$\begin{aligned} Y1(0) &= 60.6 \\ Y2(0) &= 63.2 \\ Y3(0) &= 27.1, \end{aligned}$$

the deviations of predicted system states from observed states are shown in table 2. Again, observed and predicted states agree well.

Interestingly, parameters *a* and *c* as determined from the system of equations (0.12145 and 0.18385, respectively) are both greater than when single equations are used as governing equations for their respective components (0.09104 and 0.1619). This indicates that components *Y*1 and *Y*2 have more capacity to grow than is indicated by treating them as isolated components, or not in the context of an interactive system. Clearly, component *Y*1 has observed states as shown earlier because it is comprised of the smallest measured trees in the stand, not because all 4- to 8-inch sugar maples grow as indicated by the boundary conditions. Equation (89)

$$dY1/dt = .09104\ Y1\ e^{-.01917\ Y1}$$

the single component equation, tells nothing about the relationship of trees in this class to trees in the larger size classes. On the other hand, the first equation of the system,

$$dY1/dt = .12145\ Y1\ e^{-.00957(Y1 + Y2 + Y3)} \qquad (94)$$

tells us that components $Y2$ and $Y3$ are inhibiting the growth of $Y1$. Furthermore, if all of $Y3$ were removed, one would expect a response in the growth of component $Y1$. The above equation will give a response because the inhibiting component will be smaller. Importantly, the amount of response is taken care of by the system itself.

Table 2. — *Deviations of predicted from observed sums of diameters on a plot on the Argonne Experimental Forest*

Year	D.b.h. class (inches)		
	4 to 8	8 to 12	12 to 16
	(Deviation in inches)		
1947	--	--	--
1949	0.1	0.0	-0.1
1950	.3	.5	.0
1951	.5	.7	.0
1954	- .1	.0	.0
1958	.3	- .1	.1
1962	- .8	- .7	- .1
1967	- .1	.0	.0
1968	.4	.4	.0

Although equation (93) is a realistic characterization of how forest system components interact, like all models, it involves simplifying assumptions. One could, at the expense of more computing time and a slight increase in data requirements, have a separate coefficient for each inhibiting component in each equation. Likewise, remeasurement data on other components of the forest system such as browse could easily be included as separate equations in such a multidimensional model. The upper limit of equations and unknown parameters appears determined only by available historical data and computing costs. (Determination of the six parameters in equation (93) required 7 seconds central processor time on a CDC 6600 computer.)

IMPLICATIONS

The preceding examples show how it is possible to obtain governing equation parameter estimates on the basis of observed system states. The approach used represents a constructive alternative to regression techniques for models expressed as differential equations. The attractiveness of the boundary value problem approach is clear in example 2, where parameters are determined in three nonlinear equations simultaneously.

The models and boundary conditions discussed here are of the simplest type. However, the same approach used in this paper is applicable to models involving time lags, time-dependent coefficients, partial differential equations and to boundary conditions involving linear and nonlinear relationships between the state variables.

In conclusion, this approach allows the scientist to more completely quantify his knowledge of forest development processes, to express his theories in terms of one or more governing differential equations, and to determine unknown numerical constants in the governing equations — three important steps in the study of forest dynamics.

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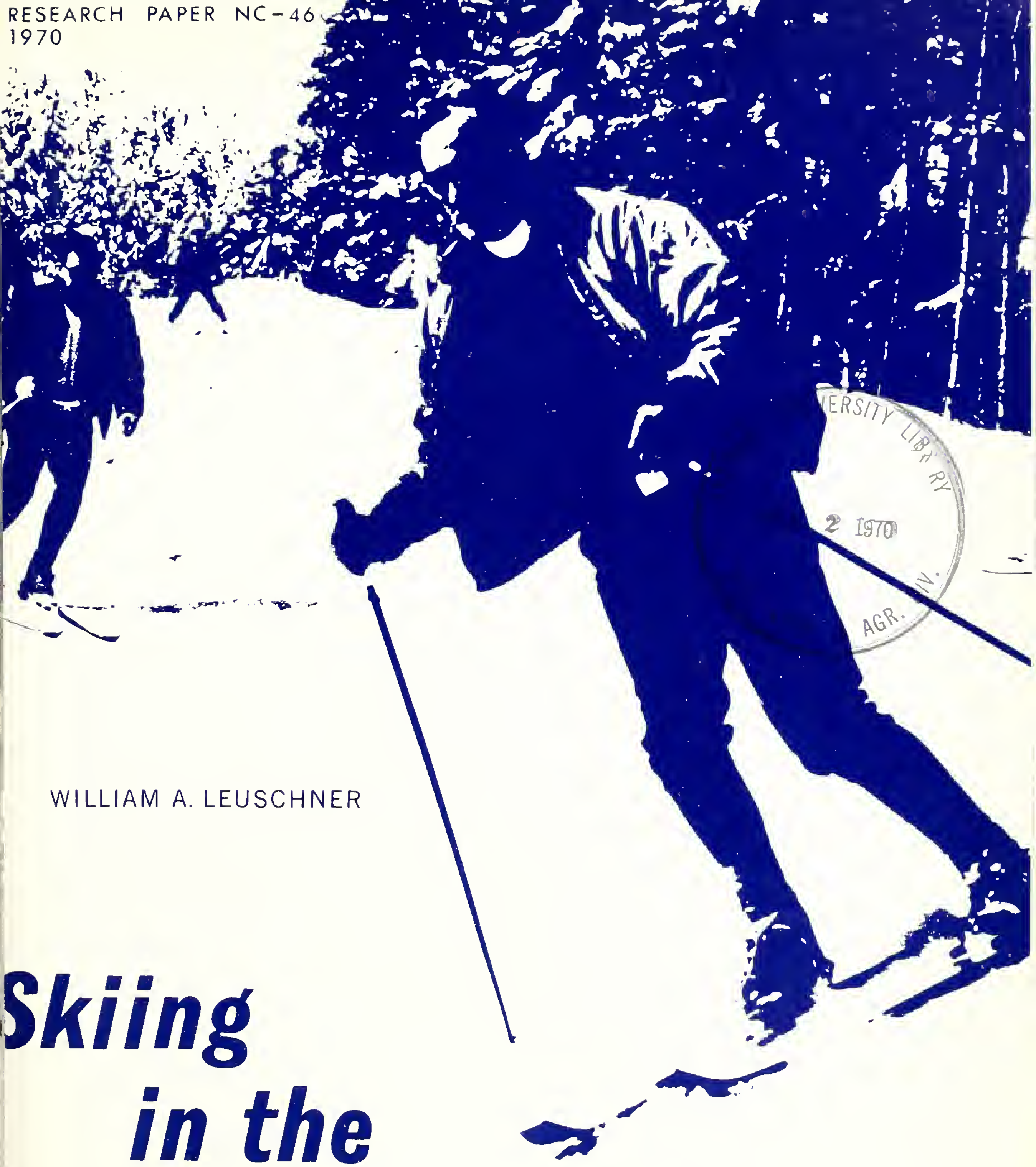

WILLIAM A. LEUSCHNER

Skiing in the Great Lake States:

THE INDUSTRY AND THE SKIER

NORTH CENTRAL FOREST EXPERIMENT STATION
FOREST SERVICE • U.S. DEPARTMENT OF AGRICULTURE

FOREWORD

Skiing has become increasingly popular in the Great Lakes Region in the past decade, heightening both private and public interest in building ski facilities. Information for investment decisions has been lacking due to the newness of the industry's expansion, the small size of ski areas, and the lack of organized data-gathering. Regional studies of the eastern and western skiing industries were made since 1960, but no comprehensive study was made in the Midwest. This report is intended to help fill that information gap by describing the industry and the skiers, and by analyzing the factors associated with financial success in ski-area operation.

The study covers the States of Illinois, Indiana, Michigan, Minnesota, and Wisconsin. Data were gathered over two skiing seasons. During the 1967-68 season about 84 percent of the Region's ski-area operators were interviewed. In 1968-69 the ski areas were again visited and a sample of skiers more than 12 years old taken. At the end of the 1968-69 season these skiers were sent questionnaires. In all, 147 ski-area operators were contacted and 2,350 usable skier questionnaires were received.

The study reported here was made by the North Central Forest Experiment Station, Forest Service, U.S. Department of Agriculture, which is solely responsible for the content of this report. The financial assistance of the Upper Great Lakes Regional Commission and the help of personnel from the Eastern Region, Forest Service, U.S. Department of Agriculture, are gratefully acknowledged.

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CONTENTS

The author is an Economist for the North Central Forest Experiment Station at the Station's headquarters laboratory in St. Paul, Minnesota (maintained in cooperation with the University of Minnesota).

Ski areas in the Great Lakes region range from small units with a few rope tows to year-round resort complexes. (Photo courtesy of the Michigan Tourist Council, Lansing, Mich.)

SKIING IN THE GREAT LAKE STATES:

William A. Leuschner

THE GREAT LAKES SKIING INDUSTRY

The earliest reported commercial ski area in h Great Lakes Region was in Minnesota in 92. Areas in Wisconsin and Michigan quickly o wed. The growth in numbers of areas cannot e traced precisely. Some 148 areas were open n 967, an estimated increase of 36 since 1960. n comparison, the western States had an in- r se of 45 areas between 1955 and 1964.

Ski areas in the Great Lakes Region differ r tly in size and facilities: some consist of a e rope tows and a "warming house," while t ers are year-round resort complexes with s ts worth several million dollars. Despite this ersity of types and sizes, describing the a erage" ski area provides a quick overall view f he Great Lakes skiing industry.

The average Great Lakes ski area has four e tows and one cable tow or lift; these serve cres of beginner slopes, 19 acres of interme- e slopes, and 12 acres of advanced slopes. n ermediate slopes predominate in both number n acreage regardless of the size of the ski a. The average vertical rise for ski areas is 4 feet. Those ski areas interviewed were open n average of 48 days during the season but the er, better equipped areas averaged 75 days.

The Great Lakes skiing industry is unique n everal ways. First, many ski areas are located e to cities, making skiing available to a large ion of the population almost daily. Further, a the areas provide night skiing (compared i 18 percent in the West in 1964), thereby ng the midwesterner an opportunity to ski f r a day's work. Perhaps even more impor- a is that the limited acreage of Great Lakes k areas allows intensive management of the ski es. The best possible snow conditions are ided for the longest possible time through the use of slope grooming and snowmaking equipment.

A commonly used measure of ski-area size or capacity is tow and lift capacity in vertical transport feet per hour (VTF). VTF is calculated by multiplying the vertical rise of each tow and lift by the number of skiers per hour it can transport (p. 37). Total reported VTF capacity in the region increased an average of 13 percent per year by 1967 from 64.3 million VTF in 1960. This can be compared with 24 percent for the 10.4 million cable-only VTF in 1960 (fig. 1).[1] Michigan maintained itself as leader in total capacity during this period, although Minnesota had the greatest percentage increase. This capacity growth was concentrated in large ski areas.

Estimated attendance increased an average of nearly 20 percent per year from 1960 to 1967 but fell off slightly to 2.3 million visits during the 1967-68 season (fig. 2).[2] Projections show a diminishing rate of growth (p. 30).

Ski areas in the two largest capacity classes,[3] constituting 45 percent of all areas, captured 78 percent of the estimated skier visits. The rope only businesses (45 percent of the ski areas) had only 15 percent of the skier visits.

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[1] "Cable" equipment in this report means any tow or lift not requiring the skier to grip a moving rope with his hands. "Combination" means both "rope" and "cable" equipment are present on a ski area. Cable-only VTF was measured separately because skiers preferred cable to rope facilities and because some people believe cable VTF can be measured more accurately than rope VTF.

[2] See page 38 for the difference between estimated and reported attendance.

[3] The ski areas were divided into five classes: rope tow only, and four combination rope and cable classes: (1) less than 300,000 VTF, (2) 300,000-699,999 VTF, (3) 700,000-1,499,999, and (4) 1,500,000 or more.

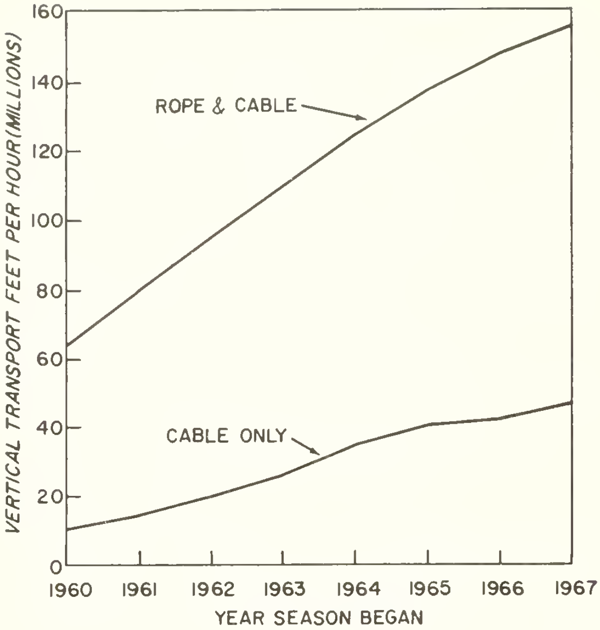

Figure 1.—Seven-year growth in ski area VTF capacity.

Capacity is one thing — use is another. If capacity is based on average number of days in the skiing season, the Great Lakes skiing industry is operating at 30 percent of its potential throughout the season, at 48 percent on weekends and holidays, and at 18 percent on weekdays.[4] However, if capacity is based on average skiable days, the season-long utilization is almost 47 percent, weekend-holiday utilization is 80 percent, and weekday utilization is 27 percent. In this case, the combination rope and cable areas use over 98 percent of their weekend-holiday capacity.

The overall utilization of western ski areas for 1963-64 was similar to the Great Lakes, but weekend-holiday utilization was much lower (49 percent). On the other hand, overall utilization in the eastern States in 1962-63 was 78 percent, with a remarkable 122 percent on weekends and holidays.

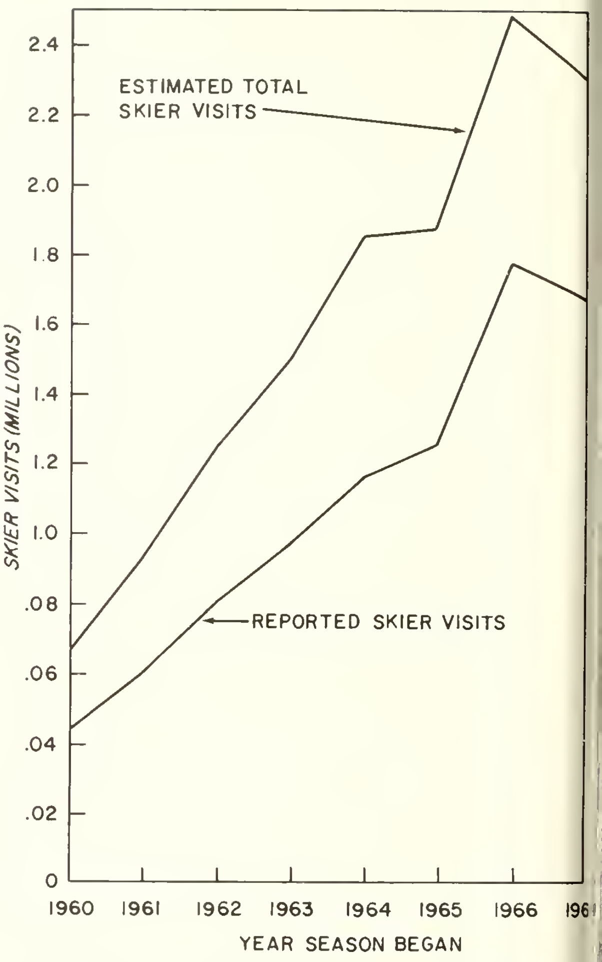

Figure 2.—Seven-year attendance trend.

Financial Trends and Market Structure

Financial data, and particularly those for t same ski areas for several years, are scarce all that were available are being presented. T few earlier studies show the skiing industry be relatively unprofitable (DuBois 1966; Fede Reserve Bank of Minneapolis 1964; Sisser 1960; Sno-Engineering 1967). These earlier fin ings were generally substantiated by the curre study. Although the average income stateme showed a loss before Federal income taxes tween 1963-64 and 1965-66, there was an u ward trend and profits were shown in the 1966- season (fig.3).

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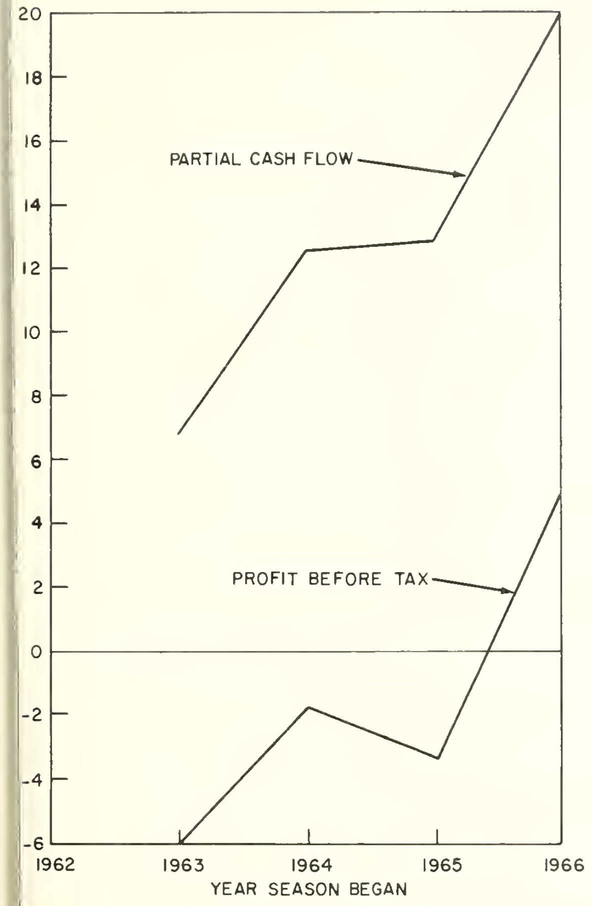
[4] See page 37 for assumptions used to calculate capacity.



igure 3.—Four-year profit trend of 27 areas (average per area).

All years would have appeared "profitable" i depreciation were not deducted.[5] This may e plain how some ski areas operate several years a a loss — they liquidate their investment.

These income statements were from 27 ski a eas for 4 consecutive years. This is a small b se from which to generalize for the industry a d the areas were not scientifically chosen. H wever, no other income statements were a ilable for 4 consecutive years.

A $1,200,000 growth in total assets — an a erage of $46,000 per ski area — was found for t same 4 years (fig. 4). Funds for this increase came from $1,400,000 of long- and short-term debt, despite a $100,000 decrease in the equity accounts, indicating that debt financing was available. Land accounted for $200,000 of increased assets, buildings and equipment for $800,000, and "working capital" for the remainder. Investment peaked in 1965-66.

These investment data are from 26 ski areas, 18 of which also furnished income statements for the preceding profitability discussion. These areas were also not chosen scientifically.

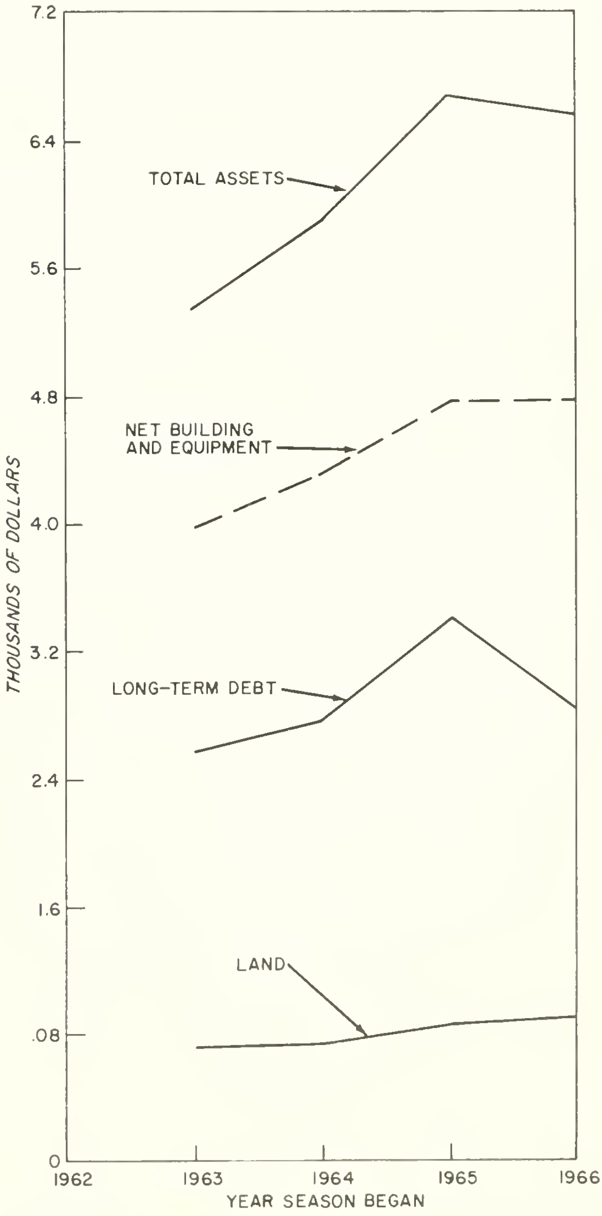


Figure 4.—Selected balance sheet accounts totaled for 26 ski areas.

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[5] In 3 years only 20 of 27 areas reported t ir depreciation; in the fourth only 19 reported it However, the above statements are correct b ause depreciation is added to profit and missi r depreciation could only make the total h her.

Both the general economic climate of the skiing industry and factors associated with individual ski area success should be examined in assessing the likelihood of financial sucess. The market structure is important because some special feature may affect the likelihood of successful investment.

The market structure is usually assessed by considering the number of buyers and sellers, the degree to which one seller can make his product appear different from others, and the difficulty of entering or leaving the industry.

Buyers and Sellers

Considering the estimated 349,100 skiers in 1968-69 and the short season, it becomes obvious there are many buyers and that no one of them can ski enough to affect an area's operations. The major part of the discussion will therefore be devoted to the sellers.

Ski areas (sellers) may be categorized by the type of skiing they offer: single-day, weekend, or vacation trips.[6]

Single-day skiers travel an average of 68 miles one way, a fact that helps define the market area for ski areas catering to them. A look at the distribution of ski areas shows that less than a dozen would probably compete for the same single-day skier.

The weekend skier travels an average of 237 miles one way, thereby increasing the number of ski areas competing for his patronage. However, a few large ski areas apparently get much of the weekend trade. These "meccas" are familiar to most midwest ski enthusiasts. Moreover, some smaller rural areas cater to local, single-day skiers and therefore do not compete strongly for the weekend skier. On the other hand, the urban-oriented areas offer less expensive alternatives so the larger areas probably consider them. It is probable that only a few ski areas are in strong, direct competition for the same skier.

The Great Lakes ski areas providing vacatio trips must compete with foreign and domest package plans, private clubs, and a host of resor accessible by air. If just the Great Lakes vacation market is considered, only a few resor compete. Nevertheless, these large resorts fa some competition because skiers may ski sever different areas during the week.

In summary, the number of sellers competin for the same skier appears to increase from th single-day market through the vacation marke but never becomes so large that the actions one ski area do not affect the business of th others. This is supported by the wide variet of tow and lift ticket prices (table 1).

Table 1.–*Summary of 1967-68 season prices*

Price category (dollars)	Ski areas	Percent of all areas reporting	Price category (dollars)	Ski areas	Percent o all area reportin
	Number			Number	
0.50	2	2	3.50	7	7
1.00	10	10	4.00	15	14
1.50	9	8	4.50	8	8
2.00	12	11	5.00	4	4
2.50	15	14	5.50	1	1
3.00	21	20	6.00	1	1

1/ This is the weighted average price of tickets throughout the 1967-68 season, including day tickets, season passes, package plan income allocated to ticket revenues, etc.

Product Differentiation

A ski area can exert more control over pric and protect a share of the market by differen ating its product. One way to differentiate by manipulating the quality of the skiing expe ience (amount and condition of snow, type tow and lift facilities, length of tow and l lines, and variability of slopes and trails). though statistical analysis did not show su quality factors to be related to financial succes skiers rated quality among the most importa reasons for going to a particular ski area. Despi the lack of substantiating evidence, it appea to be an important element of differentiatio

Differentiation by location can be importa particularly in the single-day market. In fa single-day skiers ranked location first amo reasons for choosing an area.

Advertising is unlikely to keep skiers goi to a particular area because they have am opportunity to become acquainted with all ar

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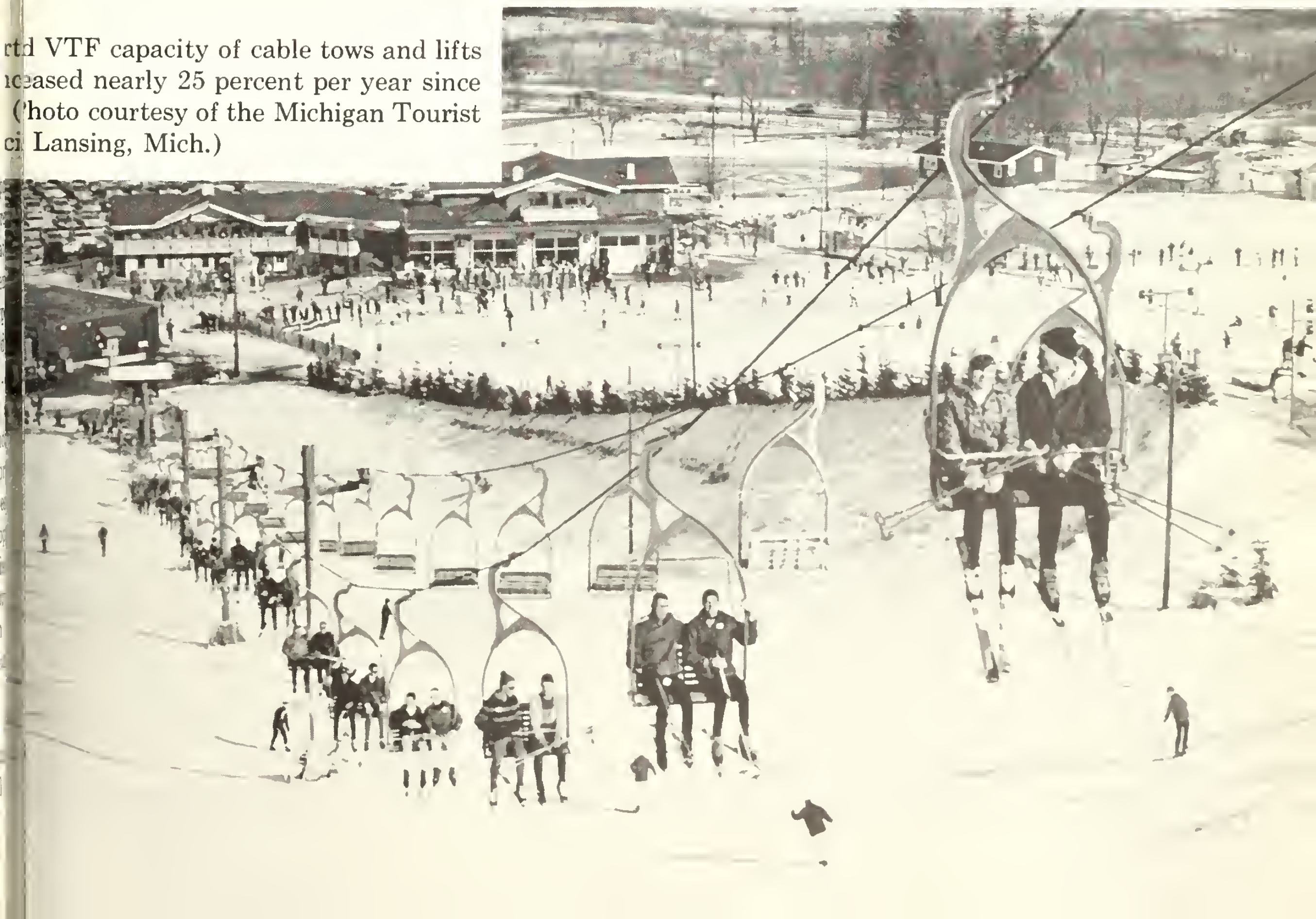
[6] A single-day trip is one where the skier traveled to the ski area and returned home the same day; a weekend trip is one where the skier is away from home at least 1 night but less than 4 nights for the primary purpose of skiing; a vacation trip is one where the skier is away from home 4 or more nights for the primary purpose of skiing.

The average Great Lakes ski area has fou rope tows, one cable tow or lift, and a max mum vertical rise of 242 feet. (Photo courtes of the Telemark Company, Hayward, Wis

rtd VTF capacity of cable tows and lifts
icreased nearly 25 percent per year since
(Photo courtesy of the Michigan Tourist
ci Lansing, Mich.)



in their single-day market, and probably in their weekend market. This is substantiated by skiers ranking advertising low among reasons for choosing an area. Such services as eating and drinking facilities are also ineffective methods of differentiation, as indicated by analysis of service factors and reported skier motives (p. 24).

In summary, there appears to be moderate to high product differentiation. This again is consistent with the observed variety of tow and lift ticket prices.

Entry and Exit

Ease of entry to the industry determines how readily the number of ski areas can increase, thereby diminishing industry profits. Ease of exit determines how readily overcapacity can decrease, thus helping to restore industry profits.

Barriers to entering the Great Lakes skiing industry are low. Product differentiation such as "good" location of established ski areas can be a barrier, but this may be ignored if the entrant thinks there is, or will be, excess skier demand. Higher costs than those paid by industry members for equipment, and all other necessary inputs could stop entry. However, equipment and other necessary inputs are readily available in the open market at no cost disadvantage although existing areas might have some slight advantage in skilled personnel. The one item most mentioned as a barrier is the possible high cost or unavailability of capital funds.

As previously seen, long-term debt has been available to the ski industry (fig. 4). The apparent low profitability of ski-area operation has been noted in print for at least a decade so there seems no reason to believe funds will be relatively more expensive for today's skiing industry entrant than for yesterday's. Further, the amount needed to enter is not an insurmountable barrier. This is illustrated by 16 areas having average total assets of $59,100 (none greater than $100,000), positive cash flows, but not currently showing profits.

A final barrier to entry would occur if the cost structure required a very large share of the market to reach an efficient cost per skier. This barrier is probably not important because some ski areas with attendance as low as 4,000 are making profits, although the profits may be slightly lower than those from higher attendance areas.

Low entry barriers are consistent with the observed overcapacity, low average profits, and fairly high number of entrants.

Leaving the industry is probably difficult because there are not many alternative uses for ski-area equipment, buildings, and land. This makes it difficult to liquidate the remaining investment or use the assets to enter another industry. In fact, some area operators reported they would sell if only they could find a buyer. Moderately high barriers to exit are consistent with reports of areas bankrupt but still operating, as well as observed overcapacity and low profits.

Summary

The investment possibilities are generally poor, due to past low profits, the constant threat of overcapacity, and moderate returns in view of the high risks. The vacation market, in particular, is a poor investment due to the decreasing cost of travel and the attractiveness of western and European ski areas. The weekend market is only slightly better because the new entrant must cope with existing ski areas with established names, underutilization of tows and lifts on weekdays, and the threat of increasing competition from eastern and western areas. An urban-oriented, single-day ski area is the best investment possibility for three reasons: (1) its product is made different from others by location (2) the high population density increases possibilities of utilizing capacity by special promotional activities (such as week-night schools) and (3) the lower number of ski areas available to any one skier allows greater market control Although profits appear to be increasing there is no guarantee that they will continue to do so or that industry overcapacity problems have been resolved.

Factors Associated with Financial Success

Financial success was measured by the rate of return on total investment (ROR) calculated from income statements and balance sheets o 27 ski areas for which both statements were available (p. 40). The inclusion of areas wa therefore based on data availability rather than a sampling procedure. Stepwise multiple linear regressions were used to relate ROR to 15 factors.

VTF capacity was the single factor most highly related to financial success. Rate of return on investment increased sharply with VTF to a point, but then leveled off (fig. 5). Changes in VTF capacity accounted for only 21 percent of the changes in financial success.

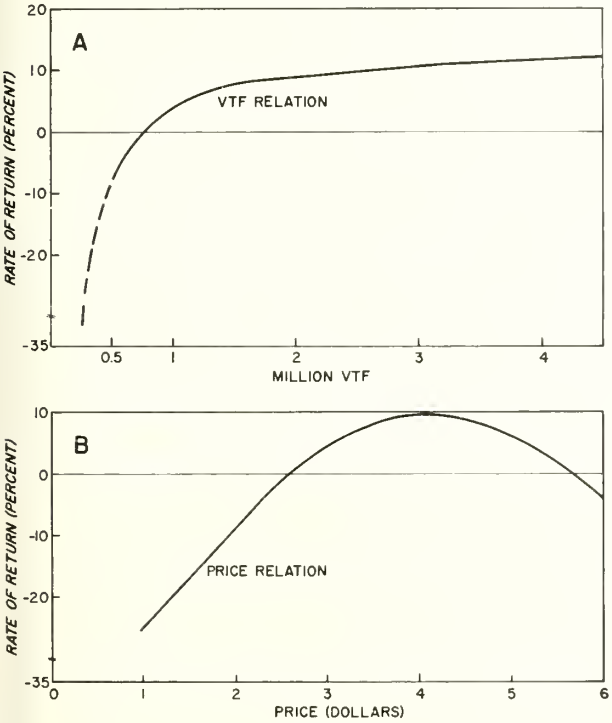


Figure 5.—Average relationship between VTF (A), average price (B) and rate of return for 27 ski areas. Solid line indicates range of independent variable data.

When VTF was deleted from the analysis the next best relationship was with average price.[7] Rate of return reached its highest when average price is just over $4.00, and then decreased as the average price rose (fig. 5). Changes in average price accounted for 36 percent of the changes in financial success, 15 percent more than VTF.

Even more of the changes in financial success were explained when ski areas were classified according to the price they charged. Areas with a price of more than $4.00 had a higher rate of return at lower VTF capacities, but their maximum rate of return was not as high as that for the $3.00 to $4.00 areas (fig. 6). Rate of return for the $3.00 to $4.00 class decreased after reaching a maximum, but tended to level off for the $4.00+ class.

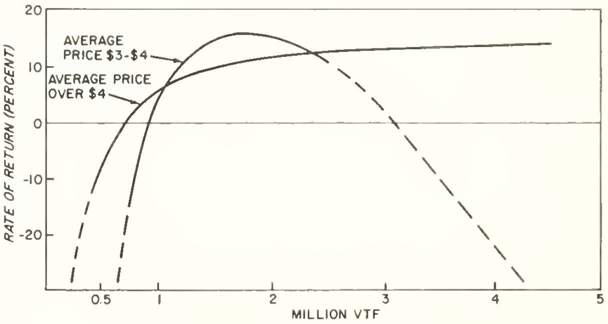


Figure 6.—Average relationship between VTF and rate of return when ski areas are classified by average price. Solid lines indicate range of independent variable data.

When only urban ski areas (those within 50 miles of a city of 50,000 or more people) were considered, several notable differences appeared (fig. 7).[8] First, rate of return increased with VTF to a maximum, and then decreased. Second, rate of return was highest at a lower price in the urban setting. Finally, the amount of snow on the ground (from Weather Bureau records) became associated with financial success for the first time. More than 90 percent of the changes in financial success were then explained by changes in VTF, average price, and amount of snow on the ground.

The analysis did not explain why the relationships were meaningful — it simply showed they existed for the 27 areas. Subject to the warnings on page 42 the following summary may be made:

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[7] This is the weighted average price of tickets throughout the 1967-68 season, including day tickets, season passes, package plan income allocated to ticket revenues, etc.

[8] The curves were calculated by assigning two variables their average value and varying the value of the third.

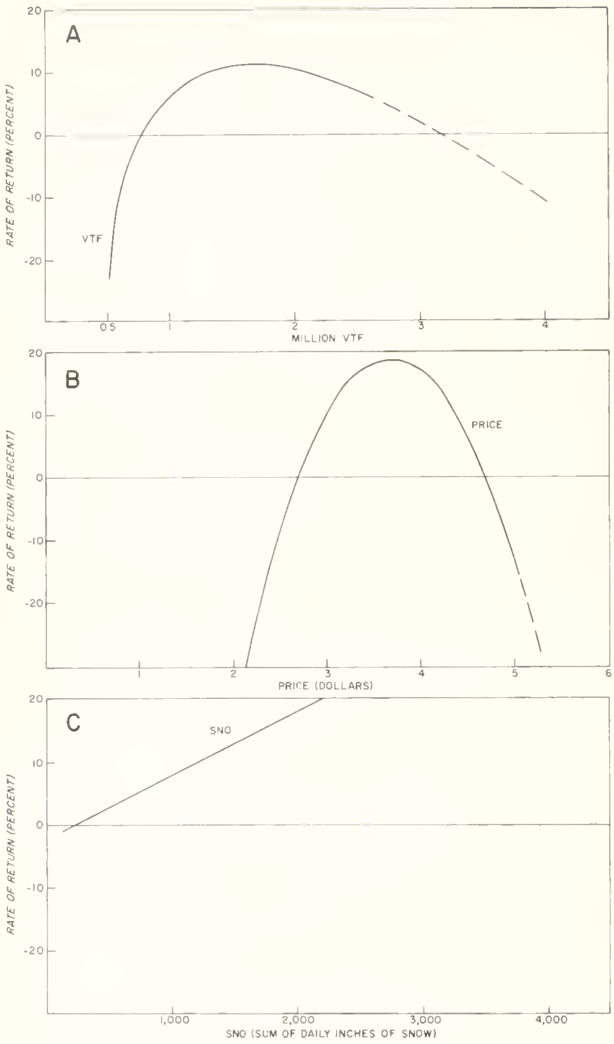

Figure 7.—Average relationships between VTF (A), average price (B), snow on the ground (C) and rate of return for urban ski areas. Solid lines indicate range of independent variable data.

1. The best recorded rate of return for the 27 areas was about 25 percent before Federal income tax. However, the average rate of return was much lower — possibly too low for the risks involved in ski industry investment.

2. Ski areas within 50 miles of a city of 50,000 or more people had a higher maximum rate of return than others; therefore, the distance from population centers should be considered when investing.

3. Rate of return varied with ticket price; therefore, each area should experiment to find its best price, starting at about $3.65 (1967-68 prices). Because this is an average price, weekend ticket prices can be higher. Rural areas may charge slightly higher prices.

4. Rate of return also varied with VTF capacity. About 1-2/3 million VTF might be the starting point for determining the best VTF for an area. Rural areas could have slightly more VTF capacity. Opening a second area rather than enlarging the present one should be considered if increased investment is planned.

Several points are important although not brought out by the analysis. First, there are few skiable days in a year and ski-area assets probably lie dormant more than two-thirds o the time. Uses for these assets on a year-round basis should be sought (Jackson 1969, p. 40).

Second, managerial ability has not been measured. But, it appears that, particularly in the smaller areas, ski area operators do not turn their full talents to management even during the season. Regardless of the reasons, this may result in failure to find the best solutions t ski-area problems.

Lack of managerial ability may be reflected in the lack of records to guide decisions. Man ski areas did not have the most basic accountin reports (and no records by cost center, such a restaurant, bar, ski rental, and slopes), nor di they even know their annual attendance. On operator responded, "We just count the mone in the cash drawer at the end of the day. there is more than we started with we are happy."

THE GREAT LAKES SKIER

Certain characteristics appear common t skiers in all regions. Skiers are predominantly young group — 55 percent of the Great Lake skiers are less than 23 years old. Compariso with age distributions from the eastern and western ski studies indicates that either the Grea Lakes skier is younger or the national skie population is becoming younger (table 2). younger national skier population is consisten with the apparent trend for skiers to have fewe years of education. Further, at least one surve of skiers in many States reported that averag age is decreasing (Pitts 1968).

More than half the skiers in the Great Lakes region are less than 23 years old. (Photo courtesy of the Rib Mountain Ski Area, Wausau, Wis.)

nked quality of slopes as one of the ry reasons for skiing at a particular Photo courtesy of the Michigan Tour- ucil, Lansing, Mich.)

A second general observation is that most skiers have been skiing only a few years. Throughout the United States one-fifth to one-third have been skiing 1 to 2 years and almost three-fourths have been skiing 8 years or less (table 2). The Great Lakes skier fits this description.

Table 2.—*Skier age, education, and years skied, by region*
(In percent of skier population)

Region	Skier age	Education beyond 12th grade	Years skied	
	13-18		1 or 2	8 or less
Great Lakes	37.0	54.8	27.8	75.0
Western1/ (1964-65)	25.0	65.0	31.0	66.0
Eastern2/ (1962-63)	19.6	83.8	20.0	70.0

1/ Source: Herrington (1967, p. 71, 78, 84).
2/ Source: Sno-Engineering (1965, p. 11, 43, 44).

About three-fourths of Great Lakes skiers report their occupation as either student or professional. The proportion of skiers who are students in the Great Lakes is much higher than in other regions, although students constitute the largest group of skiers in all regions.

The Great Lakes skier, like skiers nationwide, has a higher than average income. His median family income ($12,168) exceeds the median for the North Central States by more than $3,000 (Census Bureau 1966).[9]

Thus far the Great Lakes skier seems much like all other United States skiers. But he is different in some ways. For example, the resident Great Lakes skier travels farther than his western cousin, especially on weekend and vacation trips. The difference may be due to highway improvements in the years between the studies, or in the case of vacation trips, to the midwesterner traveling east or west to ski.

More importantly, the Great Lakes skier skis only half as many days as the western skier, and one-third as many as the eastern skier. Therefore, he has fewer opportunities to generate income for ski areas.

More than 80 percent of Great Lakes ski trips are single-day or weekend trips, indicating a limited market for vacation ski trips. Over 95 percent of all trips are by automobile or bus, indicating the importance of good roads to the Great Lakes ski industry.

Michigan residents accounted for 43 percent of the total days skied in the Great Lakes Region. In addition to accounting for more than 90 percent of the days skied in Michigan, they also accounted for two-thirds of those skied in Indiana, and one-fourth of those skied in Wisconsin. The Michigan skier, then, is an important element not only in his own State's market but in other States as well. The same holds true for the Wisconsin resident skiing in Illinois.

Skiers' Preferences

Day skiers rank proximity as the most important reason for skiing at a particular area, which helps explain the greater financial success of urban versus rural ski areas.

The physical quality of the ski slopes (not including snow quality) was ranked second by the day skier and first by skiers planning weekend or vacation trips. Cable facilities are necessary but clearly of lesser importance. The day skier next considers low ticket prices and the area's reputation with other skiers, while the weekend-vacation skier considers the area's reputation, the expected amount of crowding, and after-ski entertainment.

Skiers gave the above answers in response to questions about why they chose particular ski areas.

On the other hand, operators of ski areas were asked what factors limited attendance at their areas. Weather variables were ranked first by operators as limiting attendance. Inadequate tow and lift capacity and not enough skiable area, both of which relate to crowdedness, were ranked next. Inadequate service facilities and overnight accommodations were ranked fourth and fifth, followed by skier preference for cable

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[9] Median family income was calculated assuming an even distribution of skiers throughout the $10,000 to $14,999 income class.

versus rope facilities. It is interesting that operators ranked crowdedness variables higher than skiers did, and cable facilities lower.[10]

SPENDING AND ITS IMPACT

Great Lakes skiers spent an estimated $65 million on their sport in the 1968-69 season. More than 40 percent of this (an estimated $27 million) went for equipment, clothing, and other items purchased while not on a trip. Three-quarters of all skiers reported this type of purchase. On trips, however, the estimated average expenditure per skier per day was about $17; the amount increased from $12 for single-day trips through $22 for weekend trips, to $28 for vacation trips.

On the average, for all trips, the Great Lakes skier spends about 25 percent of his money on tow and lift tickets, 30 percent on transportation and meals, 20 percent on lodging and after-ski entertainment, 5 percent on equipment rental and repairs, and 5 percent on package plans. The remaining expenditure is on other items.

Ski areas, however, do not receive all skier expenditures – even those made while on a trip. Transportation expenditures go to someone else and the skier may also buy lodgings, meals, after-ski entertainment, and other items away from the ski area. The ski area can count on receiving as little as half of day-skier expenditures and only about a quarter of weekend and vacation expenditures. Attention to attractive food service, lodgings, and after-ski entertainment is important if increased sales to existing customers are the goal.

Available data on ski-area expenditures show wages and salaries are by far the largest single item. Goods sold in eating and drinking places are next, followed by snowmaking operating expenses, and then by goods sold in the ski shop (table 3). Not all ski areas reported complete data and the accuracy of these rankings is subject to qualifications.

Table 3.–*Selected annual expenditures per ski area, 1967-68 season*

Expenditure	Average per area	Areas in average
	Dollars	Number
Cost of goods sold:		
Eating and drinking place	13,900	66
Ski shop	7,300	45
Operating expenses:		
Snowmaking equipment	11,400	45
Tow and lift maintenance	4,700	91
Other:		
Advertising	4,300	81
Wages and salaries	27,600	96
Interest	4,500	15

The 89 ski areas that reported employment data employed a total of 2,755 people during the year, but 2,665 of these were seasonal. The seasonal employee averaged just over 4 man-days work per week and was paid an average of $1.90 an hour. An estimated 95 percent of all employees were local residents.

Those who look to ski areas to improve local economies want to sell more goods and services to outsiders (in this case, skiers entering, spending, and then leaving). They believe that the more money spent within the region, the better chance for creating additional jobs and opportunities for new investment. This is known as the "multiplier effect": the larger the "multiplier," the farther an expenditure goes before leaving the local economy. The degree to which the multiplier works depends, in part, on the structure of the local economy and the type of expenditures made. This is why statements about impact must be tied to particular local economies and why specific statements on economic impact are not made here.

How much of the Great Lakes skiers' expentures are likely to find their way into local, rural economies – those most likely to consider ski areas as an aid to growth?

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[10] The reader should use caution in interpreting these results. For example, the low price of tow and lift tickets may not attract a skier but possibly a high price will drive him away. Also, advertising may not convince a skier to attend a particular ski area, but it may be important to inform him of the days and hours the area is open or of events of special interest, such as discount evening ski schools. Finally, these results do not show why the skier goes skiing on a particular day, but why he goes to a particular ski area.

As already noted Great Lakes skiers in 1968-69 spent $27 million for equipment *while not on trips*. Similarly, only part of the $6.6 million spent on transportation found its way into local economies. Although 85 percent of the trips were by auto, local economies probably received only part of the gasoline expenditure.[11] They got little or nothing from other forms of transportation. Probably no more than a third or a half of the transportation expenditure aids the local economy. Subtracting $27 million and $3.3 million from $65 million leaves a maximum of 55 percent available for local economies.

This does not take into account those skiers who are local residents and do not bring in "new" money. If we assume a 100-mile radius for a local economy, and that skiing expenditures by residents would have been made locally on something else, probably only $16,500,000 of new money is contributed.[12]

Employment and Income

At least one man is required for each cable tow and two men for each lift. Most areas employ additional men for snowmaking and slope grooming. Fuel expenditures for snowmaking and tows and lifts probably have a small effect on local employment and personal income if motors are electric. However, purchase of fuel can cause an additional round of expenditure if the motors are diesel. Aside from labor, most maintenance expenditures are probably outside the local economy.

Common carrier transportation expenditures almost certainly have little local effect. Gasoline expenditures for private autos seem unlikely to create new jobs but may increase the income of the local service station owner and fue distributor.

Meal expenditures can have several effects Food service at the ski area can be anywhere from cafeteria style, requiring a few busboy and a short-order cook, to a formal dining roo requiring waitresses and a chef who may b "imported" from outside the region. The expenditure for unprepared food can make an additional stop within the region if there is local wholesaler, or it can leave the regio immediately.

New restaurant-connected jobs and incom may be created if weekend and vacation guest eat away from the ski area. On the other han this may simply result in fuller utilization existing employees. Of course, the owner's i come would presumably increase. A similar analysis applies to after-ski entertainment, althoug live entertainers are probably imported fro outside the region. Lodging expenditures ma be beneficial if they provide jobs for unskille labor.

Equipment purchased on the ski trip provid jobs for salesmen but the dollars spent on probably leave the region immediately to pa for the merchandise sold. Ski schools may impo a director and possibly a few instructors. Par time instructors may be local residents, there increasing income but not creating many ne jobs.

Although the preceding speculative analys indicates that ski areas do create jobs and ha a generally favorable effect on income, they a not by any means a cure-all for local econom problems. And, in view of the other econom deficiencies already mentioned, particularly the rural setting, all possible alternatives shou be sought out and evaluated before choosing ski area as a means of relieving a depressed gional economy. Ski-area investment for regio development may be sound in some cases: example, if a region is particularly well endow with snow and skiable terrain and already h a well established summer recreation indust that winter activity could supplement.

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[11] Respondents' expenditure estimates were used for all transportation costs except auto. Auto cost was estimated at 4.083 cents per mile average running cost (gasoline, oil, normal preventative and repair maintenance, washing, greasing, and tires). (Slocum Publishing Company 1968.)

[12] The average Great Lakes skier skis 4.9 days in the study area but 3.4 of these are single-day trips. About 75 percent of the single-day skiers come from within a 100-mile radius of the ski area. Now, (25 percent x 3.4 days x $10.16/day + 1.1 days x $18.07/day + 0.4 days x $22.19/day) x 349,100 skiers = $13,100,000 + $3,300,000 transportation = $16,400,000.

SUMMARY

Financial information available for this study ndicates that private investment in new ski reas is not particularly profitable, although exeptions do exist. The future may seem brighter ecause attendance and profits have shown an pward trend, but the susceptibility of the inlustry to overcapacity and its dependency on he weather create enough risk to raise serious oubt that current average returns are high nough.

Should investment be made in existing ski reas? There are several reasons why this might e prudent. An operator may profitably increase TF, as indicated in the analysis of factors afecting financial success. He may, of course, relace wornout equipment or upgrade his tows nd lifts. Or he may expand his whole operation nd try to promote off-season use of his ski area.

This study indicates low returns to local conomies, implying public investment for their elief should be made cautiously. If we assume hat the public may invest, either to help local conomies or to supply recreation activities, here are two important points to be considered. irst, the amount of overcapacity in the region hould be examined before investing funds or nd for a new ski area or increasing the capaity at an existing one. Care must be taken not o create so much overcapacity that either the ew or the old ski areas, or both, are unprofitle. Second, it may be better to give higher riority to assisting areas already operating if will help them became more profitable, rather han create new areas that dilute profits.

The preceding analyses were intended to highlight items likely to be of interest to most investors. More detailed information can be found in the Appendix.

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APPENDIX

The rows and columns of the tables may not add to their totals due to rounding errors and the presence of a dash (—) in a table indicates the information was not available or no answer was possible.

Ski Area Operator Survey, 1967-68 (Tables 4-20)

State and commercial directories listed 192 ski areas in the study area but 15 of these were excluded from the study because they were not open to the public, made no charge for skiing, or did not provide tows or lifts. In addition, nine other ski areas were out of business. Two area operators refused interviews and 19 could not be found at the end of the season. The following number of interviews was obtained in each State (noncontacts are areas that either refused to be interviewed or could not be contacted):

State	*Original number of areas*	*Noncontacts*	*Interviews obtained*
Illinois	7	0	7
Indiana	4	0	4
Michigan	73	13	60
Minnesota	34	0	34
Wisconsin	50	8	42
Total	168	21	147

Some operators did not answer all questions, either because they didn't have the answers or considered them confidential. In addition, some areas were not open during the 1967-68 season, so their information was not used in some tabulations. The number of ski areas in each tabulation is indicated.

Table 4.—*Number of visits reported by ski area, by State and ski season*
(In thousands)

State	1967-1968	1966-1967	1965-1966	1964-1965	1963-1964	1962-1963	1961-1962	1960-1961
Michigan and Indiana	881.6	918.5	691.7	653.0	593.2	488.1	356.8	273.0
Minnesota	387.0	420.9	274.6	219.6	139.5	98.9	68.8	61.5
Wisconsin and Illinois	400.7	438.5	284.2	293.6	236.4	230.4	183.8	112.7
Total	1,669.3	1,777.9	1,250.5	1,166.2	969.1	817.4	609.4	447.2
Number of areas reporting	117	117	109	102	94	92	86	85

Table 5.—*Estimated number of ski areas and visits, by State and ski season*

State	1967-1968	1966-1967	1965-1966	1964-1965	1963-1964	1962-1963	1961-1962	1960-1961
NUMBER OF SKI AREAS								
Michigan and Indiana	71	71	73	75	68	60	54	49
Minnesota	30	34	31	30	28	25	24	23
Wisconsin and Illinois	47	54	55	47	46	47	46	40
Total	148	159	159	152	142	132	124	112
THOUSANDS OF VISITS								
Michigan and Indiana	1,254	1,322	1,011	1,011	874	705	515	383
Minnesota	459	494	345	321	232	179	130	106
Wisconsin and Illinois	605	672	513	510	388	357	284	180
Total	2,318	2,488	1,866	1,842	1,494	1,241	930	669

Table 6.—*Number and VTF capacity of reporting ski areas, by season and VTF size class*

Season	Total		Rope only		Combination rope and cable							
					Size class of ski area (rated in VTF)							
					Less than 300 M		300 M to 699 M		700 M to 1,499 M		1,500 M or more	
	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF
1967-1968	140	155.5	69	48.2	2	0.2	14	7.4	30	32.6	25	67.1
1966-1967	142	148.7	71	48.8	3	.5	17	9.2	29	31.1	22	59.1
1965-1966	139	138.2	71	42.3	3	.5	16	8.6	27	29.5	22	57.3
1964-1965	132	124.2	65	39.3	5	.9	16	8.2	28	29.9	18	45.9
1963-1964	122	110.2	61	34.5	6	1.2	14	7.5	26	27.8	15	39.2
1962-1963	114	95.3	57	30.6	6	1.2	16	7.9	25	27.1	10	28.5
1961-1962	108	80.3	53	26.5	6	1.1	19	9.2	21	20.0	9	23.5
1960-1961	92	64.3	47	22.5	11	2.2	13	6.1	13	14.4	8	19.1

Table 7.—*Average annual compound percentage of increase in visits and VTF capacity, by State, 1960-61 to 1967-68*

State	Visits		VTF capacity	
	Reported	Estimated total	Total	Cable only
Michigan and Indiana	18.2	18.5	14.4	21.1
Minnesota	30.1	23.3	19.4	43.2
Wisconsin and Illinois	19.9	18.9	10.9	23.6
Total	20.7	19.4	13.4	24.2

Table 8.—*Number and VTF capacity of reporting ski areas, by season and State*

Season	Total		Illinois		Indiana		Michigan		Minnesota		Wisconsin	
	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF
1967-1968	140	155.5	5	3.0	3	2.7	60	63.4	32	38.2	40	48.2
1966-1967	142	148.7	5	3.0	3	2.7	59	59.4	33	36.9	42	46.7
1965-1966	139	138.2	5	3.0	3	2.7	59	58.1	32	29.5	40	44.9
1964-1965	132	124.2	4	2.8	4	3.7	58	50.6	29	25.1	37	42.0
1963-1964	122	110.2	4	2.6	4	3.4	54	44.1	25	20.2	35	39.9
1962-1963	114	95.3	4	2.6	2	1.8	51	37.0	23	16.9	34	37.0
1961-1962	106	80.3	4	2.6	--	--	48	30.6	21	13.1	33	34.0
1960-1961	92	64.3	3	2.4	--	--	42	25.8	20	11.0	27	25.1

Table 9.—*Number and VTF capacity of reporting ski areas, cable tows and lifts only, by season and State* [1]

Season	Total		Illinois		Indiana		Michigan		Minnesota		Wisconsin	
	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF
1967-1968	71	47.3	2	0.7	1	0.2	33	26.6	17	8.6	18	11.2
1966-1967	71	41.9	2	.7	1	.2	32	23.1	18	8.4	18	9.5
1965-1966	68	40.9	2	.7	1	.2	32	22.7	17	8.0	16	9.3
1964-1965	67	34.1	2	.7	2	.7	32	19.4	15	5.4	16	7.9
1963-1964	61	26.1	2	.7	2	.7	28	14.9	14	3.8	15	6.0
1962-1963	57	21.1	2	.7	1	.6	26	12.4	13	2.0	15	5.4
1961-1962	53	15.0	2	.7	--	--	24	8.5	12	1.4	15	4.4
1960-1961	45	10.4	2	.6	--	--	22	7.0	11	.7	10	2.1

1/ Capacities are for cable facilities only. If an area has both rope and cable facilities, the rope tow capacity is excluded.

Table 10.—*Profile of ski areas showing average values of selected characteristics, by VTF size class, 1967-68 season*

Characteristic	Number of areas in sample	Unit of measure	Total	Rope only	Combination rope and cable			
					Size class of ski area (rated in VTF)			
					Less than 300 M	300 M to 699 M	700 M to 1,499 M	1,500 M or more
All skier visits	117	Number	14,300	4,700	500	9,400	28,800	31,900
Weekend and holiday skier visits	117	Number	9,200	2,800	500	7,200	17,200	22,600
Day ticket cost	107	Dollars	2.85	2.02	4.00	3.52	3.84	3.84
Rope tows	147	Number	3.8	3.6	1.0	2.9	4.4	4.6
Cable tows and lifts	147	Number	1.0	--	1.0	1.4	2.1	2.7
Capacity of cable tows and lifts	147	M VTF/hr.	321.5	--	97.1	276.1	538.2	1,082.2
Slope grooming in season	123	Man-days	58.5	31.2	--	45.8	77.0	139.7
Snowmaking guns1/	83	Number	15.7	9.3	4.0	10.3	18.0	25.3
Novice ski slopes	144	Number	1.9	1.5	1.0	1.7	2.3	2.8
	141	Acres	7.1	3.9	2.4	6.2	8.5	15.0
Intermediate ski slopes	142	Number	2.8	2.3	1.5	2.9	3.0	3.9
	142	Acres	19.4	10.5	12.4	21.1	27.4	36.2
Advanced ski slopes	143	Number	1.9	1.2	1.5	2.0	2.2	3.3
	129	Acres	11.8	6.0	19.6	10.0	17.3	19.6
Maximum vertical rise	144	Feet	241.7	183.2	197.5	246.7	287.6	365.3
Maximum slope length	141	Feet	1,800	1,200	2,100	2,600	2,400	2,700
Restaurant seats2/	143	Number	163.3	90.0	50.0	110.4	217.6	354.1
Beds lodging2/	144	Number	27.4	4.6	--	10.8	42.0	87.9
Bar capacity2/	138	Persons	67.5	28.2	--	47.1	100.7	163.9
Children in organized ski group	136	Number	699	384	100	154	1,285	1,490
Certified ski instructors	145	Number	2.9	1.1	1.0	2.1	4.8	6.3
Advertising expenditure	112	Dollars	3,093	780	100	1,290	6,548	7,594
Days in skiing season	127	Number	75.3	64.1	70.0	78.0	82.4	91.0
Skiable days in skiing season	143	Number	47.8	27.7	10.0	48.2	75.9	75.6

1/ Average only for ski areas that make snow

2/ Includes facilities at ski area and those within walking distance

Table 11.—*Number and VTF capacity of reporting ski areas, by State and VTF size class, 1967-68 season*

State	Total		Rope only		Combination rope and cable									
					Size class of ski area (rated in VTF)									
					Total all combinations		Less than 300 M		300 M to 699 M		700 M to 1,499 M		1,500 M or more	
	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF	No. of areas	Million VTF
Illinois	5	3.0	3	0.7	2	2.3	--	--	1	0.5	--	--	1	1.8
Indiana	3	2.7	2	1.7	1	1.0	--	--	--	--	1	1.0	--	--
Michigan	60	63.4	27	17.1	33	46.4	1	0.1	5	2.8	16	17.2	11	26.3
Minnesota	32	38.2	15	15.7	17	22.5	--	--	4	2.1	7	7.4	6	13.0
Wisconsin	40	48.2	22	13.0	18	35.2	1	.1	4	2.1	6	7.0	7	26.0
Total	140	155.5	69	48.2	71	107.3	2	.2	14	7.4	30	32.6	25	67.1

Table 12.—*Use of tow and lift capacity, by weekend-holiday and weekday, 1967-68 season*[1]

75-DAY SEASON

Use	Total			Rope-only areas			Combination rope and cable areas		
	Number of visits		Capacity used	Number of visits		Capacity used	Number of visits		Capacity used
	Actual	Potential		Actual	Potential		Actual	Potential	
	Thousands	Thousands	Percent	Thousands	Thousands	Percent	Thousands	Thousands	Percent
Weekend-holiday	1,073.1	2,233.1	48.1	180.1	717.5	25.1	893.0	1,515.6	58.9
Weekday	596.2	3,349.8	17.8	124.7	1,076.3	11.6	471.5	2,273.5	20.7
Total	1,669.3	5,582.9	29.9	304.8	1,793.8	17.0	1,364.5	3,789.1	36.0
48-DAY SEASON									
Weekend-holiday	1,073.1	1,339.8	80.1	180.1	430.5	41.8	893.0	909.3	98.2
Weekday	596.2	2,232.9	26.7	124.7	717.4	17.4	471.5	1,515.5	31.1
Total	1,669.3	3,572.7	46.7	304.8	1,147.9	26.6	1,364.5	2,424.8	56.3

1/ See section on Estimating Utilization of Ski-Area Capacity for difference between 75- and 48-day season.

Table 13.—*Thousands of visits reported by ski areas, by State and VTF size class, 1967-68 season*

State	Total	Rope only	Combination rope and cable				
			Size class of ski area (rated in VTF)				
			Total all combinations	Less than 300 M	300 M to 699 M	700 M to 1,499 M	1,500 M or more
Michigan and Indiana	881.6	136.7	744.9	1.0	40.9	499.6	243.4
Minnesota	387.0	79.5	307.5	--	45.5	157.7	104.3
Wisconsin and Illinois	400.7	70.6	330.1	--	25.8	45.3	259.0
Total	1,669.3	286.8	1,382.5	1.0	112.2	662.6	606.7
Number of areas reporting	117	61	56	2	12	23	19

Table 14.—*Estimated total visits to ski areas, by State and VTF size class, 1967-68 season (Thousands of visits)*

State	Total	Rope only	Combination rope and cable				
			Size class of ski area (rated in VTF)				
			Total all combinations	Less than 300 M	300 M to 699 M	700 M to 1,499 M	1,500 M or more
Michigan and Indiana	1,254.1	150.0	1,104.1	1.0	60.0	616.9	426.2
Minnesota	459.3	100.7	358.6	--	45.5	170.4	142.7
Wisconsin and Illinois	604.6	103.8	500.8	--	40.9	55.1	404.8
Total	2,318.0	354.5	1,963.5	1.0	146.4	842.4	973.7
Estimated number of areas open	148	65	83	2	15	33	33

Table 15.—*Employment and wages by size class of ski area, 1967-68 season*

EMPLOYMENT (MAN-HOURS)

Size class[1]	Areas reporting	Yearly		Weekly	
		Total	Average per area	Total	Average per area
Rated in VTF	Number				
Rope only	37	174,540	4,720	15,560	420
300 M to 699 M	8	42,090	5,260	3,570	450
700 M to 1,499 M	26	566,270	21,400	40,230	1,550
1,500 M or more	18	465,540	25,860	30,050	1,670
Total	89	1,248,440	14,030	89,410	1,000
WAGES (DOLLARS)					
Rope only	37	303,440	8,200	27,750	750
300 M to 699 M	8	73,940	9,240	6,140	760
700 M to 1,499 M	26	1,149,100	44,190	77,020	2,960
1,500 M or more	18	1,094,860	60,820	59,400	3,300
Total	89	2,621,340	29,450	170,310	1,913

[1] The "less than 300 M" VTF size class was omitted because less than three ski areas reported.

Table 16.—*Number of different employees, mean man-days employment, and mean hourly wage, by VTF size class, 1967-68 season*

Ski-area size class[1] (rated in VTF)	Areas reporting	Yearly total			Ski season		
		Persons employed	Time employed	Mean hourly wage	Persons employed	Time employed	Mean hourly wage
	Number	Number	Man-days/year[2]	Dollars	Number	Man-days/week[2]	Dollars
Rope only	37	691	31.6	1.75	644	3.0	1.78
300 M to 699 M	8	109	48.2	1.76	103	4.3	1.72
700 M to 1,499 M	26	1,090	64.9	2.03	1,067	4.7	1.91
1,500 M or more	18	865	67.2	2.35	851	4.4	1.98
Total	89	2,755	56.6	2.10	2,665	4.2	1.90

[1] The "less than 300 M" VTF size class was omitted because less than three ski areas reported.
[2] Based on an 8-hour man-day.

Table 17.—*Annual reported ski area expenditures by VTF size class,1967-68 season*

Expenditure	Total		Rope only		Combination rope and cable areas					
					Ski-area size class[1] (rated in VTF)					
					300 M to 699 M		700 M to 1,499 M		1,500 M or more	
	Areas reporting	Expenditure	Areas reporting	Expenditure	Areas reporting	Expenditure	Areas reporting	Expenditure	Areas reporting	Expenditure
	Number	Thousand dollars	Number	Thousand dollars	Number	Thousand dollars	Number	Thousand dollars	Number	Thousand dollars
Cost of goods sold:										
Eating and drinking	66	914.3	30	273.5	7	97.9	17	262.9	12	280.0
Ski shop	45	326.4	13	72.2	4	10.6	16	80.3	12	163.3
Operating expenses:										
Snowmaking equipment	45	511.2	15	75.9	4	15.3	14	211.2	12	208.8
Tow and lift maintenance	91	430.0	52	54.5	6	8.4	18	134.9	15	232.2
Other:										
Advertising	81	346.4	31	49.5	10	12.9	23	150.6	17	133.2
Wages and salariea	96	2,646.0	44	344.1	8	73.9	27	1,163.1	17	1,064.9

[1] The "less than 300 M" VTF size class was omitted because less than three ski areas reported.

Table 18.—*Areas reporting use of snowmaking equipment, by State and VTF size class, 1967-68 season*

State	Total		Rope only		Combination rope and cable areas							
					Ski-area size class (rated in VTF)							
					Less than 300 M		300 M to 699 M		700 M to 1,499 M		1,500 M or more	
	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent	Number	Percent
Illinois	5	100	3	100	0	--	1	100	0	--	1	100
Indiana	3	100	2	100	0	--	0	--	1	100	0	--
Michigan	33	55	8	30	--	--	3	60	13	81	9	82
Minnesota	17	53	4	27	0	--	3	75	5	71	4	67
Wisconsin	19	48	6	27	1	100	3	75	3	50	6	86
Total	77	55	23	33	1	50	10	71	22	73	20	80
Number of areas reporting	140	--	69	--	2	--	14	--	30	--	25	--

Table 19.—*Percentage of ski areas reporting night skiing by State and VTF size class, 1967-68 season*

State	Total		Rope only		Combination rope and cable areas							
					Ski-area size class (rated in VTF)							
					Less than 300 M		300 M to 699 M		700 M to 1,499 M		1,500 M or more	
	Weekend nights[1]	One or more other nights	Weekend nights[1]	One or more other nights	Weekend nights[1]	One or more other nights	Weekend nights[1]	One or more other nights	Weekend nights[1]	One or more other nights	Weekend nights[1]	One or more other nights
Illinois	80	80	100	100	--	--	100	100	--	--	0	0
Indiana	33	33	0	0	--	--	--	--	100	100	--	--
Michigan	43	45	41	48	0	0	60	40	56	50	27	36
Minnesota	38	50	27	40	--	--	25	50	57	71	50	50
Wisconsin	35	38	32	23	0	0	25	50	67	83	29	43
Total	41	45	36	39	0	0	43	50	60	63	32	40
Number of areas reporting	140	140	69	69	2	2	14	14	30	30	25	25

[1] Friday, Saturday, and/or Sunday nights.

Table 20.—*Factors reported by 147 ski-area operators as inhibiting attendance*

Factor	Total all areas				
	Number of times factor was reported			Total times reported[1]	
	1st	2nd	3rd	Number	Percent
No limiting factors	8	--	--	8	1.8
Inadequate tow and lift capacity	17	12	4	33	7.5
Inadequate skiable area	12	9	2	23	5.2
Inadequate overnight accommodations	10	6	3	19	4.3
Skier prefers cable tows and lifts	4	5	2	11	2.5
Other inadequate base facilities	11	9	1	21	4.8
Poor weather, lack of snow	47	14	7	68	15.4
Other[2]	36	37	27	100	22.7
Nonresponse	2	55	101	158	35.8
Total	147	147	147	441	100.0

[1] Total times reported is the sum of the number of times the factor was mentioned among the first three factors mentioned.

[2] Any factor included in the "other" category is less than 2.5 percent of the total response.

Skier Survey, 1968-69
(Tables 21-46)

A total of 2,350 usable skier questionnaires were obtained. Some questions were unanswered, as in the case of the area operators. If another sample of skiers was taken we would not expect to obtain precisely the same answer we did this time. Standard errors indicate how different the answers might be and are calculated for tables 21, 30, and 33. The skier survey is, of course, subject to all the other types of survey error.

Wherever possible a weighted response rate is given in the skier table. A 100 percent weighted response rate is the equivalent of all 349,100 skiers' answers (or all 2,350 questionnaires) entering the table, a 90 percent rate is the same as 314,100 (.90 x 349,100) skiers' answers entering the table, and so on.

A distinction can be made between the Great Lakes skier, who is anyone who *skied* in the five study area States, and a resident skier, who is a Great Lakes skier *residing* in one of them. Further, statements can be made about resident skiers by their State of residence if it is assumed all that State's skiers skied at least once in any one of the five study area States (and therefore had a known probability of entering the sample). It is unlikely that all skiers in a nonstudy area State skied at least once in the study area; therefore, statements should not be made about them. For example, statements may be made about Ohio residents who ski in the study area and study area residents who ski in Ohio, but not the skier or skiing in Ohio.

Question 7 of the Midwestern Skier Questionnaire

(Copies of the survey questionnaires may be obtained by writing: North Central Forest Experiment Station, Folwell Avenue, St. Paul, Minnesota 55101.)

7. Which of the following factors determine WHY YOU SKI AT ONE SKI AREA INSTEAD OF ANOTHER? Indicate as many factors as are important in reaching a decision by entering the number 1 beside the most important factor, 2 beside the second most important, 3 beside the next most important, etc., for the types of ski trips listed.

a. Single day ski trip.

b. Weekend or vacation ski trip.

SINGLE DAY SKI TRIP		WEEKEND OR VACATION SKI TRIP
--------	Advertising about the ski area which you have seen	--------
--------	The reputation of the ski area and the surrounding region for after skiing (apres ski) activity.	--------
--------	Presence of cable tow and lift facilities instead of only rope tows at the ski area.	--------
--------	The closeness of the ski area to your residence	--------
--------	The number of slopes and trails including their length, vertical drop, variability, challenge to your skiing skill, and general interest while skiing.	--------
--------	The relatively low price of tow and lift tickets	--------
--------	The physical layout and general appearance of the ski lodge including the amount and quality of eating, drinking, and/or lodging facilities.	--------
--------	The number of other ski areas within one half hour's drive of the ski area you are visiting.	--------
--------	The length of lift and tow lines and the amount of congestion on slopes which you expect to find at the ski area.	--------
--------	The ski area's reputation among fellow skiers as a "good" place to ski.	--------
--------	Other (specify) --	--------
--------	Other (specify) --	--------

Table 21.—*Estimated number of skiers and percentage of State population using ski areas, by residence, 1968-69 season*[1]

Residence	Estimated number of skiers	State population2/	Percent skiers	Standard error
	Thousands	Thousands		Thousands
Illinois	54.1	11,047	0.5	36.7
Indiana	3.6	5,118	.1	1.3
Michigan	128.5	8,766	1.5	71.2
Minnesota	78.7	3,700	2.1	32.9
Wisconsin	50.1	4,233	1.2	19.8
Study area total	315.0	32,864	1.0	--
Ohio	8.6	(3/)	--	8.0
Iowa, South Dakota, or North Dakota	6.0	(3/)	--	3.7
All other U.S.	11.1	(3/)	--	5.7
Non-U.S.4/	8.4	(3/)	--	3.6
Total	349.1	(3/)	--	102.6

1/ Weighted response rate 100 percent.

2/ State population as of July 1, 1968. Source: Census Bureau. Current population report, population estimates and projections. Ser. P-25 (430): p. 2. 1968.

3/ See section on Skier Survey.

4/ Primarily Canadian.

Table 22.—*Percentage of skiers, by age class and residence, 1968-69 season*[1]

Residence	Age class							Total
	13-18	19-22	23-30	31-40	41-50	Over 50	No response	
Illinois	29.6	17.7	26.8	11.9	10.9	2.5	0.6	100
Indiana	9.8	26.1	17.4	12.7	32.0	--	2.0	100
Michigan	51.2	13.5	16.0	10.4	6.0	2.2	.8	100
Minnesota	37.1	19.6	19.0	13.2	7.2	2.5	1.4	100
Ohio	7.7	19.7	31.7	19.3	19.2	--	2.5	100
Wisconsin	28.2	16.7	19.8	19.7	9.8	3.6	2.2	100
Iowa, South Dakota, and North Dakota	14.4	25.8	20.6	18.8	6.9	--	13.5	100
All other U.S.	9.0	56.5	19.1	1.6	3.6	.7	9.5	100
Non-U.S.2/	13.2	25.1	37.6	8.7	12.8	--	2.5	100
Total	37.0	18.1	20.0	12.7	8.2	2.3	1.7	100

1/ Weighted response rate 100 percent.

2/ Primarily Canadian.

Table 23.—*Average days skied*[1] *per skier, by residence, type of trip*[2] *and location of ski area,*[3] *1968-69 season*[4]

Residence	Total				Single day			Weekend			Vacation		
	Total	In home State2/	In rest of study area3/	Out of study area3/	In home State2/	In rest of study area3/	Out of study area3/	In home State2/	In rest of study area3/	Out of study area3/	In home State2/	In rest of study area3/	Out of study area3/
Illinois	5.1	1.0	3.1	1.0	0.9	1.9	0.0	0.1	0.8	0.0	0.0	0.4	1.0
Indiana	5.8	.9	4.5	.4	.8	1.0	.0	.0	1.6	.0	.0	1.9	.4
Michigan	5.6	5.1	.1	.4	3.6	.1	.0	1.1	.0	.0	.4	.0	.4
Minnesota	6.5	4.0	1.6	.9	3.3	.9	.0	.6	.5	.0	.1	.1	.8
Ohio	6.9	1.3	3.3	2.3	1.3	.5	.4	.0	1.8	1.1	.0	1.0	.9
Wisconsin	6.1	3.5	1.6	1.0	2.9	.5	.2	.5	.8	.0	.1	.3	.8
Iowa, South Dakota, and North Dakota	6.4	1.5	3.4	1.5	1.5	1.5	.1	.0	1.6	.0	.0	.3	1.4
All other U.S.	2.1	.0	2.1	.0	.0	1.5	.0	.0	.2	.0	.0	.4	.0
Non-U.S.5/	4.5	.6	2.6	1.3	.0	.8	1.1	.6	.8	.0	.0	1.0	.2
Total	5.7	3.5	1.4	.8	2.7	.7	.1	.6	.5	.1	.2	.2	.7

1/ Zero days skied was considered a legitimate answer in any category within a State of residence as long as the person had skied 1 or more days in any one category. This means the averages within a State of residence are additive. For example, the average Illinois skier skied 5.1 days, of which 2.8 were single-day trips.

2/ On single-day trips the skier returned home each night, on weekend trips he stayed overnight 1 to 3 nights, and on vacation trips he stayed overnight 4 or more nights.

3/ "In home State" are days skied in State of residency, "in rest of study area" are days skied in other States in study area but not in State of residency, and "out of study area" are days skied in locations outside the study area.

4/ Weighted response rate 97 percent.

5/ Primarily Canadian.

Table 24.—*Percentage of skiers taking various combinations of trips, by residence, 1968-69 season*[1]

Residence	Day only	Weekend only	Day and weekend	Vacation only	Day and vacation	Weekend and vacation	Day weekend and vacation	Nonresponse	Total
Illinois	58.7	14.1	16.6	2.6	2.8	0.8	1.4	3.1	100
Indiana	21.9	24.6	13.5	22.6	5.3	2.5	9.7	--	100
Michigan	62.9	13.8	14.4	1.5	1.6	1.4	2.2	2.3	100
Minnesota	60.0	7.6	24.0	2.1	1.8	.3	1.8	2.4	100
Ohio	4.1	63.2	9.4	15.6	.3	1.9	.8	4.7	100
Wisconsin	57.0	16.2	19.0	1.6	2.7	1.0	2.1	.6	100
Iowa, South Dakota, and North Dakota	42.4	22.7	29.9	--	3.1	.9	.4	.6	100
All other U.S.	65.9	8.9	1.9	9.0	1.6	--	--	12.6	100
Non-U.S.2/	40.7	35.2	1.3	15.8	1.4	.6	.9	4.0	100
Total	58.1	14.6	17.0	3.0	2.0	.9	1.9	2.6	100

1/ Weighted response rate 100 percent.
2/ Primarily Canadian.

Table 25.—*Percentage of days skied in different States by State of skier residence, 1968-69 season*[1]

Residence	State of ski area location							Total
	Illinois	Indiana	Michigan	Minnesota	Ohio	Wisconsin	Other	
Illinois	19.5	0.3	0.4	0.2	0.0	0.3	8.5	3.4
Indiana	.9	14.9	.0	.0	.0	.0	.0	.3
Michigan	17.8	66.7	90.6	6.4	45.7	24.6	16.2	43.0
Minnesota	.4	.0	1.0	61.5	.0	2.5	22.6	17.9
Ohio	.3	.5	.1	.0	19.0	.0	.0	.6
Wisconsin	40.7	10.5	.8	17.5	.4	57.1	10.7	19.8
Iowa, South Dakota, and North Dakota	.0	.0	.0	.1	.0	.0	7.9	.5
All other U.S.	19.6	7.1	5.8	13.9	32.6	15.4	29.8	13.4
Non-U.S.2/	.9	.0	1.4	.4	2.3	.1	4.2	1.0
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

1/ Weighted response rate not available.
2/ Primarily Canadian.

Table 26.—*Percentage of skiers first skiing in indicated season, 1968-69 season*[1]

Season first skied	Percentage	Accumulative percentage
1968-1969	13.8	13.8
1967-1968	14.0	27.8
1966-1967	10.2	38.0
1965-1966	10.3	48.3
1964-1965	7.7	56.0
1963-1964	6.9	62.9
1962-1963	5.3	68.2
1961-1962	6.8	75.0
1960-1961	2.2	77.2
1959-1960	3.5	80.7
1958-1959	2.5	83.2
1957-1958	2.5	85.7
1956-1957	1.3	87.0
1955-1956	1.5	88.5
1954-1955 and earlier	8.5	97.0
No response	3.0	100.0

1/ Weighted response rate 100 percent.

Table 27.—*Motives for choosing ski areas for single-day trips, 1968-69 season*[1]

(In percent)

Motive[2]	Importance of motive					
	First	Second	Third	Fourth	Fifth	Sum of 1-3
Closeness to residence	44.7	15.9	11.4	6.5	4.6	72.0
Physical quality of slopes	26.9	20.5	14.1	7.8	3.1	61.5
Presence of cable facilities	5.9	17.2	16.1	14.1	6.5	39.2
Low price of tow and lift tickets	3.5	13.8	13.2	7.0	7.2	30.5
Area's reputation with skiers	6.3	7.9	9.6	7.6	9.0	23.8
Expected amount of crowding	2.5	7.9	11.7	9.1	8.6	22.1
Reputation for after-ski entertainment	1.4	4.8	4.3	4.6	5.4	10.5
Other	5.6	3.1	1.3	.6	.4	10.0
Advertising	1.1	1.5	3.8	3.7	6.7	6.4
On-site eating, drinking, and/or lodging facilities	.6	2.6	2.8	4.7	7.2	6.0
Number of other ski areas in vicinity	1.4	1.2	1.7	2.4	1.9	4.3
No motive mentioned	--	3.6	10.0	31.9	39.4	13.6
Total responding	100.0	100.0	100.0	100.0	100.0	300.0

1/ Weighted response rate 62 percent.
2/ See question 7 for a full statement of the motive.

Table 28.—*Motives for choosing ski areas for weekend or vacation trips, 1968-69 season*[1]

(In percent)

Motive[2]	Importance of motive					
	First	Second	Third	Fourth	Fifth	Sum of 1-3
Physical quality of slopes	54.5	17.0	6.5	6.9	3.2	78.0
Presence of cable facilities	4.4	19.2	19.6	9.9	6.2	43.2
Area's reputation with skiers	13.9	9.4	7.4	9.7	5.8	30.7
Expected amount of crowding	1.8	10.8	16.6	11.5	7.9	29.2
Reputation for after-ski entertainment	5.6	12.2	8.3	4.8	8.2	26.1
On-site eating, drinking, and/or lodging facilities	2.0	9.5	9.3	8.2	8.1	20.8
Advertising	5.5	4.2	5.8	4.8	4.2	15.5
Closeness to residence	5.3	4.9	5.2	6.7	5.8	15.4
Low price of tow and lift tickets	1.8	3.2	9.1	6.1	8.8	14.1
Number of other ski areas in vicinity	.9	5.2	4.1	4.9	5.0	10.2
Other	4.3	1.8	1.7	1.0	.9	7.8
No motive mentioned	--	2.6	6.4	25.5	35.9	9.0
Total responding	100.0	100.0	100.0	100.0	100.0	300.0

1/ Weighted response rate 35 percent.
2/ See question 7 for a full statement of the motive.

Table 29.—*Increasing winter sports participation due to snowmobiling, by residence, 1968-69 season*[1]

Residence	Percent of skiers reporting increase	Percent of skiers not responding	Mean days per year increase
Illinois	6.7	0.7	7.2
Indiana	14.2	1.5	4.6
Michigan	27.0	1.4	17.1
Minnesota	20.8	2.3	17.6
Ohio	3.3	.6	10.0
Wisconsin	21.2	1.4	6.7
Iowa, South Dakota, and North Dakota	22.5	.7	11.7
All other U.S.	.9	2.9	5.0
Non-U.S.[2]	8.9	4.0	16.7
Total	19.6	1.6	14.9

1/ Weighted response rate 100 percent.
2/ Primarily Canadian.

Table 30.—*Total skier expenditure, by item and residence, 1968-69 season*[1]

(Thousands of dollars)

Item	Residence									Total
	Illinois	Indiana	Michigan	Minnesota	Ohio	Wisconsin	Iowa, South Dakota, North Dakota	All other U.S.	Non-U.S.	
Transportation	1,375	69	2,044	1,437	273	943	174	136	109	6,563
Lodging	1,162	112	1,478	1,157	244	556	113	29	151	5,002
Meals	1,150	130	2,237	1,562	231	845	177	83	183	6,598
Equipment rental	382	29	388	209	140	169	72	60	53	1,501
After ski entertainment	666	74	1,238	781	171	433	115	51	63	3,591
Package plans	676	110	547	398	151	314	34	0	5	2,235
Ski lessons	168	32	274	189	43	87	30	5	14	842
Equipment purchased on trip	446	84	735	350	50	323	60	25	6	2,079
Tow and lift tickets	1,231	83	3,166	2,330	308	1,228	166	105	162	8,779
Other expenses on trip	219	39	292	293	44	112	28	32	27	1,085
Equipment purchased not on trip	3,757	510	10,673	6,194	841	3,791	433	339	507	27,045
Total	11,232	1,272	23,072	14,900	2,496	8,801	1,402	865	1,280	65,320

[1] An overall weighted response rate is not stated because transportation and other expenditures were estimated separately by type of trip and residence. This means there were 54 separate response rates. However, for the five study area States, the best weighted response rate was 99 percent and the worst 88 percent.

Table 31.—*Standard error of total skier expenditure by item and residence, 1968-69 season*[1]

(Thousands of dollars)

Item	Residence									Total
	Illinois	Indiana	Michigan	Minnesota	Ohio	Wisconsin	Iowa, South Dakota, North Dakota	All other U.S.	Non-U.S.	
Transportation	1,052	46	1,334	815	336	639	183	91	42	2,510
Lodging	988	75	984	714	199	329	108	3	32	2,192
Meals	716	58	1,123	746	269	380	171	39	47	2,102
Equipment rental	191	19	240	119	288	92	32	13	40	703
After ski entertainment	440	49	823	394	169	229	126	23	15	1,362
Package plans	705	53	409	211	107	260	--	--	--	1,140
Ski lessons	76	14	196	149	66	63	31	2	--	358
Equipment purchased on trip	305	93	493	280	32	175	42	1	1	782
Tow and lift tickets	810	29	1,443	1,072	476	457	98	33	40	2,607
Other expenses on trip	196	28	143	183	16	73	33	2	1	401
Equipment purchased not on trip	2,546	378	6,465	2,394	798	1,092	342	74	205	8,900

[1] An overall weighted response rate is not stated because transportation and other expenditures were estimated separately by type of trip and State of residence. This means there were 54 separate response rates. However, for the five study area States, the best weighted response rate was 99.3 percent and the worst 87.9 percent. Equipment expenditures while not on trip were estimated separately and had a weighted response rate of 96.6 percent (table 35).

Table 32.—*Average skier expenditure*[1] *per day of trip, by trip type and residence, 1968-69 season*[2]

Residence	Average for all trips	Single-day trip	Weekend trip	Vacation trip
Illinois	16.49	11.21	23.15	27.67
Indiana	25.77	18.57	30.07	28.98
Michigan	13.15	9.73	18.23	23.34
Minnesota	12.61	9.66	16.79	19.17
Ohio	19.30	16.29	21.31	20.13
Wisconsin	11.71	8.97	14.40	18.95
Iowa, South Dakota, and North Dakota	18.09	18.57	14.95	23.29
All other U.S.	11.26	10.08	12.11	21.32
Non-U.S.[3]	12.24	9.59	13.22	17.93
Total	13.70	10.16	18.07	22.19

[1] Excludes transportation costs and equipment expenditure while not on trip.

[2] Weighted response rate for single-day trips was 79 percent, for weekend trips 36 percent, and for vacation trips 14 percent. Actual response is better than indicated because all skiers did not take all kinds of trips and could not have answered expenditure questions.

[3] Primarily Canadian.

Table 33.—*Average skier expenditure*[1] *per day of trip, by type of trip and item, 1968-69 season*[2]

Item	Average for all trips	Single-day trip	Weekend trip	Vacation trip
Transportation	2.75	1.93	3.34	5.50
Lodging	1.80	.00	4.22	5.20
Meals	2.91	2.15	3.97	4.25
Equipment rental	1.11	1.32	.92	.48
After-ski entertainment	1.43	.97	2.08	2.27
Package plans	.86	.00	1.57	3.56
Ski lessons	.33	.37	.25	.37
Equipment purchased	1.03	.89	1.19	1.33
Tow and lift tickets	4.02	4.25	3.57	3.95
Other expenses on trip	.43	.32	.43	.98
Total	16.67	12.19	21.55	27.88

1/ Includes transportation, but excludes equipment expenditure while not on trip.
2/ Weighted response rate not available.

Table 34.—*Standard error of average skier expenditure per day of trip by type of trip an item, 1968-69 season* [1,2]

Item	Average for all trips	Single-day trip	Weekend trip	Vacation trip
Transportation	0.15	0.12	0.26	0.57
Lodging	.16	.00	.27	.41
Meals	.12	.13	.20	.26
Equipment rental	.11	.17	.12	.08
After-ski entertainment	.12	.12	.20	.16
Package plans	.13	.00	.28	.69
Ski lessons	.04	.06	.06	.07
Equipment purchased	.21	.32	.22	.15
Tow and lift tickets	.09	.10	.17	.57
Other expenses on trip	.06	.07	.07	.18
Total	1.44	1.34	1.97	3.18

1/ Includes transportation, excludes equipment expenditure while not on trip.
2/ Weighted response rate not available.

Table 35.—*Annual skier expenditure while not on ski trips for equipment and supplies, 1968-69 season*[1]

Residence	Skiers making purchases	Average expenditure per skier 2/	Total expenditure
	Percent	Dollars	Thousand dollars
Illinois	73.3	69.44	3,757
Indiana	83.4	142.77	510
Michigan	79.5	83.04	10,673
Minnesota	75.7	78.74	6,194
Ohio	57.5	97.61	841
Wisconsin	76.8	75.68	3,791
Iowa, South Dakota, and North Dakota	55.1	72.36	433
All other U.S.	27.2	30.45	339
Non-U.S. 3/	63.0	59.99	507
Total	74.3	77.37	27,045

1/ Weighted response rate 96 percent.
2/ Average based on all skiers.
3/ Primarily Canadian.

Table 36.—*Average one-way mileage per trip by residence and type of trip, 1968-69 season* [1]

Residence	Total			Auto trips			Commercial carrier trips		
	Day	Weekend	Vacation	Day	Weekend	Vacation	Day	Weekend	Vacation
Illinois	109	262	845	98	248	651	172	338	1,135
Indiana	74	313	487	74	278	386	0	900	1,200
Michigan	61	205	607	59	204	477	72	223	1,297
Minnesota	55	205	811	44	191	678	130	302	1,085
Ohio	109	378	498	90	332	448	360	529	673
Wisconsin	64	255	719	63	227	563	73	398	935
Iowa, South Dakota, and North Dakota	83	289	581	83	285	631	50	350	553
All other U.S.	39	269	680	39	171	583	0	321	1,200
Non-U.S. 2/	63	260	497	63	260	497	0	0	0
Total	68	237	706	62	222	564	111	345	1,035

1/ Weighted response rate 94 percent.
2/ Primarily Canadian.

Table 37.—*Percentage of skiers traveling by auto, by one-way distance class, type of trip, and residency class, 1968-69 season*[1]

One-way distance	Total		Day trip		Weekend trip		Vacation trip	
	Resident2/	Non-resident	Resident	Non-resident	Resident	Non-resident	Resident	Non-resident
Miles								
0-25	15.0	6.0	26.7	12.6	0.3	0.0	0.1	1.2
26-50	15.1	12.0	26.2	26.1	1.2	.0	.0	.0
51-75	11.8	12.3	18.2	24.4	4.3	2.4	.5	1.2
76-100	5.3	8.1	7.8	5.3	2.5	5.2	.9	2.7
101-125	6.5	10.5	6.5	11.9	8.1	11.8	.7	2.0
126-150	2.2	.7	1.5	1.6	3.6	.0	1.3	.0
151-200	9.4	6.8	3.6	5.5	19.8	10.0	5.7	1.8
201-250	10.3	5.5	3.0	.8	21.1	11.8	13.8	2.8
251-300	7.4	2.7	1.2	.4	15.9	5.5	14.0	2.4
301-350	5.1	11.0	.3	.0	11.3	23.0	11.1	13.1
351-400	2.0	1.9	.1	.0	4.4	3.7	5.7	2.9
401-500	1.5	8.9	.0	.0	2.8	17.2	6.9	14.2
501-750	1.2	6.3	.0	.9	1.3	4.2	7.5	30.3
751-1,000	.7	1.5	.0	7.1	.2	.8	3.9	7.8
Over 1,000	2.5	.9	.0	.0	.3	.0	25.1	6.7
Nonresponse	4.0	4.9	4.9	3.4	2.9	4.4	2.8	10.9
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

1/ Weighted response rate for single-day trips was 71 percent, for weekend trips 46 percent, and for vacation trips 13 percent. Actual response is better than indicated because all skiers did not take all kinds of trips and could not have answered certain travel questions.

2/ A resident is anyone living in one of the five study area States.

Table 38.—*Percentage of trips by form of transportation, type of trip, and residency class, 1968-69 season* [1]

Transportation	Total		Day trip		Weekend trip		Vacation trip	
	Resident	Non-resident	Resident	Non-resident	Resident	Non-resident	Resident	Non-resident
Auto	85.0	86.1	87.1	98.2	88.0	79.5	67.1	73.9
Bus	10.4	8.2	12.4	1.8	9.2	16.7	4.5	.8
Plane	3.2	3.2	.3	.0	1.1	3.8	22.5	10.0
Train	.9	2.4	.1	.0	.5	.0	5.7	15.3
Other	.5	.1	.2	.0	1.2	.0	.2	.0
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

1/ Weighted response rate not available.

Table 39.—*Percentage of skiers 19 years and older, by family income class and residence, 1968-69 season* [1]

Residence	Family income class							Total all classes
	Less than 4,000	4,000 to 6,499	6,500 to 9,999	10,000 to 14,999	15,000 to 24,999	25,000 or more	Non-response	
Illinois	5.5	15.6	16.9	24.0	18.3	14.5	5.1	100
Indiana	18.7	3.9	11.6	16.8	16.8	28.3	3.8	100
Michigan	6.3	6.8	24.5	24.9	20.8	10.2	6.5	100
Minnesota	12.7	6.2	18.3	30.6	19.5	10.0	2.7	100
Ohio	.1	14.8	32.6	17.8	17.4	17.2	.0	100
Wisconsin	12.8	12.9	16.8	29.9	11.0	11.9	4.8	100
Iowa, South Dakota, and North Dakota	2.0	11.9	30.7	4.3	35.2	15.9	.0	100
All other U.S.	2.3	.0	8.7	65.4	17.1	1.5	5.0	100
Non-U.S.2/	12.8	16.3	5.6	50.0	6.6	2.1	6.7	100
Total	8.6	9.6	19.4	28.6	18.0	11.2	4.6	100

1/ Weighted response rate for skiers 19 years and older was 98 percent.

2/ Primarily Canadian.

Table 40.—*Expenditure by skiers 19 years and older, by family income class and type of trip,*[1] *1968-69 season*

TOTAL SEASONAL EXPENDITURE (THOUSANDS OF DOLLARS)[2]

Type of trip	Income class							Total
	Less than 4,000	4,000 to 6,499	6,500 to 9,999	10,000 to 14,999	15,000 to 24,999	25,000 or more	Non-response	
Day	620	847	1,579	2,044	1,723	956	327	8,096
Weekend	425	803	2,162	2,761	2,075	1,656	257	10,138
Vacation	416	688	1,193	2,038	3,631	2,766	453	11,187
Total	1,461	2,339	4,934	6,843	7,429	5,375	1,037	29,421
	EXPENDITURE PER DAY OF TRIP (DOLLARS)[3]							
Day	10.03	13.44	12.99	14.29	13.38	13.83	10.66	13.20
Weekend	16.62	18.70	23.31	22.73	25.35	29.63	22.45	23.48
Vacation	16.49	28.58	25.41	24.54	33.14	30.56	38.09	28.34
Total	12.39	16.34	17.69	18.11	21.07	22.96	16.41	18.55

[1] Excludes equipment while not on trip, includes transportation.

[2] Weighted response rate approximately 97 percent for trip expenditure items and 93 percent for transportation of skiers 19 years and older.

[3] Weighted response rate approximately 72 percent of skiers 19 years and older.

Table 41.—*Average number of days skied per skier 19 years and older, by family income class and type of trip, 1968-69 season* [1]

Type of trip	Income class							Average
	Less than 4,000	4,000 to 6,499	6,500 to 9,999	10,000 to 14,999	15,000 to 24,999	25,000 or more	Non-response	
Day	3.1	2.6	3.2	2.8	3.7	3.4	3.7	3.2
Weekend	1.0	1.5	1.3	1.3	1.6	1.8	1.0	1.4
Vacation	.9	.9	.7	1.0	2.2	2.8	.9	1.3
Total	5.0	5.0	5.2	5.1	7.5	8.0	5.6	5.9

[1] Weighted response rate for skiers 19 years and older was 98 percent.

Table 42.—*Percentage of skiers 19 years and older, by trip combination and family income class, 1968-69 season* [1]

Trip type or combination	Family income class							Total all classes
	Less than 4,000	4,000 to 6,499	6,500 to 9,999	10,000 to 14,999	15,000 to 24,999	25,000 or more	Non-response	
Day only	11.3	10.3	20.5	29.5	14.5	7.9	5.9	100
Weekend only	4.7	9.9	25.4	29.7	15.3	10.5	4.4	100
Day and weekend	6.8	11.0	18.2	26.3	20.4	14.2	3.1	100
Vacation only	7.1	.0	11.3	26.7	22.7	28.3	4.0	100
Day and vacation	4.8	.9	9.0	32.7	27.2	21.7	3.7	100
Weekend and vacation	5.9	3.2	9.1	31.3	33.8	16.7	.0	100
Day, weekend, and vacation	6.1	10.6	10.5	22.4	29.3	19.1	2.0	100
Nonresponse	3.8	7.8	3.7	22.9	48.8	13.0	.0	100
Total	8.6	9.6	19.4	28.6	18.0	11.2	4.6	100

[1] Weighted response rate for skiers 19 years and older was 98 percent.

Table 43.–*Percent of skiers, by residence, sex, and marital status, 1968-69 season* [1]

Residence	Married			Unmarried			Non-response	Total
	Male	Female	Total	Male	Female	Total		
Illinois	24.8	16.7	41.5	22.4	33.8	56.2	2.2	100
Indiana	48.3	23.0	71.3	6.8	21.9	28.7	.0	100
Michigan	16.8	11.9	28.7	36.5	33.7	70.3	1.0	100
Minnesota	22.7	13.7	36.4	30.5	32.5	63.0	.6	100
Ohio	22.2	30.6	52.9	16.7	30.4	47.1	.0	100
Wisconsin	24.8	17.4	42.1	28.9	27.1	56.0	1.9	100
Iowa, South Dakota, and North Dakota	18.8	7.8	26.6	47.0	26.4	73.4	.0	100
All other U.S.	15.9	3.7	19.6	66.3	14.1	80.4	.0	100
Non-U.S.[2]	31.9	22.5	54.4	28.5	17.1	45.6	.0	100
Total	21.4	14.3	35.7	32.1	31.1	63.2	1.1	100

[1] Weighted response rate 100 percent.
[2] Primarily Canadian.

Table 44.–*Percentage of skiers, by residence and occupation, 1968-69 season* [1]

Residence	Occupation									Total
	Student	Professional	Manager or proprietor	Clerical or sales	House-wife	Craftsman or foreman	Operative or laborer	Service worker	Other	
Illinois	42.9	15.5	9.8	13.6	5.5	1.6	1.1	6.3	3.8	100
Indiana	24.5	12.7	20.3	10.8	7.8	--	--	21.5	2.4	100
Michigan	61.1	17.2	4.8	5.2	2.9	2.0	.1	6.0	.7	100
Minnesota	54.1	23.6	8.2	6.4	.8	.9	.2	5.4	.4	100
Ohio	19.4	37.6	9.3	14.8	7.9	--	--	11.1	--	100
Wisconsin	43.8	16.3	10.1	6.7	3.0	3.5	.6	13.5	2.5	100
Iowa, South Dakota, and North Dakota	49.5	28.6	1.4	3.1	4.6	1.7	5.5	.9	4.6	100
All other U.S.	70.0	21.1	5.2	3.2	--	--	--	--	.6	100
Non-U.S.[2]	35.9	29.9	11.6	--	5.4	--	--	8.9	4.4	100
Total	52.3	19.3	7.5	7.1	3.0	1.7	.4	7.1	1.5	100

[1] Weighted response rate 100 percent.
[2] Primarily Canadian.

Table 45.–*Percentage of skiers by education class and residence, 1968-69 season* [1]

Residence	Years of education						Total
	1-8	9-12	13-16	17-19	20 or more	Non-response	
Illinois	3.0	37.4	47.0	9.4	3.1	--	100
Indiana	9.8	27.6	52.4	8.4	1.7	--	100
Michigan	11.6	46.3	31.0	6.9	3.4	.6	100
Minnesota	9.0	30.1	48.7	8.5	3.5	.2	100
Ohio	1.2	13.2	70.6	12.4	2.6	--	100
Wisconsin	5.9	36.6	46.2	8.4	1.7	1.2	100
Iowa, South Dakota, and North Dakota	--	26.0	44.4	17.8	11.8	--	100
All other U.S.	2.4	12.7	75.3	9.5	--	--	100
Non-U.S.[2]	2.4	16.6	58.6	9.7	12.7	--	100
Total	7.9	36.7	43.1	8.3	3.4	.5	100

[1] Weighted response rate 100 percent.
[2] Primarily Canadian.

Table 46.—*Past and projected skier visits to the study area and past and projected study area population, income per household, and number of ski areas*[1]

Season	Projection			Estimated study area		
	Estimated actual	A	B	Population	Income per household	Areas
	Number	Number	Number	Thousands	Dollars	Number
1960-1961	669	699	777	30,171	6,753	112
1961-1962	930	960	894	30,415	6,614	124
1962-1963	1,241	1,220	1,168	30,751	6,883	132
1963-1964	1,494	1,482	1,453	31,166	7,090	142
1964-1965	1,842	1,741	1,790	31,577	7,401	152
1965-1966	1,866	1,998	2,109	31,944	7,789	159
1966-1967	2,489	2,247	2,384	32,248	8,377	159
1967-1968	2,318	2,503	2,295	32,659	8,520	148
1975-1976		4,388	4,124	34,978	10,210	212

1/ See section on Estimating and Projecting Total Attendance for discussion of estimating and projecting attendance. Projection A is made with equation 2; projection B is made with equation 3.

Financial Statements (Tables 47-56)

Each ski area was asked for its income statements and balance sheets for the last 5 years. Some operators refused to release this information and some did not have it. Although this information is not a probability sample, it does represent the largest collection of financial data on the Great Lakes skiing industry to date.

The accounts in the financial tables are intended to follow standard accounting definitions except as noted below.

Gross receipts — other seasonal. — Includes receipts from ski shops, room rental, and miscellaneous items accruing during the skiing season.

Total net income.—Includes *all* net income accruing during the year. This includes net concession receipts (e.g., restaurant or ski shop), capital gains or losses, and miscellaneous income not a regular part of operations, such as inter received or insurance claims.

Other intangible assets.—Goodwill and orga ization expense are the primary entries in t account.

Accounts payable and notes payable.—D for some ski areas included notes payable in accounts payable; therefore, accounts payable overstated and notes payable understated. Al some areas included long-term debt curren payable in notes payable. However, the net fect of these procedures does not distort t current liabilities.

Long-term debt.—In many cases the ski-a owner invested using long-term debt rather t equity. Unfortunately, this did not become parent until it was too late to create a separ account for this type of debt. Presumably, owner-lenders would be more lenient in t demands for meeting fixed obligations — a p to be considered in analyzing individual bala sheets.

Table 47.—*Total reported gross receipts, by account and receipt class*

1967-68 SEASON

Accounts	Annual gross receipts									
	Less than 25 M dollars		25 M to 49 M dollars		50 M to 99 M dollars		100 M to 249 M dollars		250 M or more dollars	
	M dollars	Number areas1/	M dollars	Number areas1/	M dollars	Number areas1/	M dollars	Number areas1/	M dollars	Number areas1/
Gross receipts										
Tow and lift tickets	72.1	20	116.1	5	179.6	4	101.7	1	345.5	2
Restaurant	15.9	11	17.2	3	33.4	4	22.4	1	90.4	2
Bar	--	--	7.6	1	1.5	1	2.6	1	37.0	1
Ski rental	5.7	8	13.6	2	38.8	4	17.2	1	54.3	2
Other seasonal	10.9	8	32.1	5	34.4	4	2.2	1	61.8	2
Non-seasonal	--	--	--	--	1.8	1	--	--	--	--
Total gross receipts	104.6	20	186.6	5	289.5	4	146.1	1	589.0	2
1966-67 SEASON										
Gross receipts										
Tow and lift tickets	126.3	28	244.6	11	212.5	6	233.0	3	1,851.7	8
Restaurant	18.5	14	49.7	8	95.2	5	45.6	3	691.8	6
Bar	.5	1	7.2	1	31.9	2	8.9	1	303.7	5
Ski rental	9.9	9	39.2	8	27.7	4	39.5	3	319.2	6
Other seasonal	9.9	11	58.8	9	22.4	4	19.6	3	398.2	8
Non-seasonal	--	--	2.1	2	67.6	1	--	--	742.6	3
Total gross receipts	165.1	28	401.7	11	457.2	6	346.6	3	4,307.2	8
1965-66 SEASON										
Gross receipts										
Tow and lift tickets	105.0	19	228.3	9	167.7	6	82.7	1	940.7	5
Restaurant	14.2	16	28.9	5	106.4	6	32.1	1	552.6	4
Bar	3.6	1	2.6	2	36.5	2	--	1	265.5	5
Ski rental	10.9	11	29.1	6	36.3	5	19.6	1	128.6	3
Other seasonal	12.6	11	33.9	7	22.4	5	41.2	1	292.9	5
Non-seasonal	--	--	2.8	1	31.3	1	--	1	635.5	3
Total gross receipts	146.3	19	325.6	9	400.6	6	175.6	1	2,815.8	5
1964-65 SEASON										
Gross receipts										
Tow and lift tickets	113.3	16	125.4	5	145.6	4	147.6	2	401.5	3
Restaurant	16.1	9	29.7	4	36.4	4	198.7	2	472.1	3
Bar	--	--	--	--	2.3	1	--	--	191.2	3
Ski rental	10.3	7	20.6	3	17.8	3	29.4	1	111.5	3
Other seasonal	11.2	8	20.3	4	10.8	4	33.1	2	176.1	3
Non-seasonal	--	--	29.7	2	--	--	--	--	468.9	2
Total gross receipts	150.9	16	225.7	5	212.9	4	408.8	2	1,821.3	3
1963-64 SEASON										
Gross receipts										
Tow and lift tickets	91.4	17	147.2	6	34.1	1	273.6	2	162.5	1
Restaurant	27.8	12	23.7	4	7.8	1	2.6	1	42.2	1
Bar	.5	1	--	--	--	--	--	--	19.0	1
Ski rental	11.8	8	21.7	5	8.4	1	17.5	1	63.9	1
Other seasonal	20.4	7	25.2	6	5.1	1	17.8	2	34.8	1
Non-seasonal	--	--	4.0	2	--	--	--	--	--	--
Total gross receipts	151.9	17	221.8	6	55.4	1	311.5	2	322.4	1
1962-63 SEASON										
Gross receipts										
Tow and lift tickets	30.5	7	80.4	5	30.2	1	77.3	1	465.2	2
Restaurant	13.5	5	17.7	3	8.2	1	51.1	1	42.7	1
Bar	--	--	--	--	--	--	51.1	1	17.9	1
Ski rental	2.1	2	12.6	3	16.5	1	--	--	71.5	1
Other seasonal	2.8	3	45.3	3	5.8	1	21.3	1	36.3	2
Non-seasonal	--	--	2.8	1	--	--	23.1	1	--	--
Total gross receipts	48.9	7	158.8	5	60.7	1	223.9	1	573.6	2

1/ Indicates the number of ski areas having the particular source of income.

Table 48.—*Total reported income statements, by receipt class*

1967-68 SEASON

Accounts	Annual gross receipts											
	Total		Less than 25 M dollars		25 M to 49 M dollars		50 M to 99 M dollars		100 M to 249 M dollars		250 M or more dollars	
	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas
Total gross receipts	1,230.0	25	83.4	11	229.0	6	376.1	5	278.8	2	262.7	1
Total net income[1]	10.9	8	3.1	4	4.2	2	--	--	3.6	2	--	--
Expenses												
Depreciation[2]	173.2	13	19.8	3	33.1	4	52.0	3	26.1	2	42.2	1
Interest[2]	67.0	15	6.6	4	8.1	4	32.1	4	14.0	2	6.2	1
All other	966.0	25	101.8	11	186.6	6	301.2	5	183.1	2	193.2	1
Total expenses	1,206.2	25	128.2	11	227.8	6	385.3	5	223.2	2	241.6	1
Profit before Federal income tax	34.7	25	-41.7	11	5.4	6	-9.2	5	59.1	2	21.1	1
1966-67 SEASON												
Total gross receipts	5,907.8	48	140.4	15	463.6	13	679.7	9	346.6	3	4,277.5	8
Total net income	202.1	19	2.8	5	1.5	4	.9	1	34.7	3	162.2	6
Expenses												
Depreciation	943.1	37	27.3	7	91.9	11	86.3	8	50.1	3	687.5	8
Interest	411.7	33	4.4	6	24.2	9	43.6	7	29.2	3	310.3	8
All other	4,740.3	48	141.5	15	393.8	13	579.2	9	248.9	3	3,376.9	8
Total expenses	6,095.1	48	173.2	15	509.9	13	709.1	9	328.2	3	4,374.7	8
Profit before Federal income tax	14.8	48	-30.0	15	-44.8	13	-28.5	9	53.1	3	65.0	8
1965-66 SEASON												
Total gross receipts	4,516.7	46	168.7	20	441.1	12	400.7	6	412.5	2	3,093.7	6
Total net income	95.7	14	2.0	3	3.1	4	2.2	2	8.0	1	80.4	4
Expenses												
Depreciation	904.0	35	33.0	10	127.4	11	68.4	6	94.1	2	581.1	6
Interest	299.7	31	7.2	8	32.4	11	23.1	4	15.1	2	221.9	6
All other	3,896.9	46	168.5	20	472.3	12	348.9	6	353.5	2	2,553.7	6
Total expenses	5,100.6	46	208.7	20	632.1	12	440.4	6	462.7	2	3,356.7	6
Profit before Federal income tax	-488.3	46	-38.0	20	-187.9	12	-37.5	6	-42.3	2	-182.6	6
1964-65 SEASON												
Total gross receipts	3,492.0	37	186.6	19	288.1	7	216.4	4	408.8	2	2,392.1	5
Total net income	20.1	15	3.8	7	1.8	2	2.7	3	5.8	1	6.0	2
Expenses												
Depreciation	666.2	26	32.2	9	62.2	6	55.8	4	76.6	2	439.4	5
Interest	254.8	26	10.1	9	19.4	6	21,1	4	36.2	2	168.0	5
All other	2,801.6	37	165.2	19	231.9	7	170.4	4	283.8	2	1,950.3	5
Total expenses	3,722.6	37	207.5	19	313.5	7	247.3	4	396.6	2	2,557.7	5
Profit before Federal income tax	210.5	37	-17.1	19	-23.6	7	-28.2	4	18.0	2	-159.6	5
1963-64 SEASON												
Total gross receipts	1,600.6	33	167.6	19	286.9	8	55.4	1	512.6	3	578.1	2
Total net income	22.8	9	.4	2	5.7	4	--	--	8.9	2	7.8	1
Expenses												
Depreciation	370.9	23	33.5	10	86.1	7	9.2	1	114.1	3	128.0	2
Interest	101.0	22	6.0	8	25.0	8	1.6	1	33.8	3	34.6	2
All other	1,323.0	33	149.0	19	266.7	8	48.0	1	351.7	3	507.6	2
Total expenses	1,794.9	33	188.5	19	377.8	8	58.8	1	499.6	3	670.2	2
Profit before Federal income tax	-171.5	33	-20.5	19	-85.2	8	-3.4	1	21.9	3	-84.3	2
1962-63 SEASON												
Total gross receipts	1,513.2	20	70.4	8	221.2	7	60.7	1	223.9	1	937.0	3
Total net income	9.2	7	2.0	2	1.5	3	--	--	--	--	5.7	2
Expenses												
Depreciation	297.2	16	12.8	5	47.3	6	13.2	1	43.2	1	180.7	3
Interest	71.8	16	2.9	5	11.9	6	1.9	1	10.2	1	44.9	3
All other	1,209.4	20	49.0	8	206.8	7	37.2	1	220.1	1	696.3	3
Total expenses	1,578.4	20	64.7	8	266.0	7	52.3	1	273.5	1	921.9	3
Profit before Federal income tax	-56.0	20	7.7	8	-43.3	7	8.4	1	-49.6	1	20.8	3

[1] Number of areas is the number of areas having net income. All other areas did not have any net income.

[2] Some areas did not report their depreciation or interest separately. These areas have all of their expenses included in "all other."

Table 49.—*Total of all reported balance sheets, by asset class, as of end of 1967-68 season*

Accounts[1]	Total		Asset class							
			Less than 100 M dollars		100 M to 249 M dollars		250 M to 999 M dollars		1,000 M or more dollars	
	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas
Current assets:										
Cash	87.6	10	1.4	2	69.8	6	3.7	1	12.7	1
Accounts receivable	25.1	4	--	--	1.9	2	20.0	1	3.2	1
Inventories	32.1	8	2.6	3	16.1	3	6.4	1	7.0	1
Other	18.6	6	.3	1	7.7	3	2.3	1	8.3	1
Total current assets	163.4	11	4.3	3	95.5	6	32.4	1	31.2	1
Noncurrent assets:										
Land	579.1	8	15.7	2	126.0	5	--	--	437.4	1
Buildings and equipment	3,147.1	10	150.9	2	1,241.4	6	352.6	1	1,402.2	1
Accum. deprec.	861.3	10	67.9	2	372.8	6	191.5	1	229.1	1
Net buildings and equipment	2,285.8	11	83.0	3	868.6	6	161.1	1	1,173.1	1
Other intangible assets	30.9	3	--	--	.4	2	30.5	1	--	--
Other tangible assets	199.4	7	1.2	1	3.5	4	39.9	1	154.8	1
Total assets	3,258.6	11	104.2	3	1,094.0	6	263.9	1	1,796.5	1
Current liabilities:										
Accounts payable	133.4	9	10.4	2	63.6	5	37.4	1	22.0	1
Notes payable	428.0	7	14.2	1	246.6	4	46.6	1	120.6	1
Long-term debt cur. pay.	126.3	6	1.0	1	45.7	3	25.1	1	54.5	1
Other	71.7	9	1.8	1	48.9	6	14.7	1	6.3	1
Total current liabilities	759.4	10	27.4	2	404.8	6	123.8	1	203.4	1
Noncurrent liabilities:										
Long-term debt	1,773.7	9	44.9	1	410.9	6	17.5	1	1,300.4	1
Preferred stock	--	--	--	--	--	--	--	--	--	1
Common stock	901.8	11	63.1	3	371.1	6	99.6	1	368.0	1
Capital and other surplus	434.2	3	--	--	56.6	1	9.6	1	368.0	1
Retained earnings	-610.5	10	-31.2	2	-149.4	6	13.3	1	-443.2	1
Total liabilities	3,258.6	11	104.2	3	1,094.0	6	263.8	1	1,796.6	1

[1] Some areas did not report their current asset and liability accounts in detail. Therefore, those amounts are included in "other" and the total accounts, and number of areas is the number reporting detail accounts. Number of areas in the noncurrent accounts is the number reporting nonzero dollar amounts in those accounts.

Table 50.—*Total of all reported balance sheets, by asset class, as of end of 1966 -67 season*

Accounts	Total		Asset class							
			Less than 100 M dollars		100 M to 249 M dollars		250 M to 999 M dollars		1,000 M or more dollars	
	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas
Current assets:										
Cash	985.3	33	44.2	10	61.6	13	50.8	4	828.7	6
Accounts receivable	320.7	17	1.7	2	30.7	7	10.4	2	277.9	6
Inventories	227.3	29	26.0	9	25.1	10	6.3	4	169.9	6
Other	200.1	22	1.8	4	21.9	10	17.0	2	159.4	6
Total current assets	1,733.4	38	73.7	14	139.3	14	84.5	4	1,435.9	6
Noncurrent assets:										
Land	1,949.1	31	92.0	9	388.4	13	420.1	3	1,048.6	6
Buildings and equipment	18,210.8	40	1,011.4	15	3,498.3	15	2,238.0	4	11,463.1	6
Accum. deprec.	6,116.6	40	326.7	15	1,150.3	15	1,152.7	4	3,486.9	6
Net buildings and equipment	12,118.2	41	708.7	16	2,348.0	15	1,085.3	4	7,976.2	6
Other intangible assets	38.2	8	.8	2	32.7	4	1.3	1	3.4	1
Other tangible assets	479.0	19	11.0	7	47.2	5	282.6	3	138.2	4
Total assets	16,317.9	41	886.1	16	2,955.7	15	1,873.8	4	10,602.3	6
Current liabilities:										
Accounts payable	930.9	33	85.5	10	221.2	14	107.3	4	516.9	5
Notes payable	1,261.2	17	86.4	5	438.3	7	30.0	1	706.5	4
Long-term debt cur. pay.	479.3	17	44.9	4	99.1	6	88.7	3	246.6	4
Other	662.9	31	12.9	9	91.7	13	51.0	4	507.3	5
Total current liabilities	3,334.1	38	229.6	13	850.3	15	277.0	4	1,977.2	6
Noncurrent liabilities:										
Long-term debt	7,704.3	37	368.1	12	1,485.5	15	1,060.6	4	4,790.1	6
Preferred stock	64.2	3	2.4	1	--	--	50.0	1	11.8	1
Common stock	4,201.3	41	413.2	16	1,302.4	15	246.0	4	2,239.7	6
Capital and other surplus	716.4	14	60.1	5	107.2	6	8.2	1	540.9	2
Retained earnings	297.6	38	-187.5	14	-789.6	14	232.0	4	1,042.7	6
Total liabilities	16,317.9	41	886.0	16	2,955.7	15	1,873.8	4	10,602.5	6

Table 51.—*Total of all reported balance sheets, by asset class, as of end of 1965-66 season*

Accounts	Total		Asset class: Less than 100 M dollars		100 M to 249 M dollars		250 M to 999 M dollars		1,000 M or more dollars	
	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas
Current assets:										
Cash	696.4	40	48.8	11	47.1	17	118.0	7	482.5	5
Accounts receivable	296.4	21	--	--	9.9	12	36.4	5	250.1	4
Inventories	228.8	30	12.6	7	26.9	11	41.0	7	148.3	5
Other	264.3	26	.3	2	23.3	12	121.4	7	119.3	5
Total current assets	1,485.9	42	61.6	11	107.3	19	316.8	7	1,000.2	5
Noncurrent assets:										
Land	2,025.4	31	56.1	5	478.8	17	455.6	4	1,034.9	5
Buildings and equipment	17,948.9	43	626.6	12	4,317.1	19	3,704.7	7	9,300.5	5
Accum. deprec.	5,488.5	43	200.2	12	1,351.8	19	1,492.4	7	2,444.1	5
Net buildings and equipment	12,460.3	43	426.4	12	2,965.2	19	2,212.3	7	6,856.4	5
Other intangible assets	40.8	9	.8	2	32.9	4	3.1	2	4.0	1
Other tangible assets	591.0	25	6.4	5	102.3	12	399.0	4	83.3	4
Total assets	16,603.5	43	551.3	12	3,686.6	19	3,386.8	7	8,978.8	5
Current liabilities:										
Accounts payable	1,657.7	35	52.8	7	736.3	17	292.8	7	575.8	4
Notes payable	1,061.7	16	27.3	4	328.7	6	148.0	2	557.7	4
Long-term debt cur. pay.	329.3	12	6.0	1	89.5	5	95.5	3	138.3	3
Other	271.9	32	5.3	7	95.8	15	72.5	7	98.3	3
Total current liabilities	3,320.7	40	91.5	10	1,250.2	18	608.9	7	1,370.1	5
Noncurrent liabilities:										
Long-term debt	8,560.0	38	241.0	7	1,831.3	19	2,164.9	7	4,322.8	5
Preferred stock	203.6	3	2.4	1	151.2	1	50.0	1	--	--
Common stock	4,372.5	42	263.7	12	1,440.3	19	453.0	6	2,215.5	5
Capital and other surplus	736.2	16	14.9	3	345.1	11	8.2	1	368.0	1
Retained earnings	-589.3	41	-62.1	11	-1,331.5	18	101.8	7	702.5	5
Total liabilities	16,603.5	43	551.3	12	3,686.6	19	3,386.8	7	8,978.9	5

Table 52.—*Total of all reported balance sheets, by asset class, as of end of 1964-65 season*

Accounts	Total		Asset class: Less than 100 M dollars		100 M to 249 M dollars		250 M to 999 M dollars		1,000 M or more dollars	
	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas
Current assets:										
Cash	721.7	38	30.3	8	88.3	17	94.5	9	508.6	4
Accounts receivable	416.5	26	2.0	1	83.3	15	73.3	6	257.9	4
Inventories	172.5	28	8.2	5	17.6	12	58.1	7	88.6	4
Other	197.1	25	1.0	2	24.6	12	99.4	8	72.1	3
Total current assets	1,507.8	41	41.5	10	213.8	18	325.3	9	927.2	4
Noncurrent assets:										
Land	1,817.7	30	47.3	5	462.2	14	503.8	7	804.4	4
Buildings and equipment	16,678.6	41	587.4	10	3,640.5	18	4,474.1	9	7,976.6	4
Accum. deprec.	4,565.9	41	154.0	10	1,239.3	18	1,214.2	9	1,958.4	4
Net buildings and equipment	12,112.6	41	433.4	10	2,401.0	18	3,259.9	9	6,018.3	4
Other intangible assets	71.6	10	.8	2	33.7	6	2.4	1	34.7	1
Other tangible assets	609.4	23	7.8	5	30.3	7	252.3	7	319.0	4
Total assets	16,119.1	41	530.7	10	3,141.0	18	4,343.7	9	8,103.7	4
Current liabilities:										
Accounts payable	1,781.6	35	59.5	5	752.4	18	434.1	8	535.6	4
Notes payable	494.2	12	39.6	5	69.7	3	42.8	2	342.1	2
Long-term debt cur. pay.	374.4	11	9.8	2	40.7	2	120.8	4	203.1	3
Other	211.1	32	8.7	6	76.1	15	96.8	9	29.5	2
Total current liabilities	2,861.3	39	117.6	8	938.9	18	694.5	9	1,110.3	4
Noncurrent liabilities:										
Long-term debt	7,853.7	37	203.2	7	1,374.7	17	2,923.1	9	3,352.7	4
Preferred stock	215.4	4	2.4	1	50.0	1	163.0	2	--	--
Common stock	4,131.9	40	202.7	10	1,267.9	18	443.3	8	2,218.0	4
Capital and other surplus	1,051.4	17	49.7	3	391.6	9	242.1	4	368.0	1
Retained earnings	5.3	39	-44.9	9	-882.0	17	-122.4	9	1,054.6	4
Total liabilities	16,119.0	41	530.7	10	3,141.0	18	4,343.6	9	8,103.7	4

Table 53.—*Total of all reported balance sheets, by asset class, as of end of 1963-64 season*

Accounts	Total		Asset class							
			Less than 100 M dollars		100 M to 249 M dollars		250 M to 999 M dollars		1,000 M or more dollars	
	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas
Current assets:										
Cash	541.7	35	35.3	14	60.0	12	28.0	7	418.4	2
Accounts receivable	194.0	23	13.7	4	32.4	11	15.9	6	132.0	2
Inventories	168.3	28	14.5	9	42.3	10	86.7	7	24.8	2
Other	217.0	22	4.5	4	38.6	9	149.0	8	24.9	1
Total current assets	1,121.0	38	68.0	14	173.3	13	279.6	9	600.1	2
Noncurrent assets:										
Land	1,127.0	31	114.9	11	369.6	10	427.8	8	214.7	2
Buildings and equipment	10,072.7	39	1,071.7	15	2,286.2	13	3,626.3	9	3,088.5	2
Accum. deprec.	3,007.6	38	300.1	14	785.8	13	939.0	9	982.7	2
Net buildings and equipment	7,072.9	39	771.5	15	1,508.3	13	2,687.3	9	2,105.8	2
Other intangible assets	161.0	10	2.0	2	138.1	5	20.9	3	--	--
Other tangible assets	277.3	16	8.8	6	17.7	3	156.9	5	93.9	2
Total assets	9,759.2	39	965.2	15	2,207.0	13	3,572.5	9	3,014.5	2
Current liabilities:										
Accounts payable	1,289.3	34	143.6	11	335.9	13	569.8	8	240.0	2
Notes payable	79.8	9	28.8	5	6.0	1	45.0	3	--	--
Long-term debt cur. pay.	205.4	12	22.1	4	64.1	4	68.2	3	51.0	1
Other	396.3	30	31.1	10	72.2	10	60.2	9	232.8	1
Total current liabilities	1,970.8	38	225.6	14	478.2	13	743.2	9	523.8	2
Noncurrent liabilities:										
Long-term debt	4,491.4	34	363.5	11	916.1	12	2,334.6	9	877.2	2
Preferred stock	253.4	5	2.4	1	50.0	1	201.0	3	--	--
Common stock	2,463.0	37	425.8	14	678.7	12	758.5	9	600.0	2
Capital and other surplus	621.2	14	229.0	5	154.5	4	237.7	5	--	--
Retained earnings	-40.6	37	-281.2	14	-70.4	12	-702.5	9	1,013.5	2
Total liabilities	9,759.2	39	965.2	15	2,207.1	13	3,572.5	9	3,014.5	2

Table 54.—*Total of all reported balance sheets, by asset class, as of end of 1962-63 season*

Accounts	Total		Asset class							
			Less than 100 M dollars		100 M to 249 M dollars		250 M to 999 M dollars		1,000 M or more dollars	
	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas
Current assets:										
Cash	399.5	23	5.6	7	64.8	9	36.3	5	292.8	2
Accounts receivable	225.8	15	.8	3	20.2	5	19.4	5	185.4	2
Inventories	128.2	16	5.6	3	11.2	5	55.1	6	56.3	2
Other	239.6	18	--	--	28.6	9	189.4	8	21.6	1
Total current assets	993.1	26	12.0	7	124.8	9	300.2	8	556.1	2
Noncurrent assets:										
Land	905.9	23	50.1	6	274.7	8	408.6	7	172.5	2
Buildings and equipment	7,646.1	26	559.1	7	1,278.4	9	3,046.4	8	2,762.2	2
Accum. deprec.	1,967.7	26	174.5	7	418.2	9	643.2	8	731.8	2
Net buildings and equipment	5,696.4	26	384.6	7	860.2	9	2,421.2	8	2,030.4	2
Other intangible assets	116.9	4	.2	1	112.4	2	4.3	1	--	--
Other tangible assets	155.8	13	1.4	3	17.2	4	39.9	4	97.3	2
Total assets	7,868.2	26	448.3	7	1,389.4	9	3,174.2	8	2,856.3	2
Current liabilities:										
Accounts payable	999.5	21	32.6	4	229.1	9	297.8	6	340.0	2
Notes payable	58.7	6	8.4	3	--	--	50.3	3	--	--
Long-term debt cur. pay.	160.1	7	5.5	1	31.1	2	72.5	3	51.0	1
Other	166.3	19	15.9	7	38.2	4	67.3	7	44.9	1
Total current liabilities	1,384.6	25	62.4	7	298.4	9	587.9	7	435.9	2
Noncurrent liabilities:										
Long-term debt	4,087.4	24	185.4	5	634.3	9	2,327.5	8	940.2	2
Preferred stock	54.2	3	2.4	1	--	--	51.8	2	--	--
Common stock	1,740.1	25	203.5	7	400.6	8	536.0	8	600.0	2
Capital and other surplus	497.3	9	23.1	3	94.9	2	379.3	4	--	--
Retained earnings	104.7	25	-28.5	7	-38.7	8	-708.3	8	880.2	2
Total liabilities	7,868.2	26	448.3	7	1,389.4	9	3,174.2	8	2,856.3	2

Table 55.–*Summary income statements for 27 ski areas*[1] *willing or able to supply them, by season*

Accounts	Season							
	1966-67		1965-66		1964-65		1963-64	
	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas
Total gross receipts	2,655.8	27	2,022.8	27	1,889.5	27	1,475.0	27
Total net income	146.0	14	52.5	11	18.4	13	22.8	9
Expenses								
Depreciation	404.3	20	435.9	20	384.3	20	350.3	19
Interest	189.4	20	121.6	21	127.8	21	97.2	19
All other	2,071.7	27	1,607.0	27	1,443.1	27	1,216.2	27
Total expenses	2,665.4	27	2,164.5	27	1,955.2	27	1,663.7	27
Profit before Federal income tax	136.4	27	-89.2	27	-47.3	27	-165.9	27

1/ Eighteen of these ski areas are also included in table 56.

Table 56–*Total balance sheets for 26 ski areas*[1] *willing or able to supply them, by season*

Accounts	Season							
	1966-67		1965-66		1964-65		1963-64	
	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas	M dollars	Number areas
Current assets:								
Cash	186.2	23	195.8	24	149.5	24	77.7	24
Accounts receivable	174.9	10	166.4	13	202.0	15	140.5	14
Inventories	42.6	18	41.9	18	57.6	18	92.8	18
Other	73.5	14	108.5	16	48.6	15	60.0	13
Total current assets	477.3	24	512.6	25	457.7	26	371.0	25
Noncurrent assets:								
Land	932.9	20	853.0	20	738.0	20	699.8	21
Buildings and equipment	7,825.3	26	7,297.8	26	6,312.7	26	5,461.1	26
Accum. deprec.	3,050.3	26	2,527.9	26	1,962.8	26	1,513.0	26
Net buildings and equipment	4,775.0	26	4,769.9	26	4,349.9	26	3,948.1	26
Other intangible assets	34.1	6	35.0	7	136.9	9	138.3	6
Other tangible assets	336.8	10	489.7	15	229.6	14	194.6	10
Total assets	6,556.1	26	6,660.3	26	5,912.1	26	5,351.8	26
Current liabilities:								
Accounts payable	366.0	20	746.0	14	713.1	22	510.6	21
Notes payable	642.0	10	462.4	4	148.1	9	71.4	8
Long-term debt cur. pay.	278.3	11	236.5	5	236.7	9	196.8	11
Other	198.8	21	122.7	11	104.1	20	94.8	19
Total current liabilities	1,485.0	25	1,567.7	25	1,202.0	25	873.7	25
Noncurrent liabilities:								
Lont-term debt	3,236.6	24	3,392.2	24	2,786.8	23	2,535.2	23
Preferred stock	52.4	2	203.6	3	203.6	3	201.6	3
Common stock[2]	2,015.2	25	1,976.6	25	1,887.5	25	1,860.1	25
Capital and other surplus	174.3	10	272.6	11	263.8	11	242.4	9
Retained earnings	-407.4	24	-752.3	24	-431.6	24	-361.2	24
Total liabilities	6,556.1	26	6,660.3	26	5,912.1	26	5,351.8	26

1/ Eighteen of these ski areas are also included in table 55.

2/ One of 26 areas is a partnership therefore no common stock appears on balance sheet.

Calculating VTF Capacity of Tows and Lifts

Vertical transport feet per hour, abbreviated s VTF throughout this report, is a commonly sed measure of tow and lift capacity (Herringn 1967, p. 38-40). It is calculated by multiplyg the vertical rise in feet for each tow or lift y the number of skiers per hour the tow or lift n transport to the top of the hill. This calculaon is made for each tow or lift on a ski area nd the results added to obtain the VTF capaty for that ski area.

Two points should be noted. First, VTF allays refers to a "per hour" figure because it is e product of vertical rise and skiers *per hour.* econd, the length of the tow or lift has little lation to its capacity for moving skiers up the ll because the concern is with how fast the w or lift will deposit skiers at the top. Once e tow or lift is "filled" with skiers (for exame, each chair occupied), the rapidity with which e chairs reach the top of the hill (skiers per ur) determines the capacity, not the length of e lift.

Estimating Utilization of Ski-Area Capacity (Table 12)

Capacity is usually based on the capacity of e limiting, or "bottleneck," piece of equipment. ki-area capacity was calculated by the method sed by Herrington (1967, p. 61-64) and others. ecause the method uses VTF in its calculations here is an implicit assumption that tows and ts are the limiting factor in a ski-area's productive capacity.

The annual capacity in number of skiers was tained by the following equation:

Equation 1

$$\frac{\text{Skier Days}}{\text{Season}} = \frac{\text{VTF}}{\text{Hour}} \times \frac{\text{Skier Day}}{\text{VTF}} \times \frac{\text{Hours}}{\text{Day}} \times \frac{\text{Days}}{\text{Season}}$$

he percentage of capacity utilization was callated by using skier days per season as the enominator and actual attendance as the numator. Industry utilization was obtained by viding the total reported attendance by the kier days per season added for reporting ski eas.

Following previous studies (Herrington 1967, Sno-Engineering 1965), it was assumed a skier uses 8,000 VTF per day and that ski areas are open 5 hours per day.[13] The number of days per season was assumed to be 48 and 75 as discussed below.

Two reasons for calculating capacity utilization are to see how fully the present investment is being used and to estimate how much new tow and lift capacity must be added to meet estimated increases in demand. Total days in the skiing season should be used when examining investment utilization because investment is made in the ski area with the hope of a return throughout the entire season. Further, the capacity of the ski area stands ready to be used at any time during this period, pending favorable weather conditions, and at least certain costs are incurred to maintain this readiness. Accordingly, the first part of table 12 uses the mean length of the Great Lakes skiing season (75 days).

The maximum capacity currently available should be considered if capacity utilization is examined to determine new tow and lift capacity requirements. This means utilizaztion should be based on capacity available to the market — the skiable days per season. Further, "hours per day" may be increased to allow for more lighting and "days per season" increased for more snowmaking.

It has been suggested that the demand for skiing shifts depending on the month, regardless of snow conditions. For example, skiers may be ready to participate in other sports by late winter or early spring. Shifts of this type should be considered (if they can be identified) and capacity calculated accordingly. Analyses of new capacity requirements were not made due to limitations of time, money, and data.

A second table was calculated to compare capacity utilization estimates of the Great Lakes industry with those of the eastern and western industries. The second part of table 12 uses the mean skiable days of reporting areas (48 days).

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[13] The Great Lakes skier may require fewer VTF per day due to the lower vertical rises. However, the author knows of no studies on the subject and uses this figure for lack of better information. If 8,000 is too large, the potential capacity is understated and the percent utilization overstated.

Estimating and Projecting Total Attendance (Tables 4, 5, 13 and 14)

The terms "skier day," "skier visit," and "visit" are all synonymous in this study. They all refer to one person visiting a ski area for all or any part of a day for the purpose of skiing.[14]

Estimating Past Attendance

Surprisingly, many ski areas did not keep attendance records, thereby requiring calculation from gross ticket receipts or outright estimation by the operator. Missing attendance data were of three types: (1) one or more years data but missing data at one or both ends, (2) two or more years data but missing data *between* them, (3) no attendance data.

With the first type of missing data, the annual percentage changes in reported attendance were used to project the existing data both forward and backward. This procedure uses the absolute attendance for each area and estimates only the changes in attendance from reported industry averages. The procedure assumes attendance at a given area changed in the same proportion as that of the industry.

With the second type of missing data, estimates were made by allocating the difference in attendance between available years to the intervening years in proportion to the industrywide changes.

With the third type of missing data, there were no attendance data available, therefore, regression equations were used to estimate 1967-68 season attendance. The attendance was then projected backward using the annual attendance changes as described.

One potential source of error is ski areas that opened and closed before the 1967-68 season may not have been included. For example, an area may have opened in 1960-61 and closed in 1963-64 and not had its attendance estimated during these four seasons. This means attendance could be underestimated in the earlier yars. If the earlier years are underestimated, the compound growth rate of skier attendance is overestimated; that is, attendance grew less rapidly than shown.

Projecting Attendance

Attendance may be projected in many ways Perhaps the simplest method is to project solel on the trend of attendance over time. Howeve time trends do not reflect changes in factors tha may affect future attendance. Estimation usin time trends is dangerous because the factors ma change causing unforeseen changes in attendanc This disadvantage may be offset by the eas of predicting the future value of the time-tren variable. That is, it is easy to predict accuratel a variable representing the year 1975, but it ma be difficult to predict accurately disposable pe sonal income for 1975.

A least squares, stepwise, multiple linear r gression was used to examine the relationshi between thousands of skier visits per millio population, year, real disposable personal incon per household, number of ski areas, and th reciprocals of year and real disposable person income per household. The first statistically si nificant equation[15] was:

Equation 2

$$\underset{(4.430)}{\text{VISITS}} = 534.588 - 30{,}684.941\ \underset{(2{,}748.548)}{1/\text{YEAR}}$$

$$R^2 = 0.9541 \qquad F = 124.629 \qquad \text{Residual d.f.} = 6$$

where: VISITS = thousand of skier visits per seaso per million study area population. Population is as of July 1 of the second year in the skiing season; e.g., 1967-68 population is as of 7/1/68.

1/YEAR = the reciprocal of YEAR, that is, one divided by YEAR.

YEAR = the last two digits of the first year in a skiing season; e.g., the value of YEAR is "67" for the 1967-68 skiing season.

This equation, which is nothing more than time trend, is the simplest to understand a use. Further, the use of only 8 years' data mak it doubtful that more sophisticated analysis justified.

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[14] This definition is identical to Herrington's (1967, p. 32), but apparently differs from Sno-Engineering's (1965, p 3).

[15] The figures in parentheses immediately neath the dependent variables are the standa errors of the estimate; those in parentheses neath the independent variables are their sta ard errors. This convention will be followed he after.

When the year and its reciprocal are not allowed to enter the regression the following significant equation is obtained:

```
                                              Equation 3
VISITS = 100.545 +0.490 NOARES -875,763.750 1/DPYHSA
(4.353)           (0.169)        (Not available)
   R2 = 0.9630    F = 65.146    Residual d.f. = 5

where:  VISITS    = as above
        NOARES    = the number of ski areas operating
                    in the industry
        1/DPYHSA = the reciprocal of DPYHSA
        DPYHSA    = disposable personal income per
                    household in the study area
                    deflated to the 1957-59 base by
                    the Consumer's Price Index--All
                    Items.  Income is as of December
                    31 of the first year in a skiing
                    season which, in effect, lags
                    income a year.
```

Total skier visits were estimated by calcuating VISITS and multiplying it by the study area population from the Bureau of Census' eries I-B projection.

Projections should be used cautiously. Equaon (2) is a time trend, and will always project ncreasing attendance, regardless of changes in ther variables. In addition, both equations are ased on past experience and relationships that ay change in the future. Although the projecons are expected to be reasonably accurate over period of years, they could be inaccurate for ny particular year. For instance, there may an extremely warm winter resulting in deeased attendance.

The Skier Sample[16]

The study area was stratified and within ach stratum a list was made of each day each i area in the sample was scheduled to be open. ne such day (including any night operation) as called an area-day. Area-days were selected ith probabilities proportionate to size of attendnce on weekdays and weekends-holidays. The ample was controlled to increase the probability that no two areas within a stratum were selected on the same day and that days selected were distributed over the entire season (table 57).

The final stage was a systematic sample, after a random start, of vehicles entering ski-area parking lots. The vehicles were treated as unequal clusters with all skier occupants 13 years and older included in the sample. A skier is anyone who skied one or more times during the 1968-69 season at a ski area in the five-State region. Occupied rooms at ski areas were treated the same way as vehicles.

Table 57.—*Distribution of selected area-days by month and day of week*

Month	Number of area-days		
	Total	Weekend holiday	Weekday
December	33	24	9
January	37	21	16
February	37	20	17
March	32	20	12
April	1	0	1
Total	140	85	55

Response Rates

There were three opportunities for nonresponse: at the ski area for a particular area-day, when skier names and addresses were requested, and when mail questionnaires were not returned. Four out of 140 area-days were missed; equaling a 97 percent response. Just under 90 percent of the selected entrants returned name and address cards and about 66 percent of the mail questionnaires were returned after an original mailing and two followups. Detailed responses to the mail questionnaire were:

Names and addresses obtained	5,893
Names deleted by unbiased methods	2,199
Questionnaires mailed	3,694
Questionnaires returned but not in population	136
Base for response rate calculation	3,558
Questionnaires returned but unusable	74
Questionnaires not returned	1,134
Usable questionnaires	2,350

The calculating formulae for population estimates and variances are available upon request.

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[16] The sample design and attendant formue were developed by Leslie Kish, Program Diector, and Martin Frankel, Research Assistant, urvey Research Center, Institute for Social Researcharch, University of Michigan. The author asumes full responsibility for any errors or misplications.

Statistical Analysis of Factors Associated with Financial Success

One dependent variable (ROR) and 15 independent variables were hypothesized. A least squares, stepwise, multiple linear regression was used to examine the relationships between the dependent and independent variables. A first round of regressions was run resulting in the curves presented in figure 5. All the observations were then classified by several categories, for example, by a series of price categories. The significant variables from the first round of equations were used to test the significance of the classifications and a second round of regressions run on the significant categories. The results of the second round of regressions are in figures 6 and 7.[17]

Measuring Financial Success

The rate of return on total investment (ROR), calculated from ski-area income statements and balance sheets, was used as the dependent variable. The number of areas analyzed was reduced to 27 because calculating ROR required both financial statements. ROR was defined and calculated by the formula:

Equation 4

$$ROR = \frac{PBT - CG - NOPY + INT}{TL - CL + LTDCP}$$

where: PBT = profits before Federal income tax
CG = capital gains
NOPY = other nonoperating net income
INT = interest
TL = total liabilities
CL = current liabilities
LTDCP = long-term debt currently payable.

Where possible, a 2-year average for the 1966-67 and 1967-68 seasons was calculated. Nine ROR's were for the two seasons and 18 for the 1966-67 season only. The range of ROR values was −25.33 percent to +25.12 percent.

Independent variables in the first round o regressions were:

BARCAP	Persons bar capacity.
COMPAR	Number of competing ski area within 20-minute drive.
NITES	Nights per week skiing is provided.
NOFSE	Number of off-season service provided.
NOINS	Number of certified ski instructors.
POPDIS	Population within 250 miles ski area decayed over distanc
PRICE	Mean price per skier visit.
RORC	A dummy variable indicatin rope only facilities versus com bination rope and cable facilitie
SLPGRM	Man-days of slope grooming p season per acre.
SLPVA	A compound variable measurir slope variability.
SNO	The sum of the daily inches snow on the ground during th skiing season.
TEMP	The sum of the daily maximu and minimum temperatures du ing the skiing season.
TOLFT	A compound variable measuri the number and types of ro and cable facilities.
VERDRP	Feet of maximum vertical dro
VTF	Vertical transport feet per hou as previously defined.

Results of the First Round of Regressions

The stepwise regression was constructed that every independent variable's relation to t dependent variable is examined before any sin variable is added to the equation. Further, t relationships are examined after each varia is added and any variables that have beco superfluous are deleted. This process continu usually, until a preset level of significance reached. Using a stepwise regression mea every factor listed above was examined for relationship to ROR.

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[17] For a more detailed discussion see Leuschner, William A. Factors associated with financial success in the midwestern skiing industry. (Unpublished Ph. D. dissertation on file at Univ. Mich., Ann Arbor.) Readers with specific questions are invited to correspond with the author.

The results of the first round of regressions were:

Equation 5

$$\begin{array}{lll} & \text{ROR} = 14.545 & - 10{,}907.371\ 1/\text{VTF} \\ & (12.620) & (4{,}261.242) \\ \text{F Level} & 0.05 & 0.05 \\ & R^2 = 0.208 & \text{Residual d.f.} = 25 \end{array}$$

When vertical transport feet were deleted a second equation was obtained:

Equation 6

$$\begin{array}{llll} & \text{ROR} = -58.407 & + 32.807\ \text{PRICE} & - 3.968\ \text{PRICE}^2 \\ & (11.565) & (9.779) & (1.341) \\ \text{F Level} & 0.01 & 0.01 & 0.01 \\ & R^2 = 0.361 & \text{Residual d.f.} = 24 & \end{array}$$

No other significant equations were obtained when PRICE and PRICE squared were also deleted.

Classifying ROR

Equations 5 and 6 showed the empirical importance of vertical transport feet and price. Next, ROR was classified by these variables to hold their effect constant. The 27 observations were placed in a series of categories depending on an area's average price and VTF. The VTF categories were:

Category 1 – 1,500,000 VTF or more
Category 2 – 700,000 to 1,499,999 VTF
Category 3 – Less than 700,000 VTF

and the PRICE categories were:

Category 1 – Less than $3.00	(PRI1)
Category 2 – $3.00 to $4.00	(PRI2)
Category 3 – Greater than $4.00	(PRI3)

Two more classifications were made. First the 27 observations were classified by whether the areas were rural (RUROR) or urban (URBROR). An urban ski area was one located within 50 miles straight line distance of a city with 50,000 or more population. All others were considered rural. The final classification was by the ski area's geographical location. Lakes Michigan and Huron form a natural barrier around Michigan's Lower Peninsula. The Mackinac Bridge forms a pecuniary and psychological barrier to the north. It seemed likely that customers of Lower Peninsula and Indiana ski area would come from a more restricted area and might differ from those using other ski areas. Accordingly the observations were classified by whether they were in Michigan's Lower Peninsula or Indiana or whether they were in the rest of the study area.

Equations 5 and 6 were then calculated by category within classification. The price and urban/rural classifications were significantly different from the first round of regressions.

Results of the Second Round of Regressions

A second round of stepwise regressions was run on each of the categories for the price and urban/rural classifications. The number of independent variables was further reduced before the second round for technical reasons. Those variables deleted were TEMP, NOINS, COMPAR, NOFSE, and, for the price classification only, PRICE.

The significant results for the price category were:

Equation 7

$$\begin{array}{llll} & \text{PRI2} = 114.269 & - 0.029\ \text{VTF} & - 80{,}338.500\ 1/\text{VTF} \\ & (7.541) & (0.011) & (21{,}732.723) \\ \text{F Level} & 0.01 & 0.05 & 0.01 \\ & R^2 = 0.666 & \text{Residual d.f.} = 10 & \end{array}$$

Equation 8

$$\begin{array}{lll} & \text{PRI3} = 17.582 & - 11{,}894.562\ 1/\text{VTF} \\ & (5.190) & (2{,}786.180) \\ \text{F Level} & 0.01 & 0.01 \\ & R^2 = 0.722 & \text{Residual d.f.} = 7 \end{array}$$

No further significant equations were obtained when VTF was deleted from both of these price categories.

The significant results of the urban/rural classification were:

Equation 9

$$\begin{array}{llll} & \text{URBROR} = -192.295 & + 0.010\ \text{SNO} & - 0.015\ \text{VTF} \\ & (5.104) & (0.002) & (0.006) \\ \text{F Level} & 0.01 & 0.01 & 0.05 \\ & & - 37{,}512.016\ 1/\text{VTF} & + 137.866\ \text{PRICE} \\ & & (8{,}404.629) & (17.888) \\ \text{F Level} & & 0.01 & 0.01 \\ & & - 18.835\ \text{PRICE}^2 & \\ & & (2.967) & \\ \text{F Level} & & 0.01 & \\ & R^2 = 0.921 & \text{Residual d.f.} = 9 & \end{array}$$

All significant equations were checked for statistical validity using residual plots except equation 8, which had too few observations to make plots meaningful. In addition, the correlation matrices and standard errors were examined as checks on multicollinearity. The author was satisfied these checks showed no serious violations of the regression model.

Warnings About the Results

All statements based on these regressions are subject to the following warnings:

1. The use of stepwise regressions means the equations should be considered untested hypotheses.

2. There is great diversity among ski areas. The regressions are an average and there can be large divergencies from the average.

3. Except for equation 9, the amount of variation explained by the equations ranges from 21 percent to 72 percent. This leaves ample room for differences in ROR due to unexplained variations.

4. The results are based on 27 ski areas that happened to supply the financial data required. Strictly speaking, generalizations cannot be made about the industry. However, inspection of these areas shows a good distribution of values over the dependent variables, States, VTF sizes, prices, and other independen variables.

5. A factor's lack of statistical significanc does not mean it also lacks practical significance

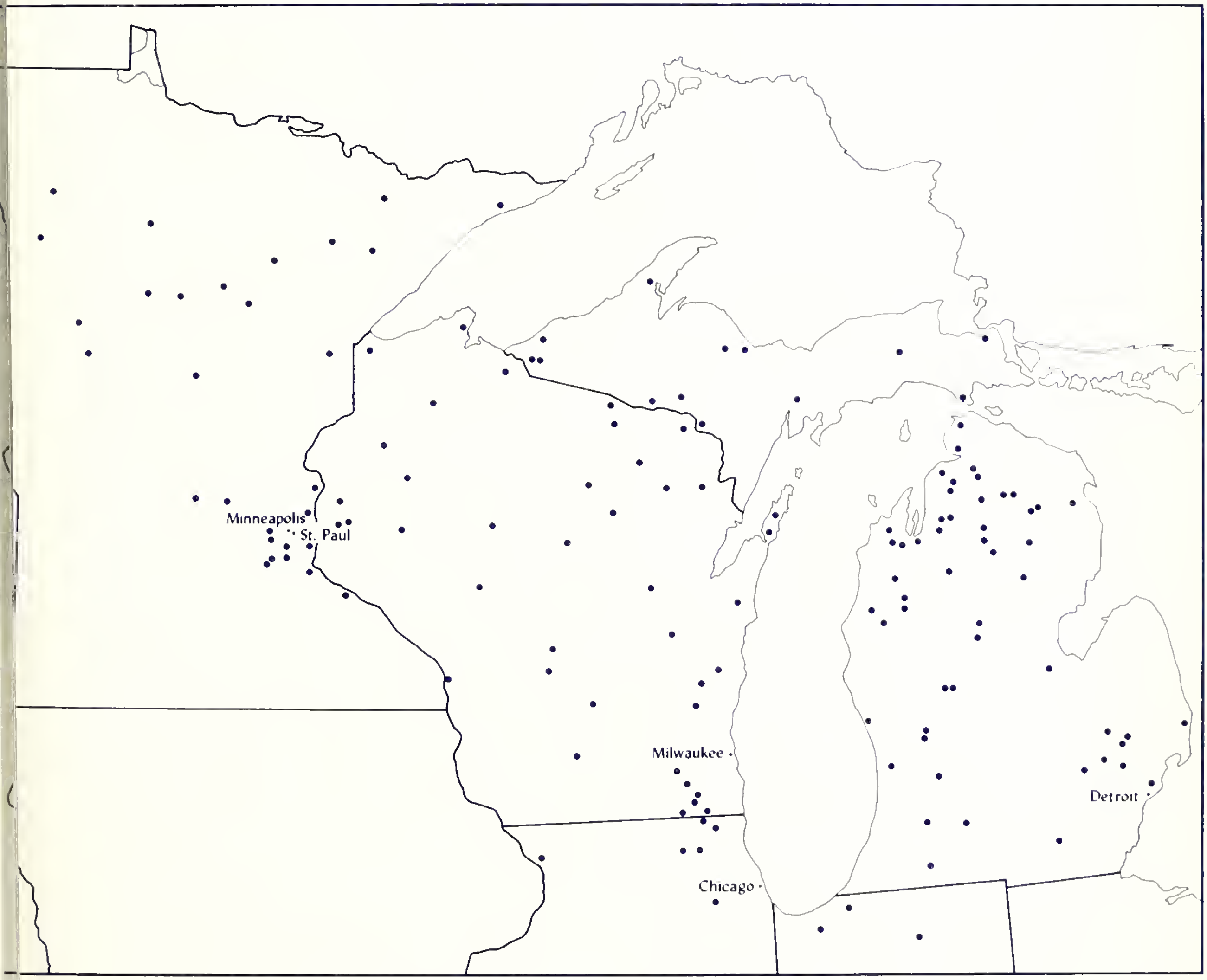
Snowmobiling and Skiing

In the year 1970 snowmobile ownership ma reach the half million mark in the Great Lake area. Does this rapidly growing sport compet with or complement skiing?

Our study showed that organized snowmobil activity is adjacent to half of the ski areas. A majority of these areas reported no effect, o an increase in skier attendance due to snowmobiling. One-fourth felt that snowmobilin increased their ski shop merchandise sales, whi almost half saw a beneficial effect on food an beverage sales. Some felt that snowmobile renta would increase skier attendance.

Although snowmobilers, on the average, a probably older than skiers, many people enjo both sports. Nearly half of all Great Lake skiers had ridden a snowmobile within the la 2 years. Many reported that snowmobiling i creased their winter outdoor activity an averag of 15 days. The compatibility of skiing an snowmobiling may encourage the developme of winter sports centers offering both.

LOCATION OF SKI AREAS, 1967-68 SEASON.



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USDA FOREST SERVICE
RESEARCH PAPER NC-47
1970

PROCEEDINGS OF THE NINTH LAKE STATES FOREST TREE IMPROVEMENT CONFERENCE

AUGUST 22-23
1969

NORTH CENTRAL FOREST EXPERIMENT STATION
FOREST SERVICE
U.S. DEPARTMENT OF AGRICULTURE

North Central Forest Experiment Station
D. B. King, Director
Forest Service — U.S. Department of Agriculture
Folwell Avenue
St. Paul, Minnesota 55101

Acknowledgments

The Ninth Lake States Forest Tree Improvement Conference was held at Itasca State Junior College and the University of Minnesota, North Central Forest Experiment Station, Grand Rapids, Minnesota, on August 22-23, 1969. The School of Forestry, University of Minnesota, St. Paul, was the host institution.

At the Conference, tree breeders, forest geneticists, and foresters from the Lake States were joined by the participants in the Lake States Post Conference Tour of the 2nd World Consultation of Forest Tree Breeding. Eleven visitors from Australia, Canada, Finland, Greece, Malaysia, Republic of South Africa, Sweden, and the U.S.A. discussed research results and breeding techniques with the Lake States tree breeders.

For the benefit of the foreign visitors, there were brief reviews of the current status and recent results of tree improvement endeavors in the Lake States, which appear in these Proceedings. Also included are descriptions of field-trip stops to the University of Minnesota's genetics tests in the vicinity of Grand Rapids, Minnesota. Descriptions of the stops at the Cloquet Forest Research Center of the University of Minnesota have not been included since most of the research and demonstrations pertained to silviculture and forest management.

The Tree Improvement Committee wishes to thank the University of Minnesota and its staff for hosting the Conference, the Blandin Paper Company for refreshments on the field trip, and the North Central Forest Experiment Station of the USDA Forest Service for publishing these Proceedings.

Scott S. Pauley, Chairman[1]
LAKE STATES FOREST TREE IMPROVEMENT COMMITTEE

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[1] *Prepared prior to Professor Pauley's death, April 18, 1970.*

CONTENTS

FOREST GENETICS RESEARCH AT THE UNIVERSITY OF MICHIGAN

Burton V. Barnes
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Tree Genetics

(Burton V. Barnes)

The purpose of the research program, as evienced by results as well as current research and ture direction, is to add to the knowledge of e ecology and genetics of forest trees. Alough we are interested in the practical gains at are possible and being realized in practical ee improvement, our contribution is in basic udies that stimulate and challenge young scintists and provide the basis for silviculture and ee improvement practice. Our main interests e in genecology and evolution of woody plants.

Major emphasis is placed on the aspens (*Poplus tremuloides Michx.*, trembling aspen and P. *randidentata Michx.*, bigtooth aspen) and birches primarily *Betula alleghaniensis Britton*, yellow irch but also associated species *B. papyrifera Iarsh.*, paper birch; *B. lenta L.*, sweet birch; *B. igra L.*, river birch; and *B. pumila L.*, bog birch). tudies of natural variation and hybridization ave been or are being conducted in other hardood genera; *Prunus*, *Fraxinus*, *Acer*, and *Querus*.

We have emphasized the clonal growth habit f aspens (Barnes 1966) as the basis of ecological nd genetic studies. This was used effectively in study supported by the North Central Forest xperiment Station by Copony (1969) who ound significant differences among clones in the ncidence of hypoxylon canker of trembling asen. The range in canker incidence for all clones as 10 to 90 percent; the range was wide in all ve sites studied. In many cases, nearby or adacent clones showed significant and striking diferences. The marked phenotypic differences beween clones and low variability within clones uggests a relatively strong genetic control.

Natural hybridization between bigtooth and trembling aspen is relatively common in southeastern Michigan, but apparently much rarer farther north. We are studying hybridization and hope to learn if introgression is subtly changing the genetic composition of the southeastern Michigan areas and away from it. Five hybrids were recently found by Andrejak (1968) in a seedling population in Washtenaw County. Analysis of leaf and bud characters indicated they were F_1 hybrids. Considerable overlap was observed in flowering of the parent species, so it is clear that flowering time is not a complete isolating barrier. Seeds of hybrids were highly germinable. Although introgression has not been adequately demonstrated there does not seem to be any genetic barrier to hybridization or backcrossing. We have found hybrids competing successfully with both parents well into fruiting age.

We are also investigating the possibility that trembling aspen in certain areas of the West is of ancient hybrid origin. Barnes (1967) hypothesized that *P. grandidentata*, or an Asian aspen such as *P. tremula* var. *Davidiana Schneid.*, which are likely members of the mid-Cenozoic forests of western North America, may have hybridized with trembling aspen.

Other active aspen research includes a study of clonal structure and suckering behavior of trembling aspen in Manitoba, Canada, by doctoral student G. A. Steneker, and a study of the significance of corticular photosynthesis by doctoral student Robert K. Shepard.

Intensive investigations of natural variation and hybridization of yellow birch and associated species are being conducted in the Lake States and the Appalachian Mountains. Dark and tight-barked birches, often observed in southern Michigan and elsewhere at the southern edge of the range of yellow birch (Dancik 1969), were investigated (Dancik 1967). Despite reports of their being sweet birch, they are closely related to the classically described yellow birch in chromosome number (6x = 84) and foliage characters. Studies in other portions of the range of yellow birch are underway to establish the pattern of phenotypic variation along ecological gradients and between widely separated populations such as those in northern and southern Michigan.

As the basis for these studies and companion ones, populations of yellow birch and sweet birch were sampled in various regions of the Appalachians exhibiting different climate, topography, soils, and vegetation. For example, in the Appalachian Mountains collections of foliage, fruit, pollen, and wood cores were made by doctoral student Terry L. Sharik along altitudinal transects in each of 5 physiographic regions from southwest North Carolina to northern Vermont. Currently, phenotypic variation is being related to site types and ecological gradients. Based upon this framework the mechanisms of genetic differentiation along certain environmental gradients will be investigated in the next phase of the study.

Hybridization in birches is also under study. Hybrids of yellow birch and bog birch are frequent in southeastern Michigan. However, evidence of intogression was not found (Dancik and Barnes, in press). Approximately 15 natural hybrids of yellow birch and paper birch have been found in Michigan, and others located in New Hampshire and Minnesota. We are studying the amount of gene flow between these species. Two individuals, putative allopolyploides of *B. X. purpusii* Schneid. (yellow birch and bog birch) have been found.

Since 1958 we have participated in an international provenance test of European larch organized by Professor R. Schober, University of Göttingen, Germany. Two test plantations were established in 1960 and 1961 in Washtenaw County involving 12 and 20 sources respectively. Survival of all sources was satisfactory to excellent. The best height growth was by plantation sources such as Schlitz, Dobris, and Neumünster. Sudeten larch sources from Czechoslovakia have done about as well. Poorest growth was exhibited by high elevation sources in the French alps. Average total height for the fastest growing source (Schlitz) was 24 feet in 11 years from seed; the slowest growing source averaged only 9.5 feet in 10 years. Results of these tests are encouraging and it is now time to test the fastest growing sources on a commercial basis.

Tree Physiology

(Robert Zahner, Professor)

Research in wood formation at the University of Michigan is centered around the influence of site and weather on cambial growth in hardwoods. Before the true effects of these external factors can be assessed, much remains to be learned about intrinsic growth patterns in both ring porous and diffuse porous types. Thus, in several studies we are attempting to sort out the effects of poor soils and adverse weather conditions on the relative production of fibers and vessels as compared with normal annual ring growth. We are working with three species of ring porous trees (*Quercus rubra L.*, *Fraxinus americana L.*, and *Carya glabra Mill. Sweet*), and with three of diffuse porous types (*Populus grandidentata*, *Betula papyrifera*, and *Acer rubrum L.*). Studies include numbers, sizes, and distribution of vessels and fibers currently produced in trees preconditioned the year before by drought, and in trees on dry, infertile sites; both as compared with those produced in "normal" trees. Results indicate that the ratio of fiber to vessel production in some species (e.g., pignut hickory and paper birch) is far more sensitive to environmental influence than in others (e.g., red oak and bigtooth aspen).

Entomology

(Fred B. Knight, Professor)

Stem Borers in Aspen

The work on two *Cerambycid* borers of small aspen stems has continued during 1968-69. We have completed partial life tables on both species *(Oberea schaumi and Saperda inornata)* and have identified the critical periods in their life cycles. These periods are associated with the adult and egg stages of the beetles. The adults appear to be sensitive to many mortality factors. Then when eggs are laid, many fail to hatch or if hatching does occur, the larvae often fil to become established. We are concentrat-ig our biological research on these critical times i the life cycles.

This year we have been doing some specialized search on clonal relationships. Early and purely ntative results show that the faster growing ones are resistant. Adults readily lay eggs on l clones but larval establishment may be rectly related to these clonal characteristics. 'e plan to continue this work during the next w years.

sects on Sugar Maple

This summer (1969) we began studying some the insects causing deformities in sugar maple. 'e will concentrate first on those feeding early the season on buds and later in the summer using leaf rolls on the trees. The species are own, but little is understood of their host references and bionomics. We know that a large ortion of bud mortality is insect caused and at therefore much of the deformity in trees ay be insect related. Further research on these sects may contribute much to the improvement f sugar maple.

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PROVENANCE TESTING AT MICHIGAN TECHNOLOGICAL UNIVERSITY

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The location of M.T.U. in the Keweenaw Peninsula of Upper Michigan provides some unique advantages and disadvantages in provenance testing and tree improvement research. Extremes in summer and winter temperatures are uncommon because of the moderating effect of Lake Superior. Near the Lake we have about 140 frost-free days while inland the frost-free season is only 80 days or less. Many exotic species of plants will grow in the Houghton locality but are prevented from naturally migrating here because of the cold belt to the immediate south.

Heavy snowfall does occur, averaging about 200 inches per year, and is a serious problem in provenance testing. Heavy snow accumulations affect even small pole-sized trees. The lower 4 feet of branches are repeatedly pulled down and out, weakening the stem at the node. Most trees in plantations have some type of basal crook.

Scotch Pine

Scattered plantings of Scotch pine are found throughout our area. The seed source is unknown but in most of these plantings, some of which are 30 or more years old, growth and reproduction is excellent. Where Scotch pine has been planted in mixtures with other conifers, the reproduction is almost all Scotch pine.

Our first effort in provenance testing was the NC-51 Scotch Pine Project initiated by Dr. Wright of Michigan State University. This was planted in 1961. Eight replicates of eighty sources were planted on the Keweenaw Peninsula and two replicates were planted at the Ford Forestry Center, 50 miles south. This report considers only the Keweenaw planting, the Ford Center planting having suffered excessive mortality. Analysis through the 1968 growing season indicates that:

(1) The varieties from western and central Europe: var. *haguenensis*, var. *hercynica*, var. *borussica*, var. *polonica*, and var. *pannonica*, are best suited for our area on the basis of height growth. The best of these are the Belgian sources, var. *haguenensis*, which have averaged two feet of height growth per year. Our results compare closely with Khalil's recommendations for central Minnesota, although his recommendations for northern Minnesota include some sources which are just average in our plantings.

(2) Among the sources tested, the Belgian group had the highest mortality, poorest form, and have produced the greatest number of lammas shoots.

(3) South European varieties had the most insect infestation but the incidence was not serious.

(4) Belgian sources had the most cones, with south European sources a close second. Only 6 percent of all trees had cones in 1969, however.

(5) Spanish sources have survived well, have slightly better than average height growth and appear to be one of the best for Christmas trees.

Tamarack

Our tamarack planting is part of the NC-51 Project initiated by Dr. S. S. Pauley of the University of Minnesota. The planting is part of the 1962 Accession and consists of 27 sources from Minnesota, Wisconsin, Michigan, Illinois, Ohio, Maine, Ontario, and Manitoba. The trees were planted in 1967 in sod and mulched with 3-foot squares of black polyethylene. Survival was over 90 percent through the fall of 1968. All of the trees were bent and about 25 percent of the trees were partially broken due to heavy snows of the past winter. We have tried to repair this damage and the results look good but height growth will likely be affected for several years.

The best source so far on a basis of height gowth is from Clare, Michigan. These trees gew about 3 feet in 1968. The trend to date ilicates the sources from central Minnesota, ctral Wisconsin, and central Michigan are best sted for our area with respect to height growth. Sme of the more southern sources are comparale to these but adjacent southern sources grow porly. Elgin, Illinois, and West Bend, Wisconsin, rnk with the best, but Waukesha, Wisconsin, hs performed relatively poorly. Poorest height gowth is from the Canadian sources from Manitoa and Ontario.

The 1964 Accession was planted in 1968 and cnsists of 21 sources with a greater geographic rnge than the 1962 Accession. Thus far surval has been over 95 percent.

Yellow Birch

The yellow birch planting consists of 30 sorces, range-wide, and was initiated by Dr. K E. Clausen of the North Central Forest Expriment Station's Institute of Forest Genetics, Rinelander, Wisconsin. The trees were outpnted in the spring of 1968 and are clean ctivated. Survival has been over 80 percent bt nearly every tree has been damaged by sow.

Sugar Maple

The sugar maple planting consists of 26 rangewde sources and is in cooperation with Dr. W. J. Gabriel, Northeastern Forest Experiment Station, Burlington, Vermont. Sugar maple has also been severely damaged by snow the past year. The stems have been broken and in many cases 1969 growth is entirely from sprouts. These trees are only slightly taller now than they were in the nursery two years ago.

Our other provenance plantings include Douglas-fir, ponderosa pine, white spruce, red pine, Austrian pine, white pine, red oak, and cottonwood. Analysis of these plantations is incomplete but information will be provided to anyone who may be interested in a particular planting.

Our other tree improvement work has been a study of phenotypic variation in specific gravity of sugar maple and in fertilization of northern hardwoods. Limited samples for the specific gravity study have been collected range-wide, and a more intensive sampling has been obtained from Upper Michigan. Preliminary results indicate as much variation within the merchantable part of the stem as among trees within a stand.

The study of fertilization of northern hardwoods is in cooperation with the Connor Lumber Company and is under the direction of Dr. Stephen G. Shetron and graduate student Ronald Heninger. These are first-year results in small and medium-sized sawlog stands for 600 trees stratified as to site. Sugar maple, red maple and basswood responded with about a 10 percent increase in diameter growth. There was no response with yellow birch.

NATIONAL FORESTS: RECENT TREE IMPROVEMENT DEVELOPMENTS

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This paper discusses the current status of a number of projects now under way on the National Forests in the Eastern Region of the USDA Forest Service.

Selection of Superior Trees

Initial selections are complete for one 50-clone orchard and nearly complete for a second orchard of white spruce. We are actively searching for superior tree candidates in black spruce, jack pine and red pine in the conifers, and in yellow birch, sugar maple and red oak among the hardwoods.

This year we revised our record form so that all of our superior tree records are now processed by ADP. We now have records on over 1,500 selections in all species. Keeping track of changes, records of grafting, etc., was becoming a big chore. This new system can be kept current with a minimum of effort.

Grafting

We had another busy year with our grafting program in white and black spruce and eastern white pine. We do all our grafting in greenhouses at the Toumey and Eveleth nurseries. This year we completed over 9,000 grafts with very high success. Our "take" on white pine has averaged 99 percent for the past 3 years. The spruces, while somewhat lower at 85 to 95 percent "take," are much higher than those reported in the literature. No small part of this success is due to the skill of the nursery crews and the nurseryman.

Oconto River Seed Orchard

Perhaps our biggest news item is the purchas of some farm land near Langlade within the Nicc let National Forest (Wisconsin). Nearly 500 acre of this tract has been under cultivation. We pla to develop most of our seed orchards within th area, making it one of the largest seed orchar developments in the U.S. Topographic work ha been completed on the open areas. Soil analys shows excellent structure. A contract has bee awarded for a deer exclosure fence. We plan establish two seed orchards and a large clon breeding arboretum here in 1969.

Our seed orchards are computer-designed the University of Wisconsin, using a progra built by Dr. G. Stairs. This program randomiz the placement of ramets on a hexagonal patte but with conditions. It maximizes the crossi pattern between clones and equalizes the nu ber of times each clone appears in the orchar We're using this same program with an 85-ac seed orchard for Missouri shortleaf pine.

Rust-Resistant White Pine Program

Selection of Candidates

The selection data are now complete for cooperators except the State of Michigan a the University of Wisconsin. As of this writi 946 selections have been reported, 921 have b screened, 686 have been accepted, and 25 sel tions remain to be checked.

Grafting

The cooperation we received in the scion collection work was again excellent this year. We collected scions from 219 selections, completing the grafting phase of the program for all cooperators except the State of Michigan and the University of Wisconsin. The few remaining trees will be grafted next year.

Seed Orchard

This spring all of the white pine grafts made during 1966 and 1967 will be outplanted in the Oconto River Seed Orchard. Five grafts from each accepted candidate will be planted in a clonal breeding arboretum where they will receive intensive care to help stimulate flowering. When the grafts begin to flower, we will start a controlled pollination program to produce seed for testing. This will supplement the field pollinations already in progress.

We also intend to plant an interim seed orchard consisting of 29 clones that we received from Dr. C. Heimburger in Canada, and five clones from Dr. R. Patton of the University of Wisconsin. These clones have all been tested for resistance to blister rust. The seed produced in this orchard will be tested to determine the clones that transmit the rust-resistance to their offspring.

Hybridization

Many of the scions grafted this winter produced conelets, suggesting that there will be a good cone crop for our pollination work this spring. We hope to repeat the crosses that did not produce the required amount of seed the first time, and begin pollination work on as many new selections as possible.

Additional work is being carried out in the development of black cherry on the Allegheny and Monongahela National Forests. We are also cooperating with Region 8 in the development of shortleaf pine.

TEN YEARS OF PROVENANCE RESEARCH AT MICHIGAN STATE UNIVERSITY AND THE NEXT STEPS IN TREE IMPROVEMENT

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Michigan State University's first provenance test was started in 1958, a little more than a decade ago. Tests in three more species were started in 1959. Others have been added until the list of species covered now totals 28. There are such experiments in eight species of pine, six of elm, three of spruce, two each of larch and the true firs, and one each of Douglas-fir, poplar, birch, cherry, walnut, arborvitae and oak. Some were started by Michigan State University. In other cases we received planting stock or seeds from outside the State. Almost all the experiments are part of the NC-51 regional tree improvement project and involve cooperation with others.

The decision to place major emphasis on provenance research was based on the supposition that the major portion of the genetic variability in any species is associated with geographic origin of the seed and that some superior nonlocal types could be found.

Genetic differences among trees originating from different parts of a species' range have been very large in Scotch pine, jack pine, ponderosa pine, southwestern white pine, Douglas-fir, white spruce, and white fir. In each of these species there have been two-fold or three-fold differences in growth rate between trees from different geographic areas. There were equally striking differences in other traits such as foliage color, hardiness, leaf length and terpene composition. In fact the list of variable traits is governed mostly by time available for study.

Important but less marked differences were found in another group of species. Eastern white pine trees from Tennessee and southern Ontario grew 10 to 20 percent faster than trees from more northern areas in southern Michigan plantations. (Southern Appalachian trees were not suited, however, to the northern Lake States.) Austrian pine from Greece outgrew the more commonly planted Austrian variety. Yellow birch from the southern Appalachians leafs out late than more northern types but so far in the Michigan tests all types have grown equally fast. Differences in growth rate were minor in Japanese larch but trees from Mt. Fuji produced cones earliest and trees from the northeastern part o the range were slightly more hardy than others.

The provenance research has uncovered virtually no differences in two species only. Many years ago Paul Rudolf and Ashley Hough o the USDA Forest Service established red pin experiments in the Lake States and Pennsylvania. The trees are now more than 40 feet tall. According to the last measurements, the best and poorest origins were almost alike. We have younger study which includes more origins but the results are the same. Northern white-ceda may fall into the same category. Scott Paule sent seed from all parts of the range. The seed germinated well and produced an excellent nursery experiment. The origins were indistinguishable at age 3.

Most experiments include several differen plantations, often in different States. Whe

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[1] *Results of a Wisconsin test of the sam material is discussed in the paper by R. M. Jeffers on page 18.*

possible, the same seedlots were planted in each plantation. Generally speaking, an origin which grew well in one plantation grew well at many other test sites. For example, eastern white pine from Tennessee grew most rapidly when planted in North Carolina, Tennessee, southern Michigan, and Iowa; white pine from the southern parts of the Lake States grew well when planted in several parts of the northern Lake States. This tendency was much more pronounced than was the tendency for the local origin to be superior. Seed procurement rules must constantly be revised as the provenance experiments provide data.

Reliability of the Results

The following practical recommendations can be made for tree planters in southern and central Michigan. Similar lists can be made for other areas.

When planting this species	*Use seed collected from natural stands in these areas*
Eastern white pine	Tennessee, southern Ontario
Scotch pine	Spain (for Christmas trees) Belgium and northern France (for fast growth)
Red pine	Most parts of range
Jack pine	Michigan's Lower Peninsula
Southwestern white pine	Central parts of Arizona and New Mexico
Ponderosa pine	Eastern Washington and Oregon
Austrian pine	Parts of Greece
White spruce	E. Ontario, S. Manitoba, also parts of Wisconsin and Michigan
Douglas-fir	Northern Idaho or central parts of Arizona and New Mexico
White fir	Central parts of Arizona and New Mexico
Japanese larch	Almost any part of natural range

The red and jack pine recommendations are based on 18 to 30 years growth in plantations of the North Central Forest Experiment Station. The others are based on much younger experiments and further results can be expected soon from experiments which are now very young. How reliable are such data?

A student, Warren Nance, studied this question during the past year. He remeasured our oldest plantations of Scotch pine, ponderosa pine, and eastern white pine. The nursery data on growth and hardiness were excellent indicators of future growth rate. As it turned out, our tentative recommendations at age 2 were almost as good as our most recent ones. The jack and red pine stories have changed little with increasing age. Few surprises have been forthcoming in other experiments up to 8 years old.

Pest damage complicates the picture, however. Several insects are now active in 10-year-old Scotch pine plantations, and a wait will be necessary to learn the resistance of different varieties. A 15-year wait may be necessary when selecting elms for disease resistance.

Time has not helped cure mistakes made in the nursery. If uneven germination or uneven watering produced uneven seedbeds, the experiment is still uneven and relatively unproductive of results after a lapse of 5 to 10 years. Similarly, the results of poor weed control or poor planting are nearly as evident now as the year after planting. The moral is clear – do an excellent job from the start.

Upsets in Theory

Theory also received a great deal of attention. A decade ago I hoped that the provenance research would lead to a very clear understanding of the processes of genetic formation of races and clines, and that with such an understanding we would be able to forecast what would happen under any set of circumstances. The answers are still far from good.

As of 10 years ago there were theoretical generalizations that races from warm climates grew fastest and that races from cold climates were hardiest. Both generalizations proved true in general, but there are some very embarassing exceptions. Eastern white pine from warm Virginia grows more slowly than does the same species from colder Pennsylvania and New York. The parental stands in Virginia were not selfed, were not on particularly cold microsites, or on poor soils. In the grand-white fir complex of the Rockies, grand fir from high elevations in northern Idaho suffered extreme winter injury (in southern Michigan) whereas there was no such injury on white fir from Arizona or on Douglas-fir and ponderosa pine from lower elevations in Idaho.

In each of four Rocky Mountain species an Arizona-New Mexico race grew rapidly and was distinct from races to the north. Migration between these races was inhibited by a wide treeless barrier. And in each of the four species the slowest growing trees came from Utah. But there the similarity in patterns ended. Selection pressure operated to produce a large amount of genetic variability within the Colorado population of Douglas-fir whereas there is little difference between Colorado and Alberta limber pine. Different responses to the same selection gradient were also evident in two European species. Spain produces Scotch pine with exceptionally short and dark green needles; Spain produces Austrian pine with exceptionally long and yellow-green needles.

Theoretically, 24 origins of white spruce, well scattered over the entire natural range, should cover the gamut of genetic variability in the species. Not so. We planted Hans Nienstaedt's white spruce experiment at Kellogg Forest in southern Michigan and used border stock of unknown origin (probably some place in Wisconsin or Michigan). The border trees have been unique in producing many cones and suffering heavy attack by a gall aphid although the crowns have not yet closed.

Progress on cause-and-effect relationships has been slow. There is as yet no good explanation for the resistance of the Ural Mountain variety of Scotch pine to the black-headed pine sawfly, for the earliest flower production on slow-growing sources of eastern white pine but fast-growing sources of Scotch pine, or for the high content of four different elements in the foliage of the coastal variety of ponderosa pine.

These problems are challenging, not frustrating. Evolution has been complex. Much more work is required before we can forecast whether a genetically unknown species will behave like eastern white pine (considerable geographic var ation) or red pine (almost no geographic vari tion), the effect on height growth of a genetic i crease in nitrogen assimilation rate, or wheth the genetic variation in a particular region wi be continuous or discontinuous.

Many of these theoretical questions have in portant practical implications and need solutio In general the solutions will come only fro further experimental work because there are to many gaps in existing population genetic theor

The Future

Insofar as Michigan is concerned, the job starting large range-wide provenance tests nearly completed. Maintenance and measur ment will continue on those now in the groun but I believe that most of the important resul will be forthcoming in the first 10 to 15 years each experiment.

An interdisciplinary approach promises to b successful in the solution of some of the the retical problems mentioned earlier. Dr. Ha over is bringing his physiological talents to be in some of the provenance experiments and studying differences in terpene chemistry, phot synthetic rates, and other internal characteri tics. He hopes to learn why some of the trees a fast growing. This would help when attempti to breed new trees for specific purposes.

The provenance experiments were prelimina and were meant to point to best regions fro which to obtain trees for more intensive breedi work. Two followup experiments have bee started in eastern white pine. Seeds were co lected in 1960 from 125 single trees located various parts of Michigan; seeds were collect in 1964 from 170 single trees or stands in t southern Appalachians. Similar progeny tes have also been started in jack, red, and Scot pines.

These followup experiments have alrea yielded some information on local variation pa terns and on the amount of genetic variability important growth traits. Differences amo stands have generally been more pronounced tha differences among trees within stands. The co relations between characteristics of the paren and of their offspring have been disappointing low. Thus, the practicing forester interested good seed for the immediate future need not practice stringent selection in his cone collection work. And the tree breeder interested in a better strain for the future needs to progeny-test his selections.

For the past 3 years Dr. H. D. Gerhold of Pennsylvania State University and I have been using one plantation of the Scotch pine provenance experiment as a breeding arboretum. We have made crosses between distinct varieties, hoping to produce hybrids with hybrid vigor or with a combination of the best characteristics of different varieties. The crosses are easily made but we still have to wait for the results. Much more of this work will be done as other species flower.

The provenance experiments also offer new opportunities in hybridization between species. Many successful hybrid combinations are now known. Among them are Japanese red pine X Austrian pine, Japanese red pine X Scotch pine, eastern white pine X western white pine, Japanese larch X European larch, Japanese larch X Korean larch and white spruce X Engelmann spruce. Average parents were used to make most of the hybrids available now. Even so, many exhibit desirable growth characteristics. It is virtually certain that even better hybrids can be produced by crossing selected types of the parental species.

RESEARCH IN FOREST GENETICS AND TREE BREEDING AT THE UNIVERSITY OF WISCONSIN [1]

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The genetics and breeding group in the Department of Forestry now has two faculty members and five graduate students. We are now a part of the Plant Breeding and Plant Genetics Group which includes six departments and twenty-six faculty members in an interdepartmental program of graduate teaching and research. Close cooperators are the Department of Plant Pathology at the University of Wisconsin, the Department of Natural Resources, and the U.S. Forest Service, North Central Forest Experiment Station, Institute of Forest Genetics, Rhinelander, Wisconsin. Our research materials include four coniferous genera and one hardwood genus. Current research is summarized below for each genus.

Abies

Geographic Variation in Balsam Fir

Provenance and one-parent progeny studies were initiated with seed collections in 1960-62. Three-year-old seedlings were distributed for nursery transplanting in Michigan, Minnesota, and Wisconsin, and were measured after two seasons in nursery transplant beds at three locations.[2] For the provenance material the rang of mean total height was moderate (13.1 to 23. cm.). Effects of nursery location and seed sourc were statistically significant (99 percent proba bility). The geographic pattern of variation wa not clear, except for the concentration of easter provenances as the five tallest. The shortest provenances were from the western portion of th range yet variable performance of collections fro Manitoba and western Ontario precluded an interpretation of an east-west axis of variation.

Analysis of variation in frequency of frost damage was made possible by an early May frost i the central Wisconsin transplant beds during th fifth growing season. No other station reporte frost damage. Fortunately, the seedlings in central Wisconsin were planted in three replicate The range of variation expressed as the percen age of undamaged seedlings was large (12 to 9 percent). Replicate and seed source effects we statistically significant (99 percent probability The pattern of variation for frost damage wa markedly different than the pattern for heig growth. The five least damaged provenances we from Wisconsin and Michigan. The five mo damaged were from Manitoba and western O tario. Simple linear regressions of frequency frost damage on latitude and frost-free peri of the seed origin were not statistically signi cant.

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[1] *Research reported is funded by Federal Hatch Act and McIntire-Stennis appropriations, the Wisconsin Department of Natural Resources, the University of Wisconsin Graduate School, and the University of Wisconsin School of Natural Resources.*

[2] *Mr. Richard Jeffers, Dr. Scott Paule and Dr. J. W. Wright assisted with heig measurements.*

Seed source height growth potential, as measured by total height of seedlings whose terminal shoot was undamaged, showed no association with frequency of frost damage within the population. This suggests that increased height growth may not necessarily involve increased danger of frost damage. For example, among the three tallest seed sources one ranked eighth (out of 59) in frequency of undamaged seedlings (58 percent); another ranked 48th (13 percent).

Variation in total height and frost damage was clarified somewhat by periodic measurements on longating shoots during the fifth growing season. The least frost-damaged provenance (Lower Peninsula of Michigan) was shown to have escaped rost by flushing about 10 days later than other measured provenances. Whether late flushing is ommon to other Lake States provenances is nknown. The value of late flushing as a frost-scape mechanism is uncertain because observations were limited to one growing season. Spring rosts are not uncommon in late May for central nd northern Wisconsin, so that the results noted ere may be largely incidental. The patterns of hoot elongation in one season suggested that ate rather than duration may be the principal eterminant of differences in total height.

A nested sampling of maternal progenies within x Wisconsin and Michigan stands showed unxpectedly high variation. The range of variation as similar to the range-wide provenance variaon. Progeny effects were greater than provenance effects for total height, whereas provenance fects were greater for frost damage as the result one extensively frost-damaged provenance om the eastern Upper Peninsula.

The transplants in central Wisconsin were eld planted at three Wisconsin locations in 969. The transplants in Minnesota were field anted at one location and the material in ichigan is scheduled for field planting in 1970. urvival has been excellent in all field plantings date.

Larix

Iterspecific Hybridization
th Japanese Larch

Interspecific hybrids of Japanese larch with estern larch, tamarack, Siberian larch, and Dahurian larch have been produced to identify species combinations with high growth potential. Crosses between Japanese larch and tamarack seem especially promising in terms of growth rate. Hybrid seedlings will be grown one more season in the seedbed before field planting. The seedlings will provide material for studies on techniques for hybrid identification with emphasis on terpenes and other biochemical constituents.

Picea

Mutation Breeding in
White and Norway Spruce

The effects of pollen irradiation on genetic variation are being studied in controlled crosses and selfings made with irradiated and nonirradiated pollen. The seedlings are completing their first season of growth. A first evaluation of radiation effects will be made in the nursery next year and subsequent evaluations will be made on field plantings.

Enzyme Analysis of White Spruce

Electrophoretic techniques for enzyme analysis are being used on leaf extractions to isolate enzyme systems suitable for genetic study and for demonstration of Mendelian segregation for isoenzymes. The objective of this approach is to provide techniques for population analysis of heterozygosity. The work is being done in cooperation with Dr. James King and Mr. Richard Jeffers of the USDA Forest Service, North Central Forest Experiment Station, Institute of Forest Genetics.

Soil Ecotypes of White Spruce

The natural occurence of white spruce in southeastern Ontario on soils of widely differing calcium content has prompted a study of potential ecotypic differentiation based on soil differences. Seeds from trees growing on five calcareous and five noncalcareous soils were provided by Mr. Mark Holst, Canadian Department of Fisheries and Forestry. The seedlings are being grown in hydroponic culture at three different levels of calcium concentration and two levels of acidity.

Response to cultural conditions will be measured by dimensional characters and by chemical analysis of foliage.

Pinus

Seedling Seed Orchards of Red Pine

A combined research and seed-orchard development project for the Wisconsin Department of Natural Resources was initiated with seed collections in 1963-65 from 310 trees throughout Wisconsin.[3] The sampling was designed to allow the estimation of how much variation in growth of red pine is attributable to maternal progenies within stands, to stands within climatic provinces, and to climatic provinces.

The seeds were sown in 1967 in a manner designed to minimize nursery site heterogeneity. A row of 5 seed spots was sown with 10 to 12 seeds in each of 35 randomized complete blocks. Border rows were also sown. The intent was to thin, by hand, each seed spot to one seedling soon after germination. Excellent germination was achieved but damping-off fungi assisted in the thinning despite seedbed fumigation and fungicide applications. Mechanical thinning was postponed until the end of the first growing season to minimize the amount of transplanting required. Plot mortality was then alleviated by transplanting seedlings from multiple seedling spots to empty ones. The overall result was adequate representation of seed lots in about 90 percent of the plots but seedling variability within plots was greater than intended.

Measurements of total height are currently in progress. The seedlings will be tagged by plot, lifted by replicate, and field planted at three locations in 1970. Field measurements will be made at plantation ages 8 and 15. Then the slowest growing 225 progenies will be rogued, as will all but the largest tree in each plot of each remaining progeny. The result will be about 800 trees per plantation to be used as sources of seed for the Wisconsin state nurseries.

The biological basis for an expected improvement of up to 5 percent in wood yield comes from progeny tests established in Wisconsin several years ago (Lester and Barr 1965). Whether significant improvement is achieved wi depend on the variation pattern revealed and th success of relatively early selection. The latte will be resolved by juvenile-mature correlatio studies (Lester and Barr 1966) before converting the progeny test to a seed orchard.

Interspecific Hybridization With Pitch Pine

Controlled pollinations have been made o pitch pine using pollen of sand pine and spruc pine. The objective of this study is to identif species combinations with a high growth potential for low quality sites. Seeds will be availab for testing in 1970.

Genetics and Biochemistry of Variation in Needle Tip Burn of Eastern White Pine

Ninety trees from throughout southern Wi consin have been phenotypically classed as r sistant or susceptible to needle tip burn. Sever grafts from each selection have been made an are being used in studies of response to ozon fumigation. Repeatability analysis of reactic to ozone fumigation will be used to estimate th extent of genetic control. A survey of possib biochemical differences between susceptible ar resistant types will be made.

Ulmus

In 1957 Dr. Eugene Smalley, Department Plant Pathology, began a program of screenii world-wide seed collections of elms for resistan to the Dutch elm disease. About 400 collectio have been screened and more than 1,000 resista individuals representing 8 species and seve putative natural hybrids are now growing in elm arboretum near Madison. Many of the trees have begun flowering in the past few yea and a program of genetics and breeding has be developed jointly between the Departments Forestry and Plant Pathology to determine pa terns of inheritance for disease resistance and ornamental traits. At present our interest concentrated on four species, American el Japanese elm, slippery elm, and Siberian el

Genetics of American Elm

Three crossing studies have been made wi American elm. The first was an attempt to redu

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[3] *D. T. Lester. Proposal for genetic improvement of red pine in Wisconsin. 16 p. (Unpublished.)*

the chromosome number (56) to the same level as other species in the genus (28) so that transfer of disease resistance might be possible through interspecific hybridization. Pollinations with highly irradiated pollen and with pollen of species which rarely, if ever, cross with American elms resulted in production of a few seeds but all seedlings had the chromosome number of American elm. Self-fertility is the most likely cause of these results.[4]

The frequent assumption that elms are self-sterile as a consequence of early development of the stigma (protogyny) has been examined in selfing studies over a three-year period. Forty-seven out of 57 selfed trees have produced seed in one or more years. Unfortunately, several of the 10 trees which failed to produce seed after one selfing have been sacrificed to highway improvement. At present, I have no individual of American elm in which self-incompatibility is certain. These results raise doubts about reports of interspecific hybridization with American elm (Britwum 1961, Johnson 1946, Smucker 1944). The existence of one authentic hybrid between American and Siberian elm indicates that crossing can occur but self-fertility seems to be common.[5]

Our most recent crosses in American elm have the objective of determining general and specific combining ability for resistance to Dutch elm disease. The resistant trees are survivors of artificial inoculations on 10,000 seedlings contributed by private nurserymen. Six of the 36 survivors flowered in 1969 and were used in the crossing scheme shown in table 1. Excluding he selfed combinations, an adequate number of eedlings was obtained for all but two crosses. Analysis of variation in growth and morphology will be made after one growing season and disease esistance will be tested by artificial inoculation in the second growing season.

Genetics and Breeding of Diploid Elms

From our collections of resistant diploid individuals 15 trees have been selected as potentially useful ornamentals. These trees are being propagated by root-sprout cuttings for evaluation in landscape settings. Included among these selections are individuals of Japanese elm, Siberian X Japanese elm hybrids, Siberian X slippery elm hybrids, and multispecies hybrids of European origin.

In our diploid crosses we are presently concentrating on Japanese, slippery, and Siberian elms. Some Japanese elms have the desirable features of red autumn leaf color, open branching, and moderately large leaves. Disease resistance seems to be high but we have not tested large numbers of Japanese elms. Slippery elm has open branching, a large leaf, and is known to produce vigorous hybrids with Siberian elm. Disease resistance is thought to be low, but has not been adequately tested. Siberian elm has high disease resistance and drought tolerance. These species and some hybrids have been crossed using single tree pollens and pollen mixes (table 2). Hybrids, authenticated by morphological comparison of progenies from intra- and interspecific crosses are now one or two years old. Each tree will be artificially inoculated and scored for reaction to inoculation within the next 2 years. Survivors will be tested again and then selected for ornamental traits.

Dutch plant breeders have been highly successful in developing genetic resistance to Dutch elm disease, and we have evidence from open-pollinated progenies of our Siberian elm X Japanese elm hybrid family that resistance to the Dutch elm disease can be maintained through breeding. This result coupled with the apparent ease of crossing among the three diploid species of current interest suggests that a high degree of resistance to the Dutch elm disease can be maintained while a broad range of variation in ornamental traits is being developed.

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[4] *Lester, D. T. Unpublished data.*

[5] *Personal correspondence with F. S. Santamour, Oct. 14, 1968.*

Table 1.—Crossing scheme for disease-resistant and susceptible American elms (R = resistant, S = susceptible)

	R1	R2	S1	S2	R3	R4	S3	S4	R5	R6	S5	S6
R1	X	X	X	X								
R2	X	X	X	X								
S1	X	X	X	X								
S2	X	X	X	X								
R3					X	X	X	X				
R4					X	X	X	X				
S3					X	X	X	X				
S4					X	X	X	X				
R5									X	X	X	X
R6									X	X	X	X
S5									X	X	X	X
S6									X	X	X	X

Table 2.—Summary of elm families from crosses 1968-69

	U. japonica	*U. pumila*	*U. rubra*	*U. pumila X japonica*	*U. rubra X pumila*
U. japonica	X			X	
U. pumila	X	X	X	X	X
U. rubra	X	X	X	X	X
U. pumila X *japonica*	X	X	X	X	X
U. rubra X *pumila*		X	X	X	X

FOREST GENETIC RESEARCH AT THE INSTITUTE OF PAPER CHEMISTRY

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Forest genetics research is carried on in the Genetics and Physiology Group at The Institute of Paper Chemistry. The Institute is located in Appleton, Wisconsin, and has a staff of approximately 300 people. It is a nonprofit research and educational institution that was started in 1929 and is affiliated with Lawrence University. The Institute is a graduate school granting M.S. and Ph.D. degrees. Research at the Institute encompasses not only work in the area of pulp and paper technology but also includes studies in biology, chemistry, chemical engineering, mathematics, and environmental control.

Work in forest genetics began in 1954 under the direction of Dr. Philip Joranson and emphasizes poplar species, primarily aspens, and includes some studies involving cottonwood and closely related species. Several species of *Larix* have also been outplanted with the view to future work with this genus.

Predictions made by the USDA Forest Service indicate that pulpwood consumption is expected to increase from 50.7 million cords (1966) to 127 million cords in the year 2000. Loss of good quality forest land to agriculture, recreation, and urban and industrial development suggests increased production will need to be accomplished on less-productive lands. The magnitude of the increased production suggests several approaches will be required to prevent serious raw material shortages. The most promising include (1) improved utilization, (2) intensive forestry, and (3) forest genetics. Studies under way that speak to these problems include the following research projects.

1. *Aspen genetics and tree improvement project.* — The objectives of this long term investigation include (a) developing trees of exceptional growth rate and wood quality and (b) developing trees for several types of soils including those that will do well on sandy soils of low fertility and respond to such intensive forestry practices as fertilization and irrigation. Forest genetics procedures employed include selection, hybridization, and polyploidy. Naturally and artificially produced triploids have been obtained and show considerable promise in the area of improved wood quality and improved growth rates.

2. *Investigation of methods for the production of maximum growth in natural and improved aspen.* — This project has the overall objective of demonstrating the biological potential of aspen and aspen hybrids. More specifically, the program emphasizes the intensive aspects of forest management and has the goals of (a) exploiting presently available genetically improved species of *Populus*, (b) developing rotation age and harvesting system information, and (c) establishing the biological feasibility of such intensive forestry practices as fertilization and irrigation.

3. *Investigation of methods of separating chip and bark mixtures.* — Utilization of small-size trees and better utilization of the limbs and tops of trees hinges upon chipping in the woods, bulk handling of the chips, and separating chip/bark mixtures prior to pulping. The objective of this project is to develop chip/bark separation procedures.

4. *Tissue culture research.* — Research on the use of tissue culture in forest genetics began at the Institute in 1962 under the direction of Dr. Martin Mathes and was taken over in 1964 by Dr. Lawson Winton. The objectives of this work were to investigate the usefulness of tissue culture techniques in studying growth and differentiation with the ultimate objective of starting with a single cell and developing an intact, normally functioning tree. Several trees have been produced from callus tissue; this apparently is a "first" with woody plants. Dr. Winton acknowledges the assistance of Dr. Karl Wolter, of the Forest Products Laboratory, in suggesting several chemicals which have helped make this feat possible.

RESEARCH AT THE INSTITUTE OF FOREST GENETICS RHINELANDER, WISCONSIN

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The Institute of Forest Genetics at Rhinelander, Wisconsin, was formally opened June 6, 1957, by the North Central (then Lake States) Forest Experiment Station, USDA Forest Service.

Originally, only the Forest Genetics Project was located at the Institute. Today, there are in addition: Pioneering Research and Radiobiological Studies of Northern Forest Communities. Information presented here will pertain only to recent findings of the Forest Genetics Project. The objectives of this project are fourfold:

1. Increase knowledge of the genetic constitution and the variation in populations of several forest tree species through basic research.
2. Develop guidelines that will enable tree breeders to plan realistic and efficient tree breeding programs.
3. Breed trees for local use in pilot size operations.
4. Develop efficient means for vegetative propagation of forest trees.

The genetics project has concentrated most of its research efforts on white spruce (*Picea glauca*), jack pine (*Pinus banksiana*), red pine (*Pinus resinosa*), and yellow birch (*Betula alleghaniensis*). In addition, a large number of other native and exotic species such as tamarack, balsam fir, northern white-cedar, Norway spruce, Serbian spruce, Engelmann spruce, and eastern white pine are being studied less intensively.

Variation of White Spruce Seed Sources

To date, the Institute has established provenance tests for white spruce, jack pine, red pine, eastern white pine, tamarack, balsam fir, northern white-cedar, Engelmann spruce, Norway spruce, and yellow birch.

Results of a white spruce seed source test have been published recently by Nienstaedt (1969). White spruce (*Picea glauca* (Moench) Voss.) seed from 28 sources was collected over the entire range of the species from Alaska to Labrador and south to the limits of the species. They were field tested for 5 years in 14 field plantings in a region from 42° to 48° N. latitude from North Dakota to New Brunswick.

Survival exceeded 80 percent in all but three test locations. There were significant differences in seedling survival among seed sources. This was due largely to poor survival of seedlings from three Alaskan sources and one Yukon source. Seed from other sources showed only minor differences in seedling survival.

Seed-source differences in seedling height growth after 5 years were highly significant. The interaction between seed sources and plantation environment was also significant. A rank correlation analysis suggested that this significant genotype-environment interaction resulted mainly from the relative magnitude of response of seed from various sources to the changing environments and not to systematic changes in the ranking of seed sources.

An analysis suggested that the seed from the southeastern portion of the range of the species, including the Lake States, southern Ontario, and parts of Quebec and New England, were well adapted to all the test sites, and that some were growing better than the average for the plantations. Seedlings from sources on the Chippewa National Forest; McNally Lake, Quebec; and Ashley Mines and particularly Beachburg, Ontario, are among the 10 best in all 14 outplantings. Seedlings from some sources, particularly the northern ones, showed less than average adaptive stability and grew particularly poorly on the best sites. It is of interest to note that seedlings from a Douglas, Ontario, seed source still maintain a 22 percent superiority in height growth over those from seven other sources after 29 years in a plantation in northern Wisconsin. They are 17 percent taller than the local white spruce (King and Rudolf 1969).

The best individuals of the Beachburg source in four of the field plantings are being clonally propagated. Seed and seedlings from these grafts will be compared to seed and seedling progenies produced from white spruce seed orchards now being established in the Lake States area.

The data from this study suggest that individual tree selection, testing, and breeding of individuals from the southeastern portion of the species range may lead to improved new strains adapted to the test region used in this study. Selection in southeastern Ontario and adjacent areas in Quebec may yield particularly good genetic types.

Heritability Studies of White Spruce

Breeding for Delayed Budbreak

In the spring of 1958, 9 early-flushing and 16 late-flushing trees were selected in a white spruce plantation near Rhinelander, Wisconsin. These selections were cloned in 1962 and ultimately field planted in a replicated test in 1965.

Spring frost injury was recorded in the nursery in 1963 and again in the field in 1965. In 1963 an average of 80 percent of all buds on the early clones and 13 percent of the buds on the late clones were killed by frost. In 1965 the frost injury on these two types was 61 and 13 percent, respectively.

Flushing was recorded in the years without frost injury. On the average, the late clones were delayed 15 days in flushing compared to the early clones; the overall difference between the earliest and latest clones was 21 days.

Beginning in June 1968, terminal elongation of the clones was measured twice weekly until elongation ceased. Late clones completed 95 percent of their elongation 10 days later than the early clones. On the average, late-flushing parental trees were 42 percent taller than early-flushing parents at the end of the 1968 growing season. Total elongation of the late clones in 1968 exceeded that of the early clones by 25 percent. Since the average growth period was essentially the same for the early and late types (35.3 and 31.7 days, respectively), the superiority in total height elongation of the late clones resulted from a greater growth rate. The daily growth rate of the late clones was more than 50 percent greater than that of the early clones.

In 1967, early-flushing, late-flushing, and average individuals were crossed with 10 of the original and three additional selections (nine late-flushing and four early-flushing). The flushing and growth of the 27 progenies from the 9 late-flushing parents were studied in the greenhouse with day-night temperatures of 75°-65° F., and in growth rooms with temperatures of 65°-55° F., and 60°-50°F. The flushing differential was greatest in the greenhouse and flushing was delayed by lower temperatures. Heritability estimates (h^2) varied with growth conditions and reached 0.705 for flushing in the greenhouse. The heritability of height growth, based upon a combined analysis of all 3 growth conditions, was $h^2 = 0.445$.

In northern Wisconsin, selection and breeding of the two latest-flushing clones could increase frost avoidance by as much as 43 percent (Nienstaedt and King 1969). The data suggest that simultaneous selection for late-flushing and rapid growth is possible in white spruce.

Half-Sib Studies

Since 1957 over 100 individual white spruce trees have been selected throughout the Lake States of Michigan, Minnesota, and Wisconsin. Open-pollinated seed from 32 of the selections was sown in the nursery in the fall of 1963; resulting seedlings were transplanted in the fall of 1965 and field planted in 1968. In the nursery, the 2-2 seedlings were measured for total height and current height growth. Two of the parents from Menominee County, Wisconsin, produced seedlings that made 63 percent more growth in 1967 than all of the 28 progenies included in the 1967 measurements.

The average annual height growth of the parents was strongly correlated with the current annual growth and total height of their respective progenies (r = .80 and .81, respectively). When the progenies from the 11 parents of similar age (36 to 42-year-old) were compared, those from the 5 fastest growing parents were growing at a yearly rate (in 1967) nearly 21 percent better than the average for their age group. The data indicate the general feasibility of phenotypic selection in white spruce (Jeffers 1969).

To date, 103 white spruce half-sib families have been field planted near Rhinelander, Wisconsin. Included in these plantings are 31 selections from the Ottawa Valley in Ontario, Canada. Some of the same families have also been outplanted in three additional areas including one southeast of Rhinelander (39 families), one in the eastern Upper Peninsula of Michigan (55 families), and one in Ontario, Canada (25 families).

Disease and Insect Resistance Breeding in Jack Pine

In a number of studies at the Institute, observations have been made on the differential susceptibility of jack pine to a variety of insects and diseases. One study includes trees from 30 sources in Michigan, Minnesota, and Wisconsin that have been in the field for 15 years in 17 locations throughout the Lake States. During the first 10 years the trees in these plantings showed significant seed-source-related differences in susceptibility to white pine weevil, red-headed pine sawfly, needle rust, jack pine needle-cast, bark beetles, eastern pine shoot-borer, and eastern gall rust[1] (King and Nienstaedt 1965).

The general approach in the study of pest resistance at the Institute is as follows:

1. Parents are selected in the seed source studies from sources that have demonstrated variation in pest incidence. This should insure genetic variation in the progenies. Occasionally, parents are seleted within natural stands, provided that pest incidence has been severe enough to suggest that undamaged trees did not escape infection by accident.
2. Selection of parents is followed by grafting to establish breeding arboreta.
3. The selected clones are crossed with several pollen testor parents to produce full-sib families.
4. Resulting full-sib progenies will be tested for pest resistance.
5. On the basis of the testing in step 4, a new cycle of selection will begin.

Six seed sources were selected in a Lake States jack pine seed source test on the Argonne Experimental Forest, Hiles, Wisconsin, on the basis of white pine weevil incidence – three low incidence and three high incidence sources. Four individual trees within each of the six sources were selected on the basis of good form and growth in the resistant sources and poor form and growth in the susceptible sources. All 24 individuals were clonally propagated in 1965 and outplanted near Rhinelander in 1968. Six additional parents, one from each of the six original sources, were selected for use as pollen parents. Many of the grafts flowered in the same year they were field planted, and controlled pollinations were started. The goal is to cross the six pollen parents with all of the 24 white pine weevil selections. These 144 full-sib families will be used to study the inheritance of variation in white pine weevil resistance.

Trees selected in the Institute's seed source studies, on the basis of eastern pine-shoot borer were grafted in 1966 and were field planted in 1968. Individual jack pines have also been selected in natural stands for resistance to jack pine budworm and pine tortoise scale. They have been grafted, and all the clones will be used in controlled pollinations as soon as they begin to bear female strobili in abundance.

Birch Genetics

A rangewide study of natural variation in yellow birch including 55 seed sources has been underway at the Institute since 1963. Ten test plantings have been established in the Lake States, New York, and New England, and three plantings in Canada. Second- and third-year height growth in the Rhinelander nursery varied greatly and was essentially random. Total height was not correlated with latitude, longitude, length of growing season, annual precipitation, average January temperature, or average July temperature. Diameter was only weakly correlated with latitude and length of growing season.

In contrast to its random variation in height and diameter, yellow birch exhibits a gradual north-south trend or clinal variation in growth initiation and cessation. In general, more northern sources begin growing earlier and cease growth earlier than more southern sources.

To determine whether the height growth variability noted in the seed source study might be due to individual tree variability within the sources, a study of 199 individual tree progenies representing 20 of the original 55 seed sources plus one additional source is now in progress. At the end of the first growing season there were highly significant differences in seedling height among the sources and among the individuals within each source (Clausen and Garrett 1969).

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[1] *King, James P. Pest susceptibility variation in Lake States jack pine. (Manuscript in preparation.)*

Variation in DNA Content of Several Gymnosperms

A series of experiments has been conducted to determine the nuclear volumes and the amounts of DNA per cell in 13 coniferous species. The correlation between these two factors was determined, and the relationship between these factors and the distribution of the species was studies.

Slides of root meristems were prepared. The amount of DNA per cell was determined by Feulgen microspectrophotometry and biochemical analysis. The slides were also used to determine the nuclear volumes. Volumes were found to vary by a factor of 11.3 while DNA per cell varied among the 13 species by a mean value of 3.2. Red pine (*Pinus resinosa*) had the greatest nuclear volume and northern white-cedar the smallest. These two species also had the greatest and least amounts of DNA per cell.

The data suggest that nuclear size and DNA/cell may have an adaptive value. In the species studied, those with small nuclear volumes and less DNA/cell tend to have a wider distribution (Miksche 1967).

Intraspecific Variation of DNA per Cell in White Spruce and Jack Pine

The amount of DNA per cell was established chemically and cytophotometrically for 17 seed sources of *Picea glauca* and cytophotometrically for 11 sources of *Pinus banksiana*. DNA Feulgen absorption per cell varied from the lowest to the highest amount by factor of 1.6 and 1.5 for *P. glauca* and *P. banksiana*, respectively. Intraspecific variation of histone was similar to the observed DNA variation.

Intraspecific DNA also varied directly with intraspecific nuclear volumes; i.e., seedlings from sources with smaller nuclear volumes have less DNA per cell while seedlings from sources having larger nuclear volumes possess more DNA per cell.

A regression analysis between DNA per cell and latitude provided evidence that eastern and western population series of *P. glauca* exist in the seed sources studied. In the western series, DNA content per cell increased with increasing latitude. This relationship was not found for the eastern series. Two-year seedling height growth results also demonstrated that eastern sources are different from western sources. Seedling heights in the western provenances varied inversely with DNA content; i.e., seed from sources with small amounts of DNA per cell displayed greater growth. Seedlings from eastern sources, on the other hand, did not display the inverse relationship between DNA amount and 2-year growth (Miksche 1968).

Tree Improvement Opportunities in the North Central States Related to Economic Trends — A Problem Analysis

To determine the orientation and emphasis that tree improvement programs in the north-central States should take, a problem analysis exploring the economic needs for forest products has been developed by David H. Dawson of the Institute of Forest Genetics, and John A. Pitcher, Region 9, USDA Forest Service. The analysis indicates that most emphasis on tree improvement programs should be placed on *Populus* spp., white spruce, black spruce, *Betula* spp., *Larix* spp., shortleaf pine, jack pine, white pine, and black walnut. The analysis will be published soon by the North Central Forest Experiment Station, St. Paul, Minnesota (Dawson and Pitcher 1970).

Radiation Research

Until recently the radiation research at the Institute was conducted within the Forest Genetics Project. A significant portion of the research was devoted to the study of the relative radiosensitivity of various species of forest tree seed and seedlings. This type of research is being expanded to include a comprehensive study of the response of natural forest communities to gamma radiation. All radiation research is now under a new project entitled "Radiobiological Studies of Northern Forest Communities." Studies ranging from cell biology and genetics through general forest ecology will be conducted within the project. The program will include several seasonal exposures as well as a single long-term study of chronic irradiation. Support for this project will be provided by the Atomic Energy Commission as well as the USDA Forest Service. A wide variety of research in the program will be undertaken by University cooperators.

Leader of the new project is Dr. Thomas D. Rudolph, Principal Plant Geneticist, who has been on the Institute staff for more than 10 years.

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INTER- AND INTRASPECIFIC GRAFTING AND BREEDING OF FIVE-NEEDLE PINES

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The forest tree improvement work at the enter was started in 1949 with the introduction f some of Dr. Riker's grafted eastern white pine elections. Subsequently a program of selecting, rafting, breeding and testing for resistance was egun. The work was initiated at Basswood ake (15 miles NE of Ely, Minnesota) rather han elsewhere on a more accessible site for two ain reasons. First, the high incidence of rust n the area along with large numbers of infected *ibes* bushes affords good material for testing or resistance to blister rust. Secondly, the area s situated on a peninsula. The resultant cool, oist condition is conducive not only to graft urvival but also to rust inoculation and infection.

In our first field grafting, Riker's white pine elections were grafted on 4- to 16-year-old native hite pine rootstocks. The grafts produced cone-ets which were used in our breeding program. any of the grafts have now grown beyond each of ladders and are still too small for climb-ng for controlled pollination work. For these, record of staminate and pistillate cone produc-ion is being kept, open-pollinated cones are athered, and seeds are used in our resistance esting program.

An interspecific grafting program was begun nvolving all combinations of the rootstock species astern white pine (*Pinus strobus* L.), red pine *P. resinosa* Ait.), jack pine (*P. banksiana* amb.), mugho pine (P. *mugo* Turra), and cotch pine (*P. sylvestris* L.); with scion species astern white pine, Swiss stone pine (*P. cembra* .), Korean pine (*P. koraiensis* Sieb. and Zucc.), acedonian white pine (*P. peuce* Griseb.), and imalayan pine (*P. graffithii* McClelland). Over period of 17 years, about 2,000 grafts have een made, with a good sample of each scion-ootstock combination and about 65 percent otal survival. These combinations were set up o investigate the possibility of using interspecific grafting to stimulate cone and pollen production and to provide breeding material low to the ground on hardy rootstocks.

Both staminate and pistillate cones are produced on most graft combinations. However, a difference in scion species response to rootstock species exists in some cases. For example, Korean pine grafted on red pine rootstocks produces only staminate cones and the graft combination is not too compatible. On eastern white pine rootstocks, grafts of this species are very compatible and both male and female cones are produced. However, Swiss stone pine tends to produce more pistillate cones on red pine than on white pine. Conelet production on other rootstock-scion species combinations also differs, but the grafts are still quite young. Differences in survival on various rootstock species also exist. In addition, time of pollen production is influenced by rootstock. We find, for example, that Swiss stone pine grafted on red pine produces pollen first each year, followed by Swiss stone pine on Scotch pine, mugho pine, and finally on white pine rootstocks. Differences in primary growth and needle length also exist among the various graft combinations.

Mugho pine shows promise as a rootstock species. It has the advantages of being low and convenient for work and of having numerous terminals for grafting. Consequently, many grafts can be made on one tree. To date, survival of all of the 5-needle pine species with which we are working is good on mugho pine. The grafts produce viable pollen and pistillate conelets. Controlled pollination of these have produced viable seed. Furthermore, we find that the rootstock on which the graft is growing tends to thicken and grow more or less in pace with the scion. Thus, a firm base develops which will support the normally more rapidly growing graft for some time. If such results are consistently obtained, mugho pine could serve as a very useful rootstock for experimental pine seed orchard work.

As the grafts produce staminate and pistillate conelets, both inter- and intraspecific pollinations are made. Every combination is repeated 3 years before it is abandoned as incompatible. Reciprocal crosses are made whenever possible. Seeds produced are grown in our nursery for resistance testing and outplanting.

Pollen is collected and kept separate by scion and rootstock species for determination of the effect of rootstock species on pollen viability. Each sample is tested for viability as soon as collected. Pollen is frozen over silica gel in a dessicator and sealed in a vacuum equivalent of 1 to 2 mm. mercury for one half hour. To date, we have been able to keep pollen viable for five years using this method.

The seeds are stratified and planted in nursery beds, grown for two summers and then inoculated. Best results have been obtained by wrapping the infected *Ribes* leaf around the needles of the seedling, thereby creating a miniature moisture chamber in which high infection results. If these *Ribes* leaves are too small – for example, when using *R. hirtellum* – a larger herbaceous leaf, such as one from *Rubus idaeus* is wrapped around the outside to provide pinning material and create the moist condition. Nursery boxes are covered with burlap and kept moist for 3 days. A consistent inoculation within each nursery box is indicated by the even distribution of yellow needle lesions found the next spring.

Some indication of variation in the rust is indicated by the differences in amount of infection obtained on identical samples of several white pine selections when inoculated separately using three different species of *Ribes* as inoculation sources. Also, a difference in lesion types is frequently observed. Both small discrete lesions and large coalescing lesions with some color differences are found.

Seedlings which survive inoculation in the nursery bed are transplanted into another bed for 1 year and then planted in the field test areas where rust infection is maintained at high levels by the cultivation of *Ribes* bushes.

TREE GENETIC AND IMPROVEMENT RESEARCH AT THE UNIVERSITY OF MINNESOTA

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Forest Genetics, University of Minnesota
St. Paul, Minnesota

The School of Forestry's Tree Improvement Research Project was initiated in 1955. Studies in this area during the past fourteen years have been designed to accumulate information on genetic diversity in native and exotic tree species and isolate genetically superior lines for direct use in Minnesota forest plantings or for further selective breeding. Nursery facilities and outplanting cooperation have been provided chiefly by the University's North Central Experiment Station and the Blandin Paper Company at Grand Rapids, the Cloquet Forest Research Center at Cloquet, and the USDA Forest Service Nursery at Eveleth, Minnesota. Indispensable cooperation has also been provided by the University's Departments of Horticulture, Plant Pathology, Entomology, Fisheries, and Wildlife; the North Central Forest Experiment Station of the USDA Forest Service; The Quetico-Superior Wilderness Research Center; the Minnesota Conservation Department, and other institutions.

Most of the studies carried on under the Tree Improvement Project are conducted by graduate student assistants as part of their graduate training program. In most cases the results obtained from these investigations are also used as the basis for their degree dissertations. Aside from purely research objectives, the Tree Improvement Project thus serves a useful graduate training and educational function. Twenty-nine graduate students have thus far participated in the Tree Improvement Project.

Major effort to this point has been directed to provenance experiments. During the past 10 years this work has been greatly facilitated by our participation in a cooperative Regional Research Project (NC-51: Forest tree improvement through selection and breeding) sponsored by the U.S. Department of Agriculture. Seed source experiments of 14 North Temperate Zone species have been established in more than 30 permanent outplantings throughout the State.

Results from the older experiments (e.g., white spruce, Scotch pine and Japanese larch) have provided information on the best adapted seed sources for direct use in various localities throughout Minnesota. Some of these species have now reached sexual maturity and our future research will be increasingly directed to selective breeding studies within and between the most promising sources.

Details regarding some of the studies are presented in the material describing the Conference Field Trip.

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[1] *This was prepared by Professor Pauley prior to his death, April 18, 1970.*

FIELD TRIPS
TREE IMPROVEMENT PROJECT
SCHOOL OF FORESTRY, UNIVERSITY OF MINNESOTA

D. M. Gunn Memorial Park

D. M. Gunn Memorial Park is located on the west side of Prairie Lake, about four miles north of Grand Rapids on State Highway 38. The park was established in 1956 by the Charles K. Blandin Foundation and was designed primarily for recreation. However, the north portion of the park was designated as a tree improvement research and demonstration area. The first outplantings were made in 1956.

Japanese Larch *(Larix Leptolepis)*
Seed Source Study

This outplanting of seven Japanese larch sources was established in cooperation with Michigan State University in 1960 as a part of the North Central Region Forest-tree Improvement Project (NC-51).

Initial mortality of the 2-0 seedlings was high (46 percent) necessitating replacements with 2-2 stock in 1962 which resulted in high survival. A single border row of European larch (*L. decidua*) surrounds the plantation and a number of failed spots have subsequently been replaced with sources of native tamarack (*L. laricina*).

Height measurements have indicated significant differences among sources but no apparent relationship between growth and latitute, longitude, or elevation of parent stands was found. Such a random pattern of geographic variation is not unexpected since the native range of the species is restricted to approximately 140 square miles on Honshu Island where it is found in small discontinuous populations.

Several trees were damaged or killed by sunscald in the winter of 1967-68 and Japanese larch is susceptible to the endemic larch sawfly (*Pristiphora erichsonii*) which was found and controlled in the plantation during the current season. In spite of these shortcomings the rapid growth of Japanese larch gives it considerable potential for use in Minnesota.

White Spruce Seed Source Study

This test plantation was established in 1962 in cooperation with the North Central Forest Experiment Station of the USDA Forest Service. The study consists of 25 white spruce seed sources and a single Itasca County black spruce source. The stock was 2-2 when planted. Mortality has been less than 1 percent.

Height growth of the sources represented clearly indicates that high latitude or high altitude short growing season sources (e.g., Alaska and Montana) are the slowest growing. Best growth has been made by an eastern Ontario source (Beachburg, Ontario) which was significantly better than the next best which are local (Itasca Co.) sources (table 1).

The plantation has been sprayed annually in late June or early July during the past several years to control the yellow-headed spruce sawfly (*Pikonema alaskensis*). This insect is a locally serious pest of open-grown white spruce in this area. A duplicate of this plantation established at Cloquet where no control was practiced indicated no apparent variation in susceptibility among the white spruce sources. The black spruce plots in this planting were, however, ignored by the sawfly.

Forest Tree Improvement Arboretum

During the period 1956-57 the School of Forestry established a Breeding Arboretum at Gunn Memorial Park. The collections consist chiefly of seed sources of *Populus* and *Betula* species, including varieties and hybrids and serve as a reservoir of potentially useful genes.

North Central Experiment Station Nursery
University of Minnesota

A portion of the North Central Experiment Station Nursery has been used as the principal propagation area of the Tree Improvement Project since 1955. The Nursery is under the supervision of Professor William H. Cromell.

Dwarf Jack Pine *(Pinus banksiana)* and Eastern White Pine *(Pinus strobus)*

This study of seed-transmitted dwarfism in jack pine and white pine is being carried on in cooperation with Albert G. Johnson of the University's Horticulture Department, St. Pau Campus.

Table 1.—*Gunn Park, Plantation B (1962) white spruce seed sources (ranking by Duncan's Multiple Range Test based on height in autumn 1966 at 9 years from seed)*[1]

NCFES: Acq. No.	Seed Source Location	Height
		cm.
1663	Beachburg, Ontario	140
1647	Third River Rd.,Itasca Co.,Minnesota	127
3512	Itasca Co., Minnesota	125
1644	Adirondack Mountains, New York	119
1669	Grand Rapids, Minnesota	119
1662	Ashley Mines, Bannockburn, Ontario	117
1645	Monico, Wisconsin	116
1649	Coos County, New Hampshire	111
1655	Bangor, Maine	111
1660	Maniwaki, Quebec	111
1676	Huron National Forest, Michigan	110
3511	Itasca Co., Minnesota	105
1631	Spruce Woods Reserve, Manitoba	104
1659	Edmonston, New Brunswick	104
1661	Chicoutimi + St. Joan's City, Quebec	104
1687	Kakabeka Falls, Ontario	101
1664	Flin Flon, Manitoba	84
1628	Black Hills, South Dakota	83
1686	Moosonee, Ontario	83
1658	Lake Melville, Labrador	71
1665	Stony Rapids, Saskatcbewan	66
1677	Summit Lake Region, Fort McLeod, B.C.	61
1657	Port Hope Simpson, Labrador	57
1654	Fort Yukon, Alaska	42
1653	Gerstle, Alaska	41
1630	Lewis & Clark National Forest, Montana	39

1/ All seed sources are wbite spruce except 3512 which is black spruce.

A normal-dwarf ratio of 1:1 characterizes the segregation ratio of plants grown from open pollinated witches' broom seed of jack pine and white pine (table 2). In the absence of any evidence that the witches' brooms tested were due to a pathogen, the conclusion has been made that the observed segregation was genetically determined since such a 1:1 segregation ratio was that to be expected from a simple Mendelian dominant gene for dwarfism when fertilization is accomplished by normal pollen. This hypothesis is supported by the observed total absence of male strobili on the brooms studied.

In all segregating populations the distinction between normal and dwarfed trees was sufficiently clear to permit classification during the first season of growth. Aside from the gross differences characterizing dwarf and normal seedlings, there is a distinct difference between the progeny of different brooms. These differences are reflected chiefly in the height and crown density of the seedlings.

Blackberry Experimental Area

This experimental area, established by the Blandin Paper Company in 1960, is located about 5 miles southeast of Grand Rapids on the south side of U.S. Highway 2.

Scotch Pine Seed Source Study

Scotch pine (*Pinus sylvestris*), a native tree of Europe and Asia, has the most extensive natural distribution of any pine species in the world. It is the most important pine species throughout most of its natural range, especially in northern Europe, where it is used chiefly for lumber, piling, and pulpwood. Scotch pine has been grown in Minnesota for many years as an ornamental and in recent years has gained wide acceptance as a Christmas tree.

The seed source outplanting at the Blackberry Experimental Area was established in 1962 with 2-1 stock supplied by Michigan State University. The planting represents one of the approximately 50 similar outplantings of Scotch pine seed sources in the North Central Region of the United States established under a Cooperative Regional Research Project (NC-51) sponsored by the U.S. Department of Agriculture. A summary of results based on measurements made in September 1966 (at which time the trees were eight years old from seed) is shown in table 3. The results may be briefly summarized

Table 2.—Chi-square tests of goodness of fit to a 1 : 1 ratio for normal - dwarf segregates of open-pollinated jack pine brooms

Broom No. - Year tested - Nursery (N) or Greenhouse (GH)	Survival	Normal 1-0 seedlings	Dwarf 1-0 seedlings	Chi-Square	Probability (1 d.f.) greater tban:
	Percent	Number	Number		
1-1957-N	--	42	48	0.400	0.50
1-1961-N	--	146	132	.705	.30
1-1962-GH 1/	55	120	98	2.220	.10
2-1962-GH	75	73	77	.107	.70
3-1962-GH	70	81	59	3.457	.05
4-1962-GH	68	68	67	.007	.90
5-1962-GH	57	55	59	.140	.70
1-1962-N 1/	51	240	269	1.652	.15
2-1962-N	54	128	143	.830	.30
3-1962-N	52	125	133	.248	.50
4-1962-N	68	181	158	1.560	.20
5-1962-N	61	136	168	3.368	.05
All tests:	--	1,395	1,411	.091	.70

1/ Combined samples of seed from broom No. 1 collected in different years.

Table 3.—Relative height of Scotch pine varieties in autumn 1966 (8 years from seed) Blackberry Plantation A (1962)

Variety and seedlot numbers	Mean height 1/ in September 1966		Percent of Plantation mean	Origin
	Cm.	In.		
North European and Siberian varieties				
1. Altaica: 227	111	44	73	Altai Mts.,Siberian USSR
2. Rigensis: 223, 224,550,3513	154	61	101	Latvia, Sweden
3. Septentrionalis: 201,222,228, 230,273,274, 276,521,522, 543,544,545	126	50	82	Norway, Sweden, Finland
Central European varieties				
4. Borussica: 209	186	73	122	Northeast Germany
5. Haguenensis: 252, 253	179	71	117	West Germany
6. Hercynica: 203,208,248, 305,306,308, 312	187	74	122	Southwest Germany, Czechoslovakia
7. Polonica: 211, 317	198	78	130	Poland
South European varieties				
8. Iberica: 219	86	34	56	Spain

1/ Plantation mean = 153 cm. (60 inches)

by stating that in terms of growth rate and survival the best adapted sources for north central Minnesota are of Central European origin. Spanish sources are not frost hardy in this area. Extreme northern sources are slow growing an from the Christmas tree growers' standpoint ar undesirable because of a foliage color chang from green to various shades of yellow in autumn

COMMON AND SCIENTIFIC NAMES OF WOODY SPECIES MENTIONED IN THE TEXT

Ash, white	*Fraxinus americana* L.
Aspen, bigtooth	*Populus grandidentata* Michx.
Aspen, quaking (trembling)	*P. tremuloides* Michx.
Aspen, Asian trembling	*P. tremula* var. *Davidiana* Schneid.
Basswood	*Tilia americana* L.
Birch, bog	*Betula pumila* L.
Birch, river	*B. nigra* L.
Birch, sweet	*B. lenta* L.
Birch, white	*B. papyrifera* Marsh
Birch, yellow	*B. alleghaniensis* Britton
Birch, yellow-bog birch hybrid	*B.* x *purpusii* Schneid.
Douglas-fir	*Pseudotsuga menziesii* (Mirb.) Franco
Elm, American	*Ulmus americana* L.
Elm, Japanese	*U. japonica* (Rehd.) Sarg.
Elm, Siberian	*U. pumila* L.
Elm, slippery	*U. rubra* Muhl.
Fir, balsam	*Abies balsamea* (L.) Mill.
Fir, grand	*A. grandis* (Dougl.) Lindl.
Fir, white	*A. concolor* (Gord. &. Glend.) Lindl.
Hickory, pigment	*Carya glabra* (Mill.) Sweet
Larch, Dahurian	*Larix gmelini* (Rupr.) Litvin.
Larch, European	*L. decidua* Mill.
Larch, Japanese	*L. leptolepsis* (Sieb. & Zucc.) Gord.
Larch, Korean Dahurian	*L. gmelini olgensis* (Henry) (Ostenf. & Syrach)
Larch, Siberian	*L. sibirica* Ledeb.
Larch, western	*L. occidentalis* Nutt.
Larch, eastern	*L. laricina* (Du Roi) K. Koch
Maple, red	*Acer rubrum* L.
Maple, sugar	*A. saccharum* Marsh
Oak, red	*Quercus rubra* L.
Pine, Austrian	*Pinus nigra* Arnold
Pine, eastern white	*P. strobus* L.
Pine, Himalayan	*P. griffithii* McClelland
Pine, jack	*P. banksiana* Lamb.
Pine, Japanese red	*P. densiflora* Sieb. & Zucc.
Pine, Korean	*P. koraiensis* Sieb. & Zucc.
Pine, southwestern white	*P. strobiformis* Engelm.

Pine, limber	*P. flexilis* James
Pine, Macedonian white	*P. peuce* Griseb.
Pine, mugho	*P. mugo* Turra
Pine, pitch	*P. rigida* Mill.
Pine, ponderosa	*P. ponderosa* Laws
Pine, red	*P. resinosa* Ait.
Pine, sand	*P. clausa* (Chapm.) Vasey
Pine, Scotch	*P. sylvestris* L.
Pine, shortleaf	*P. echinata* Mill.
Pine, spruce	*P. glabra* Walt.
Pine, Swiss stone	*P. cembra* L.
Pine, western white	*P. monticola* Lamb.
Spruce, black	*Picea mariana* (Mill.) B.S.P.
Spruce, Engelmann	*P. engelmannii* Parry
Spruce, Norway	*P. abies* (L.) Karst.
Spruce, Serbian	*P. omorika* (Pancic) Purkyne
Spruce, white	*P. glauca* (Moench) Voss
Walnut, black	*Juglans nigra* L.
White-cedar, northern	*Thuja occidentalis* L.

BY-LAWS LAKE STATES FOREST TREE IMPROVEMENT COMMITTEE — 1968

I. *NAME OF ORGANIZATION*

The name of this organization shall be the Lake States Forest Tree Improvement Committee.[1]

II. *PURPOSE*

The purpose of the Lake States Forest Tree Improvement Committee shall be to encourage and coordinate forest tree improvement activities in Michigan, Minnesota, and Wisconsin.[1]

III. *MEMBERSHIP*

A. *Representation*

Membership in this Committee shall consist of representatives of:

1. Agencies within the Lake States that are conducting or strongly interested in tree improvement activities, and
2. Parties with bonafide interest in the subject matter with which the committee is concerned.

B. *Organizations Represented*

The number of committee members shall be limited to one from each of the following agencies or institutions: The University of Michigan, Michigan State University, The University of Minnesota, The University of Wisconsin, The Michigan Department of Conservation, The Minnesota Department of Conservation, The Wisconsin Department of Natural Resources. The Lake States Council of Industrial Foresters, The Institute of Paper Chemistry, The Northeastern Area State and Private Forestry, the U.S. Forest Service – Eastern Region, The Forest Products Laboratory, and the North Central Forest Experiment Station. (It is expected that these agency representatives normally will also represent the subject matter areas of silviculture, genetics, and wood technology.)

C. *Special Interest Membership*

Specific interest and subject matter areas will be represented by specialists in these areas. There shall be one member each representing the areas of forest entomology and forest pathology. Specialists in other subject matter areas shall be considered for membership at the discretion of the committee. Candidates for these positions shall be nominated by the Executive Committee and elected by the committee at large.

D. *Nominations*

Members of the Committee shall be nominated by the Head of the agency which they represent. The nominations will be called for by the Chairman at least 2 months before each Biennial Conference.

E. *Term of Membership*

Members shall serve for four years beginning January 1 following appointment and ending December 31 of the fourth year. Members may serve more than one consecutive term.

IV. *COMMITTEE OFFICERS*

A. *Officers*

The officers of the Lake States Forest Tree Improvement Committee shall be a Chairman, a Vice-Chairman and an Executive Secretary.

B. *Term of Office*

The term of office for the Chairman and Vice-Chairman shall be two years beginning January 1 following election. The Executive Secretary, who shall be the representative of the North Central Forest Experiment Station, shall serve a continuing term.

C. *Nominations for Office*

At least 2 months preceding each biennial conference, the Executive Secretary shall call for nominations from all members of the Lake States Forest Tree Improvement Committee for Chairman and Vice-Chairman candidates.

D. *Election*

The Committee shall elect a new Chairman and Vice-Chairman by majority vote on a written ballot at the Biennial Regional Conference or by mail immediately preceding this conference. Results will be announced at the conference.

V. *DUTIES OF COMMMITTEE OFFICERS*

A. *Chairman*

The Chairman shall preside over the meetings of the Lake States Forest Tree Improvement Committee. He shall appoint members to standing and special subcommittees.

B. *Vice-Chairman*

The Vice-Chairman shall preside in the absence of the Chairman and he shall be responsible for organizing the program of the Biennial Tree Improvement Conference during his term. He shall be selected from the State in which the Biennial Conference will be held.

C. *Executive Secretary*

The Executive Secretary shall keep the records of the Committee. He shall collect manuscripts of all papers on the Biennial Conference programs and help to prepare them for publication. He shall be the executive officer of the Committee.

VI. *MEETINGS*

A. *Committee*

The Lake States Forest Tree Improvement Committee shall meet in conjunction with the Biennial Lake States Forest Tree Improvement Conferences and at such other times and places as may be decided by the Committee or by the Executive Committee. Meetings other than those in conjunction with the Biennial Conference, shall be called by the Chairman. They shall be announced by mail to all members of the Committee.

B. *Conference*

The Biennial Tree Improvement Conference shall be held in odd-numbered years. The place of meeting shall rotate between the three states in this order: Michigan, Wisconsin, and Minnesota.

C. *Procedure*

Parliamentary procedures of all meetings shall be conducted in accordance with Robert's "Rules of Order."

VII. *COMMUNICATIONS*

A. *Newsletter*

At irregular times, but approximately annually, the Newsletter (Lake States Trebredinews) shall be issued. The various members of the Committee shall take turns in assembling, preparing, and issuing these Newsletters as arranged by the Executive Secretary.

B. *Proceedings*

Proceedings of the conferences and complete papers presented thereat shall be made available for publication.

C. *Special*

Announcements of conference meetings, dates, and places shall be sent to all committee members, and all attendees at previous conferences not less than 60 days before the conference is to be convened. The Executive Officers of the Committee may at any time issue communications of a general nature which they determine will be of interest to the membership of the LSFTIC and the Conference.

VIII. *COMMITTEES*

The terms of office for members of standing committees appointed by the Chairman shall be as specified in the appointment or for an indefinite period. The Chairman of a standing committee shall serv until a successor is appointed. Special subcommittees shall function and serve as directed by the Committee.

IX. *FINANCE*

A. *Non-profit Nature*

The LSFTIC is a non-profit organization, which collects no dues, maintains no treasury and issues no disbursements as a committee.

B. *Conferences*

Registration fees, not to exceed actu costs, may be collected in advance cover incidental costs in connectio with the Biennial Conferences.

C. *Communications*

Costs of special communications are borne by the supporting agencies. Publication of conference proceedings, correspondence, and minutes of committee meetings may be handled on a no-cost basis by one or more member agencies.

D. *Officers*

All expenses in connection with the duties of the LSFTIC committee shall be borne by the member's employer or by the member. The Committee shall make no reimbursement to the officers or members for expenses incurred in any regard.

X. *AMENDMENTS*

These by-laws may be amended by a majority vote of the members of the Lake States Forest Tree Improvement Committee voting at the time of any regular election of the Committee.

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[1] See Proceedings, Lake States Forest Genetics Conference, March 20 - April 1, 1953. Lake States Forest Exp. Sta. Misc. Rep. 22, 1953, p. 81.

MEMBERSHIP LIST
LAKE STATES FOREST TREE IMPROVEMENT COMMITTEE

Name	*Address*	*Appointed Through*
Scott S. Pauley [1] CHAIRMAN	School of Forestry University of Minnesota St. Paul, Minnesota 55101	1971
Burton V. Barnes VICE CHAIRMAN	School of Natural Resources University of Michigan Ann Arbor, Michigan 48104	1973
Hans Nienstaedt EXECUTIVE SECRETARY	Institute of Forest Genetics North Central Forest Experiment Station Star Route #2 Rhinelander, Wisconsin 54501	
Dean Einspahr	Institute of Paper Chemistry Appleton, Wisconsin 54910	1971
Paul R. Flink	State of Michigan Forestry Division Department of Natural Resources Stevens T. Mason Building Lansing, Michigan 48926	1971
David W. French	Dept. of Plant Pathology and Botany University of Minnesota St. Paul, Minnesota 55101	1971
Clyde M. Hunt	USDA Forest Service, Northeastern Area State & Private Forestry 6816 Market St. Upper Darby, Pennsylvania 19082	1972
Erick Kurki	State of Minnesota Division of Lands & Forestry Dept. of Conservation St. Paul, Minnesota 55101	1973
Donald T. Lester	Department of Forestry University of Wisconsin Madison, Wisconsin 53706	1973
H. L. Mitchell	Forest Products Laboratory Madison, Wisconsin 53705	1971
John A. Pitcher	USDA Forest Service 633 W. Wisconsin Ave. Milwaukee, Wisconsin 53203	1971
Thomas J. Rausch	State of Wisconsin Dept. of Natural Resources Box 450 Madison, Wisconsin 53701	1972
Richard Schantz-Hansen	Lake States Council of Industrial Foresters The Northwest Paper Company Cloquet, Minnesota 55720	1973
Louis F. Wilson	North Central Forest Experiment Station 215 Natural Resources Building Michigan State University East Lansing, Michigan 48823	1971
Jonathan W. Wright	Department of Forestry Michigan State University East Lansing, Michigan 48824	1973

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[1] *Professor Pauley was Chairman until his death April 18, 1970.*

Publications Issued for the Lake States Forest Tree Improvement Committee

Proceedings, Lake States Forest Genetics Conference, March 30 - April 1, 1953. Lake States Forest Exp. Sta. Misc. Rep. 22, 83 p. 1953.

Proceedings, Lakes States Forest Tree Improvement Conference, August 30-31, 1955. Lake States Forest Exp. Sta. Misc. Rep. 40, 108 p., illus. 1955.

Forest Genetics in the Lake States, an Annotated Bibliography, by William J. Libby, Burton V. Barnes, and Stephen H. Spurr. Univ. Mich. School Natur. Resources (no series), 74 p. 1956.

Guide for Selecting Superior Trees and Stands in the Lake States, by Paul O. Rudolf. Lake States Forest Exp. Sta., Sta. Pap. 40, 32 p., illus. 1956.

Forest Tree Seed Collection Zones for the Lake States. Mich. Dept. Conserv., Forest. Div. (no series), 13 p. illus. 1957.

Proceedings, Third Lake States Forest Tree Improvement Conference, Sept. 17-18, 1957. Lake States Forest Exp. Sta., Sta. Pap. 58, 87 p., illus. 1958.

Forest Tree Improvement Research in the Lake States; A Survey by the Lake States Forest Tree Improvement Committee. Lake States Forest Exp. Sta., Sta. Pap. 74, 56 p. 1959.

Registering and Marking Selections in the Lake States; A Report of a Subcommittee of the Lake States Forest Tree Improvement Committee, by Paul O. Rudolf and H. E. Ochsner. USDA Forest Serv. N. Cent. Reg. (no series), 9 p. 1959.

Forest Tree Seed Certification in the United States; A Report of a Subcommittee of the Lake States Forest Tree Improvemnt Committee, by J. W. Macon. Consolidated Water Power and Paper Co. (no series), 3 p. 1959.

Proceedings of the Fourth Lake States Forest Tree Improvement Conference, Oct. 6-7, 1959. Lake States Forest Exp. Sta., Sta. Pap. 81, 60 p., illus. 1960.

Proceedings of the Fifth Lake States Forest Tree Improvement Conference, Sept. 19-20, 1961. Lake States Forest Exp. Sta., Sta. Pap. 98, 42 p. 1962.

Report of the Seed Certification Subcommittee of the Lake States Forest Tree Improvement Committee, by W. H. Brener. Wis. Conserv. Dept. (no series), 4 p. 1963.

Proceedings of the Sixth Lake States Forest Tree Improvement Conference, Sept. 9-10, 1963. Lake States Forest Exp. Sta. (no series), 90 p. 1964

Forest Tree Improvement Research in the Lake States, 1965, by Paul O. Rudolf. USDA Forest Serv. Res. Pap. NC-1, 54 p. N. Cent. Forest Exp. Sta., St. Paul, Minn.

Joint Proceedings, Second Genetics Workshop of the Society of American Foresters and the Seventh Lake States Forest Tree Improvement Conference, Oct. 21-23, 1965. USDA Forest Serv. Res. Pap. NC-6, 110 p., illus. N. Cent. Forest Exp. Sta., St. Paul, Minn.

Proceedings of the Eighth Lake States Forest Tree Improvement Conference, Sept. 12-13, 1967. USDA Forest Serv. Res. Pap. NC-23, 60 p., illus. N. Cent. Forest Exp. Sta., St. Paul, Minn.

ABOUT THE FOREST SERVICE . . .

As our Nation grows, people expect and need more from their forests — more wood; more water, fish, and wildlife, more recreation and natural beauty; more special forest products and forage. The Forest Service of the U.S. Department of Agriculture helps to fulfill these expectations and needs through three major activities:

- Conducting forest and range research at over 75 locations ranging from Puerto Rico to Alaska to Hawaii.
- Participating with all State forestry agencies in cooperative programs to protect, improve, and wisely use our Country's 395 million acres of State, local, and private forest lands.
- Managing and protecting the 187-million acre National Forest System.

The Forest Service does this by encouraging use of the new knowledge that research scientists develop; by setting an example in managing, under sustained yield, the National Forests and Grasslands for multiple use purposes; and by cooperating with all States and with private citizens in their efforts to achieve better management, protection, and use of forest resources.

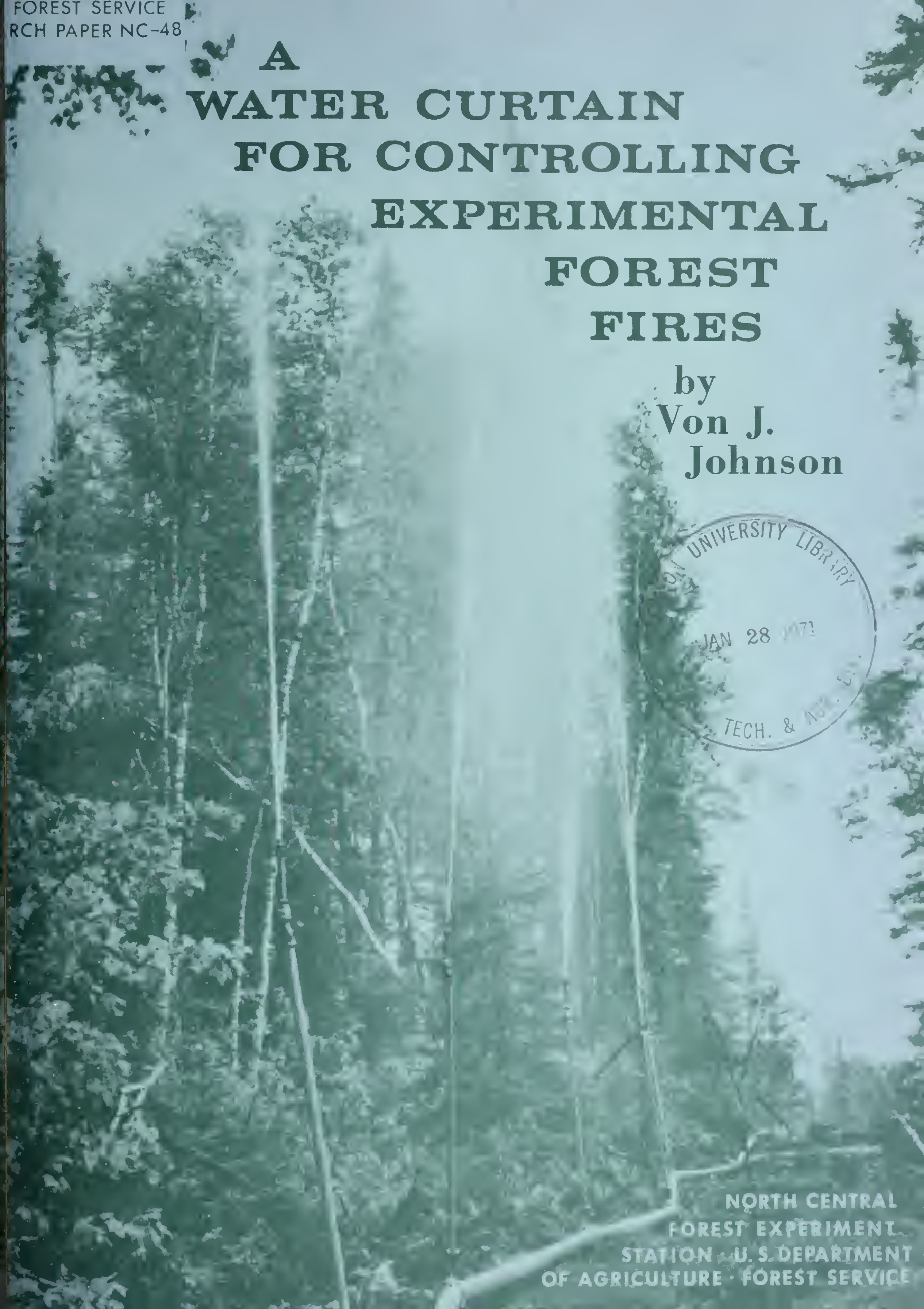
Traditionally, Forest Service people have been active members of the communities and towns in which they live and work. They strive to secure for all, continuous benefits from the Country's forest resources.

For more than 60 years, the Forest Service has been serving the Nation as a leading natural resource conservation agency.

FOREST SERVICE
RCH PAPER NC-48

A WATER CURTAIN FOR CONTROLLING EXPERIMENTAL FOREST FIRES

by
Von J.
Johnson



NORTH CENTRAL
FOREST EXPERIMENT
STATION · U. S. DEPARTMENT
OF AGRICULTURE · FOREST SERVICE

The author, Principal Fire Control Scientist for the Station, heads the Research Work Unit on research of fire control methods. He is stationed at the Headquarters Laboratory in St. Paul, which is maintained in cooperation with the University of Minnesota.

North Central Forest Experiment Station
D. B. King, Director
Forest Service — U.S. Department of Agriculture
Folwell Avenue
St. Paul, Minnesota 55101

A WATER CURTAIN FOR CONTROLLING EXPERIMENTAL FOREST FIRES

Von J. Johnson

Experimental forest fires are often required studying fire behavior, effects, and control hniques. Fires set by North Central Forest periment Station researchers for this purpose ge from 10 to 40 acres in size in stands where average tree height seldom exceeds 60 feet. ny of the fire sites are within ½ mile and 100 t elevation of a natural water source. Confine-nt of these simulated wildfires to specific areas essential, and for this reason a high-capacity ter pumping and distributing system was de-oped to provide a three-dimensional water cur-n barrier around fires. The following is a de-iption of the water curtain and an evaluation its preliminary performance.

WATER CURTAIN SPECIFICATIONS

The water curtain delivery system was as-nbled from commercially available components. e initial design called for lifting water 100 feet er a horizontal distance of 2,500 feet with an erage total discharge rate of 1,200 gallons per nute at two-thirds of maximum power capa-y. This discharge requirement was based on eliminary tests conducted by the Michigan De-rtment of Conservation, which showed that optimum spray height could be obtained at 0 p.s.i. by moving about 11.4 gallons of water r minute through a ¼-inch orifice.[1] One hun-d and twenty nozzles spaced at 20-foot inter-s were used around the perimeter of a 10-acre rning block. A discharge of 1,200 gallons per nute furnishes 10 g.p.m. for each nozzle at average pressure of 35 p.s.i. Reducing the pipe size toward the downstream end reduces power requirements and increases nozzle pressure up to 100 p.s.i.

Power needed for the system was estimated to be 130 usable horsepower. Five Model VG4D Wisconsin[2] air-cooled gasoline engines met this requirement when operating at about 70 percent maximum capacity. Horsepowers at 60° F. and barometric pressure of 29.92 inches of mercury for the Wisconsin Model VG4D air-cooled engine are as follows:

R.p.m.	*Horsepower*
1,400	25
1,600	29
1,800	32
2,000	34
2,200	36
2,400	37

Each engine has a total displacement (four cylinders) of 154 cubic inches. Close-coupled to the engines were 6-inch end-suction centrifugal Model S30Z FM 6/B Jacuzzi pumps. The empty weight of each pump unit, equipped with a 12-volt battery, starter, and 4-inch iron-pipe skids, is about 750 pounds.

Engine exhaust primers, installed on two pump units (fig. 1), facilitate priming the centrifugal pumps through 20 to 60 feet of 8-inch suction pipe. Long suction distances are usually necessary to reach suitable water sources from a riverbank (fig. 2) or lakeshore.

To partially compensate for friction loss, maintain needed pressure, and increase efficiency, the pump units are used in series along the 2,500-

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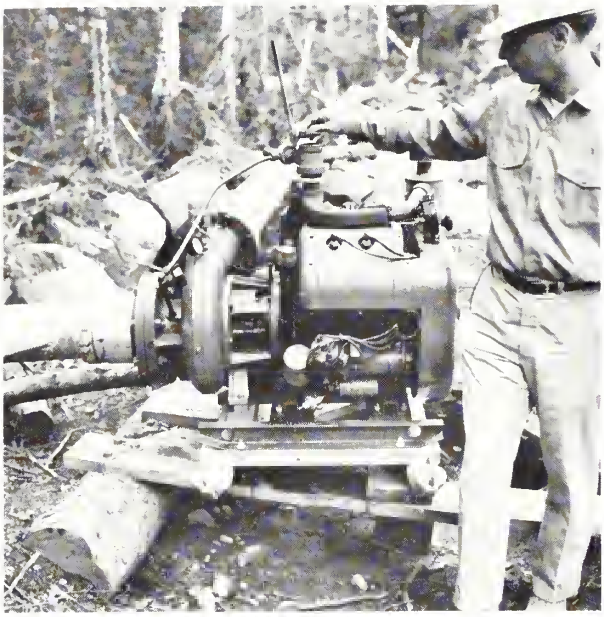
[1] *Personal correspondence with Steve Such, chigan Department of Conservation, Roscom-on, Michigan, 11/16/67, on file at the North ntral Forest Experiment Station, St. Paul, nnesota.*

[2] *Mention of trade names does not constitute endorsement of the product by the USDA Forest Service.*



F-520037
Figure 1.–Exhaust primer used to evacuate 8-inch suction line. Engines having a displacement of not less than 20 cubic inches are required for this conversion.

foot main line. Several safety mechanisms prevent damage to the units during unattended operation. A type 154MP11 loss-of-prime protecto on the volute of each pump (fig. 3) prevents impeller cavitation and damage to the packing glanc by opening the ignition circuit whenever lin pressure drops below a specified setting. A typ YC-48-51 oil pressure switch on each engine (fig 4) protects against damage whenever engine oi pressure falls below a specified amount. A Wisconsin Model YC-66D-S1 high-temperature safety switch on a cylinder head boss of each engin automatically shorts out the distributor time whenever the cylinder head exceeds safe temperature. About 10 minutes elapses before th switch cools sufficiently to restart the engine.

The 2,500-foot main line consists of 20-foo sections of 8-inch, smooth, aluminum irrigatio pipe (0.072-inch wall thickness). Quick-connec couplers[3] welded to each section have a saf working limit of 150 p.s.i. and allow for abou 11 degrees of leak-free lateral movement. Eac

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[3] *Series* 1000 *John Bean Division, FMC Corp, Lansing, Michigan.*

F-520038
Figure 2.–Forty feet of 8-inch suction line being used to reach a water sourch over an eroded bank.

F-520039
igure 3.—Loss-of-prime protector installed on the volute. A pressure gage taped in at the base of the protector assists in setting the "cutout" mechanism whenever line pressure drops below 10 p.s.i.

F-520040
'igure 4.—An oil pressure switch opens the electric circuit in case of pressure failure.

20-foot section weighs 60 pounds. A 10- to 18-foot telescoping intake elbow is used with the in-line series pump units (fig. 5). This elbow permits a pump to be moved up to 8 feet for positioning to avoid obstacles.

F-520041
Figure 5.—A telescoping elbow attached to the intake of an in-line pump unit. The 1½-inch gate valve allows for drainage of the downstream line and an outlet for standard 1½-inch fire hose.

The lateral line is comprised of 2,400 feet of 5-inch, heavy-duty aluminum pipe (0.052-inch wall thickness) in 20-foot sections. Each section weighs 32 pounds. Quick-connect couplers[4] welded to each section with a ball check riser coupling attached permit about 11 degrees of lateral leak-free movement. Flow from the main line is regulated through a valved reducer-tee. The risers, which are 60 inches long, are made from 1-inch heavy-duty aluminum pipe (fig. 6) and can be rotated 360 degrees in the ball-check coupler while under pressure.

Nozzles are assembled on the threaded end of the riser from iron pipe fittings and a ¼-inch brass nozzle head. They are adjustable on a 360-degree vertical arc (figs. 7 and 8).

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[4] *Series 400, John Bean Division, FMC Corp., Lansing, Michigan.*

F-520042
Figure 6.—Aluminum riser installed in quick couplers on a section of lateral pipe.

F-520043
Figure 7.—Nozzle assembly consists of two iron pipe elbows, two bell reducers, two bushing reducers, one 8- by ½-inch nipple, and a brass nozzle head.

F-52004
Figure 8.—The nozzle is adjustable through a 36 degree vertical arc.

The system is also equipped with a fertiliz injector[5] to add chemical fire retardants such diammonium phosphate (DAP), low-sudsing bi degradable detergents, or water-soluable dye The injector venturi is installed near the dow stream end of the main line so that corrosi materials are not circulated through the pum units.

PERFORMANCE

Preliminary testing of the water curtain sy tem was done to determine approximate rates discharge, friction losses in the lateral line, a spray patterns at various windspeeds. The pr liminary test assembly consisted of three pun units, 480 feet of main line, and 1,000 feet lateral line. Suction lift at the test site was feet through 20 feet of 8-inch tube. Total d charge lift was 20 feet. Pump units were po tioned at 0, 250, and 490 feet along the ma line. The 1,000-foot lateral line, fed from one e of the reducer-tee, was laid along an abandon

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[5] *Dragon Model 20, 8-inch venturi. Drag Engineering Company, Oakland, California.*

roadbed. Sensitive pressure gages were tapped to the end of nozzle assemblies at 20 and 980 feet. Six rain gages were spaced at 10-foot intervals midway between risers at 770 feet on an axis perpendicular to the lateral line. Approximate rates of discharge were determined from 18 runs of 20 minutes each at four average pressures (table 1). Water collected in the rain gages was measured after each run. This gave a conservative estimate of the total water volume discharged through the 48 nozzles. Pressure head values (table 1) were derived from mean readings of the two gages located on the first and last riser. As head pressure or windspeed increased, additional water atomized and drifted beyond the collection gages. Under a head of 50 p.s.i. and winds less than 5 m.p.h., this drift was about 3 gallons per minute. At 70 p.s.i. of head and windspeed greater than 10 m.p.h., 144 g.p.m. or about 20 percent of the total volume drifted downwind.

Table 1.—Measured rates of discharge for forty-eight ¼-inch nozzles compared with theoretical rates

Average pressure head (p.s.i.)	Replications	Average windspeed 1/	Total discharge (q)		Differential
			Measured	Theoretical 2/	
		M.p.h.	---- G.p.m.	----	
50	5	<5	598	601	-3
60	4	5	606	659	-53
70	4	>10	568	712	-144
88	5	5	711	798	-67

1/ 10-minute observation at a height of 6 feet and direction parallel or within 45° of rain gage axis.

2/ $q = 29.85 \cdot Cd^2 \sqrt{P}$ (Addison 1964)

Where C = 0.95, coefficient of velocity

d = 0.25 inch, diameter of nozzle

P = pressure in pounds per $(\text{inch})^2$

Assuming no loss from friction, the 5-inch conduit can supply sufficient water volume for about 400 ¼-inch nozzles. However, friction is a major source of loss and for a given pipe size it is about proportional to the square root of the pressure. At an average of 88 p.s.i., maximum variation between the pressure gages located at 20-foot and 1,000-foot risers was 10 percent (15 p.s.i.). Computed friction losses, using Scobey's coefficient of discharge equation for various pressure heads, were consistently lower than observed losses (Scobey 1930) (table 2). This was due in part to our measurement techniques and to the roughness of the couplings (Scobey's coefficient of 0.32 is applicable to smooth, new iron pipe).

Table 2—Computed vs. measured friction loss for 960 feet of 5-inch aluminum conduit with outlets at 20-foot intervals

Computed 1/ discharge			Measured 2/ loss p.s.i.	Differential
P.s.i.	G.p.m.	P.s.i.-loss		
50	598	9.6	10.4	-0.8
60	606	9.9	10.0	0
70	568	8.6	10.0	-1.4
88	711	13.8	15.4	-1.6

1/ Table 1 and Christiansen 1942

$$H_f = \left(\frac{K_s L V^{1.9}}{1{,}000 \ D^{1.1}} \right) F$$

H_f = loss in feet of head (p.s.i. = $0.43352 \ H_f$)

K_s = 0.32, Scobey's coefficient of retardation

L = 960, length of pipe in feet

V = mean velocity in feet/sec. from g.p.m.

D = 0.4167, diameter of pipe in feet

F = friction recovery factor for 48 outlets

$$= \frac{1}{m+1} + \frac{1}{2N} + \frac{\sqrt{m-1}}{6N^2}$$

$$= 0.355$$

2/ Mean difference between first (20-foot) and last (1,000-foot) riser gages during each of 18 20-minute runs.

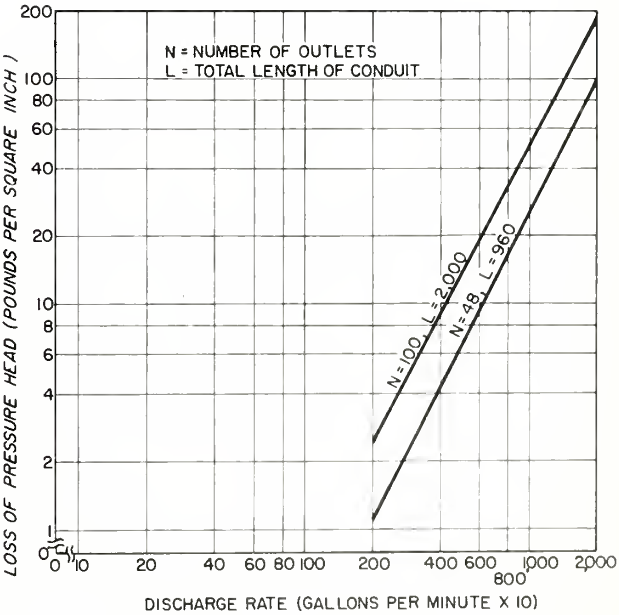

Figure 9.—Friction loss for 5-inch aluminum conduit with outlets at 20-foot intervals.

A chart was prepared from which friction losses for 5-inch aluminum conduit having outlets at 20-foot intervals can be estimated within 10-percent accuracy (fig. 9).

Heights of the visible spray column at the first and last nozzle were measured with an Abney level during each run. Average heights ranged from 35 feet at 70 p.s.i. with a 10 m.p.h. wind to 50 feet at 88 p.s.i. with 5 m.p.h. wind (fig. 10). An additional 5 to 10 feet of mist above the spray was visible against a clear sky. Injection of Rhodmine B or Methylene blue dyes into the line did not appreciably improve the visibility of the mist.

F-520045

Figure 10.—Performance tests of the water curtain being conducted at 60 p.s.i. pressure head, under 5 m.p.h. winds. Visible spray height is 45 feet. A hand-held anemometer is used to determine average windspeed at height of nozzle.

Total horizontal dispersal of the spray was much more difficult to measure. After each run, vegetation beyond the rain gages was examined for water droplets. Visible dispersal ranged from 45 feet at 50 p.s.i. and calm conditions to 130 feet at 88 p.s.i. and 10 m.p.h. winds. The average spray dispersal dimensions for all runs, including mist height, were 55 feet high and 60 feet wide.

OPERATIONAL TEST

Subsequent to the preliminary performance tests, all components of the system were assembled for use on an experimental 15-acre burn near August Lake in northern Minnesota. A main line of 2,520 feet of 8-inch pipe and 450 feet of 5-inch pipe was required for a 172-foot vertical lift over a horizontal distance of 2,450 feet. A lateral distribution line of 1,000 feet with 50 nozzles was used (fig. 11). Pump units were located at 20, 470, 850, 1,410, and 1,750 feet from the water source. By operating the five pump units at 2,200 r.p.m., an average nozzle pressure of 80 p.s.i. was maintained without serious malfunction. Under calm wind conditions, this assembly provided a water curtain 55 feet high, 60 feet wide, and 1,000 feet long. However, the fire burned discontinuously in surface fuels and the low intensity and rate of spread provided a less-than-adequate test of the system. Further tests on experimental fires are planned.

F-520046

Figure 11.—Lateral distribution line at August Lake experimental burn area.

POSSIBLE USES

The system is designed to control prescribed fires and is not sufficiently mobile for controlling fast-spreading wildfires. However, it could be effective for extended "mop-up" operations on large fires when used in conjunction with relay tanks and portable fire pumps. Under a 100-foot pressure head, it can provide sufficient water volume to supply 20 type "Y" 1½-inch portable fire pumps.

Mobility of the system depends on accessibility of the water source, topography, and physiography of the site. Where water sources are accessible by road, the pump units may be mounted on trailers for increased mobility. Pipe handling time is minimized whenever the lines are located near roads or trails. Approximately twelve 8-hour man-days are required to assemble the full system under average field conditions. Three men equipped with mobile radios are adequate to operate the system. Under average operating loads each pump unit burns 3 gallons of regular automotive fuel per hour. Total initial investment for the components is about $20,000. Maintenance cost based on two seasons operations (including fuel) is about $10 per hour of running time for all pumps. This includes lubrication of units and replacement of line valves and gaskets due to corrosion and breakage.

The water curtain appears to be particularly suited for controlling prescribed fires in areas having adequate water sources nearby and minimal topographic relief. However, its effectiveness in reducing spotfire propagation during high-intensity burning requires further testing.

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- Conducting forest and range research at over 75 locations ranging from Puerto Rico to Alaska to Hawaii.
- Participating with all State forestry agencies in cooperative programs to protect, improve, and wisely use our Country's 395 million acres of State, local, and private forest lands.
- Managing and protecting the 187-million acre National Forest System.

The Forest Service does this by encouraging use of the new knowledge that research scientists develop; by setting an example in managing, under sustained yield, the National Forests and Grasslands for multiple use purposes; and by cooperating with all States and with private citizens in their efforts to achieve better management, protection, and use of forest resources.

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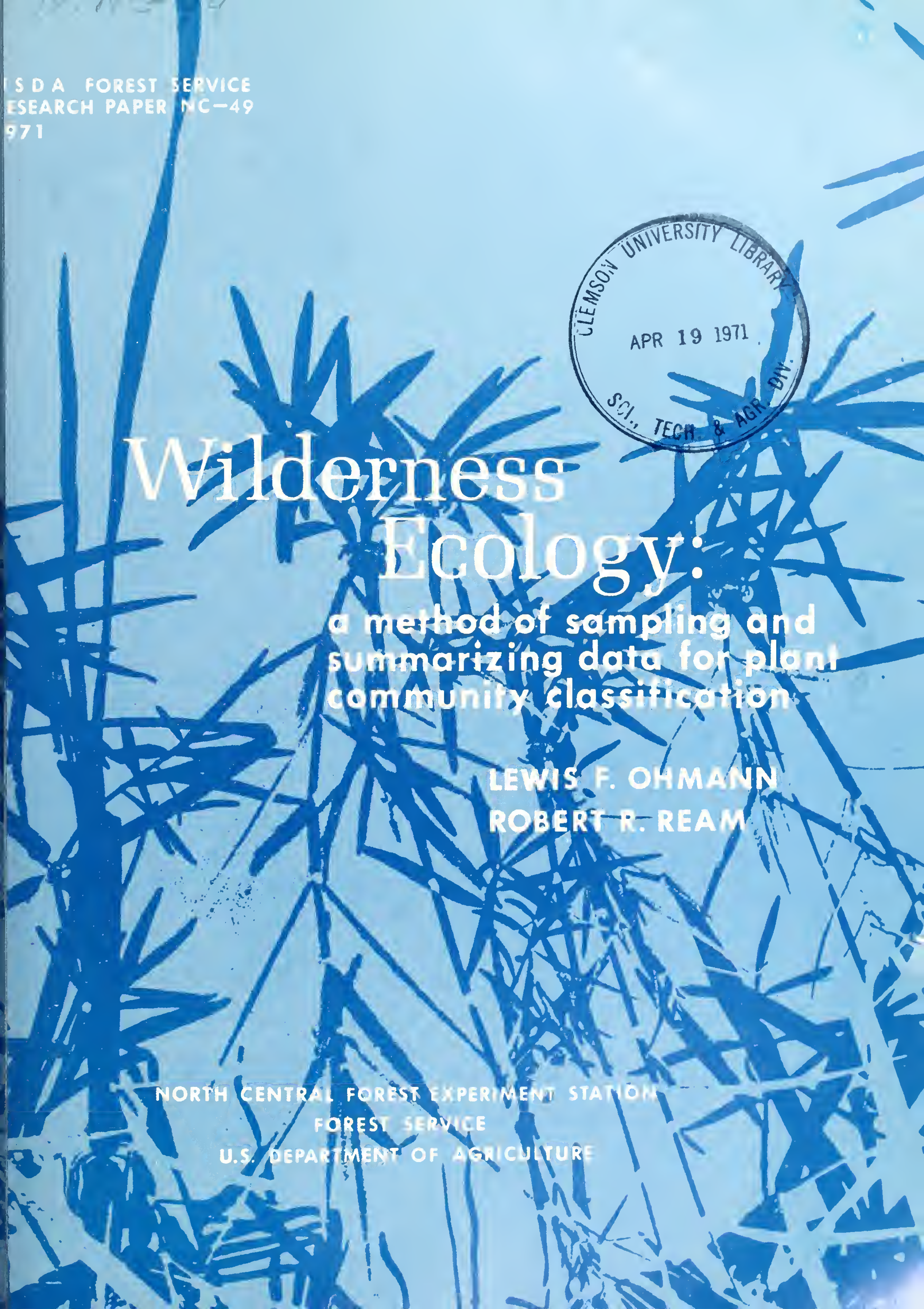

Wilderness Ecology:

a method of sampling and summarizing data for plant community classification

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ROBERT R. REAM

NORTH CENTRAL FOREST EXPERIMENT STATION
FOREST SERVICE
U.S. DEPARTMENT OF AGRICULTURE

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CONTENTS

Wilderness Ecology: A Method of Sampling and Summarizing Data for Plant Community Classification

Lewis F. Ohmann and Robert R. Ream

In 1966 a program of ecological research as initiated by researchers at the North Central orest Experiment Station, USDA Forest Serice, within the Boundary Waters Canoe Area 3WCA) of the Superior National Forest in northastern Minnesota. The mission of this program to provide basic ecological information on ant and animal communities occurring in e relatively undisturbed environment of this Vilderness Area. This information is needed to rovide a basis for management decisions to chieve specific goals, such as maintenance and storation of biotic communities. Knowledge the makeup of plant communities and the cological factors that influence them, espeally quantitative descriptions of the vegetional and environmental components, is a asic part of this information.[1] To collect such uantitative data in the BWCA, we developed sampling scheme that offers much flexibility nd should have widespread application. Thus is presented here so that researchers and land anagers may use it in surveying and classifyg plant communities of other forest lands.

The applicability of these methods for any ecific forest area is related to the general anagement goal of that forest. Even within the BWCA this is important. According to the Wilderness Act of 1964, the general goal for the management of the BWCA and other Wilderness Areas is "the preservation of their wilderness character." The BWCA is unique among the units of the National Preservation System, however, in that it is divided into two zones: the Portal Zone, where the vegetation is managed for timber harvest along with primitive recreation values, and the Interior Zone, where timber harvest is not permitted. The management of vegetation for commercial timber harvest in the Portal Zone necessitates emphasis of silvicultural goals and techniques. The survey methods described here were designed for the Interior Zone, where wilderness goals and techniques are applicable. The Interior Zone occupies about 500,000 acres, or roughly one-half of the BWCA.

The sampling scheme can be used by those with a botanical background and knowledge of any regional flora. Specific knowledge of the structure and composition of the local vegetation, while helpful, is not necessary.

This paper describes field sampling procedures and sample summary methods used in the survey of one unit of vegetation, the upland natural vegetation of the Interior Zone of the BWCA. Ream and Ohmann (1970) show how data from these survey techniques can be further used in examining interrelationships of plant communities and their environments.

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[1] *Concurrent cooperative research is being conucted by other Federal, State, private, and Univerty personnel on the other components of the biotic ommunities of this Wilderness Area.*

SAMPLE DESIGN

In designing the sample scheme, information already available for the Interior Zone of the BWCA was examined. This consisted principally of reviewing the scientific literature, reading popular accounts of voyageur travel through the area during the fur trade era, reviewing published histories of the area, and collecting official documents, maps, and aerial photographs of the area.[2] Study of this material suggested that about 300,000 acres, or 60 percent, of the Interior Zone was occupied by natural vegetation. The balance of the area was logged, either in the "cut-out and get-out" period of the early 1900's, or under controlled National Forest or State timber harvest programs of more recent years. This has been confirmed by Heinselman (1969). Because of the large area involved, and because of the limited field time and manpower available, this initial study was restricted to the natural upland vegetation and environment. For the purposes of this survey, "Upland" is defined on a topographic basis as sites upon which rainwater never accumulates, due to runoff and percolation. They are normally thought of as mineral soil sites.

In applying these survey methods to other forest units, stratifications based on general management goals, historical influences, topographical conditions, and other information should be considered.

The general extent of upland natural vegetation was outlined on U.S. Geological Survey quadrangle maps. Using a grid overlay, areas to be sampled were randomly located. To ensure representative sampling of the entire area of natural vegetation, the number of sample areas (stands) located on any one quadrangular map was proportional to the number of square miles of natural vegetation represented on the map. Random points that fell within lakes, streams, or marsh areas were immediately eliminated. To ensure a sufficient number of areas to allow rejection of those not meeting the criteria for sample acceptance in the field, about twice as many areas were established as anticipated for use. During the initial stages of study it was thought there might be an over-sampling of the most common community types. In this survey it was not a problem, however. Users elsewhere might encounter this problem and the possibility of stratifying samples on the basis of topography, moisture, geology, o some other scheme should be considered. Because the survey area was roadless, over 10 miles east to west, and up to 30 miles north t south, sample points were restricted to withi 1,000 meters of the nearest canoe route or estab lished foot trail.

Criteria for field acceptance of the randoml established sample areas were established i advance as follows:

1. The area must be upland and consist o natural vegetation.

2. The area must be a minimum of 5 acres o forested vegetation or a minimum of 100 squar meters of nonforested vegetation, provided ther is a margin of at least one tree height as a buffe between the nonforested vegetation and th nearest forest.

3. The stand must be located on a unifor topographic site; i.e., located entirely on one ex posure, slope position, and geological substrat

4. The sample area must be relatively hom geneous in vegetational composition as dete mined by visual inspection of airphotos and the site itself. The idea is to ensure that th sample represents only one kind of plant co munity, not two or more. Stands with disco tinuities in vegetation will be rejected. Criteri three above usually ensures this.

These criteria may be modified for use other vegetation types or regions according the purpose of the survey.

Two major considerations were used in d signing field data forms: (a) to make the for simple, requiring a minimum of writing, a thus reducing chance for omission of data a recording errors, and (b) to allow direct punc ing of the data from the field sheets onto co puter cards (ADP) for automatic data processi

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[2] *Heinselman, M. L., Ohmann, L. F., Ream, R. R., and Brown, C. A problem analysis of research in the ecology of wilderness biotic communities in the Boundary Waters Canoe Area. (Unpublished report on file at North Central Forest Experiment Station, St. Paul, Minnesota.)*

[3] *Natural vegetation is defined as that composed of native species, resulting primarily from environmental factors present in the ecosystem prior to settlement (and potentially still effective, including such factors as windstorms, insect and disease outbreaks, and fire) or from gradual successional change.*

This resulted in greater efficiency as well as n reduced transcription errors. Forms were rinted on high-quality paper with moisture-esistant ink to resist deterioration under wet ield conditions. Where possible, species likely o be encountered in the regional flora were rinted on the form.

SAMPLING METHODS

A field reconnaisance of each sample area vas made to determine if it met the criteria or acceptance. When the area did not meet the riteria, it was rejected and the nearest sample rea previously plotted was visited. If the area vas satisfactory, the following data were colected.

eneral Information — Data Form EI-1 (Fig. 1)

(5)[4] A three-digit stand number.

(8) Airphoto number (the stand was outlined n the airphoto).

(17-24) The legal description of the stand as etermined from the topographic map.

(25) A numerical code to identify the sampling crew. This may be important when more han one crew is involved to help answer quesions that may arise concerning the data.

(26-29) Date of sampling.

(31) Elevation of sample stand as determined rom the topographic map.

(35) Percent slope (five readings were taken vithin the sample area and averaged).

(37) Azimuth (aspect) of the stand.

(40) Position on the slope recorded as: idgetop, upper slope, midslope, lower slope, r valley.

(41) Location of the stand in relation to the many lakes of the area, recorded as either interior, island, peninsula, or lakeshore.

(42) Distance to the nearest water body of over 5 acres.

(45) Geologic bedrock type as indicated on geologic bedrock maps of the area.

(47) Surface material recorded as: ground moraine, outwash plain, end moraine, lacustrine deposit, or exposed bedrock outcrop.

(49) Annual insolation received by the site as determined from appropriate tables in Frank and Lee (1966).

(53-75) Height and age of five dominant trees of forested stands. Height was determined by use of distance tape and clinometer; age by increment boring. Many of the communities in the BWCA are of postfire origin and represent single-aged canopies. This information provides a means of looking at the communities as a time series in order to evaluate succession following a disturbance.

The remaining portion of the data sheet EI-1 was used to record a general description of the sample area. There was normally enough room left on the sheet to record a brief description of the soils.

Soils

A soil pit was dug in a representative part of the stand (representative in the sense that localized swells or humps were avoided) and the following data were recorded:

(a) Litter depth. Litter depth was also recorded in each of the ground cover plots.

(b) Depth of the fermentation (02) and humus (01) layers.[5] Soils of the BWCA have little humus and so the two layers were treated as one unit.

(c) Depth of each of the major mineral soil horizons (A1, A2, B, and C) was measured and horizon texture and color were briefly described.

(d) Volume of cobbles and stone (greater than 3 inches in diameter) in the profile was estimated.

(e) A 1-quart sample of the combined fermentation and humus layer, and a quart sample of the B horizon were collected for laboratory analysis.

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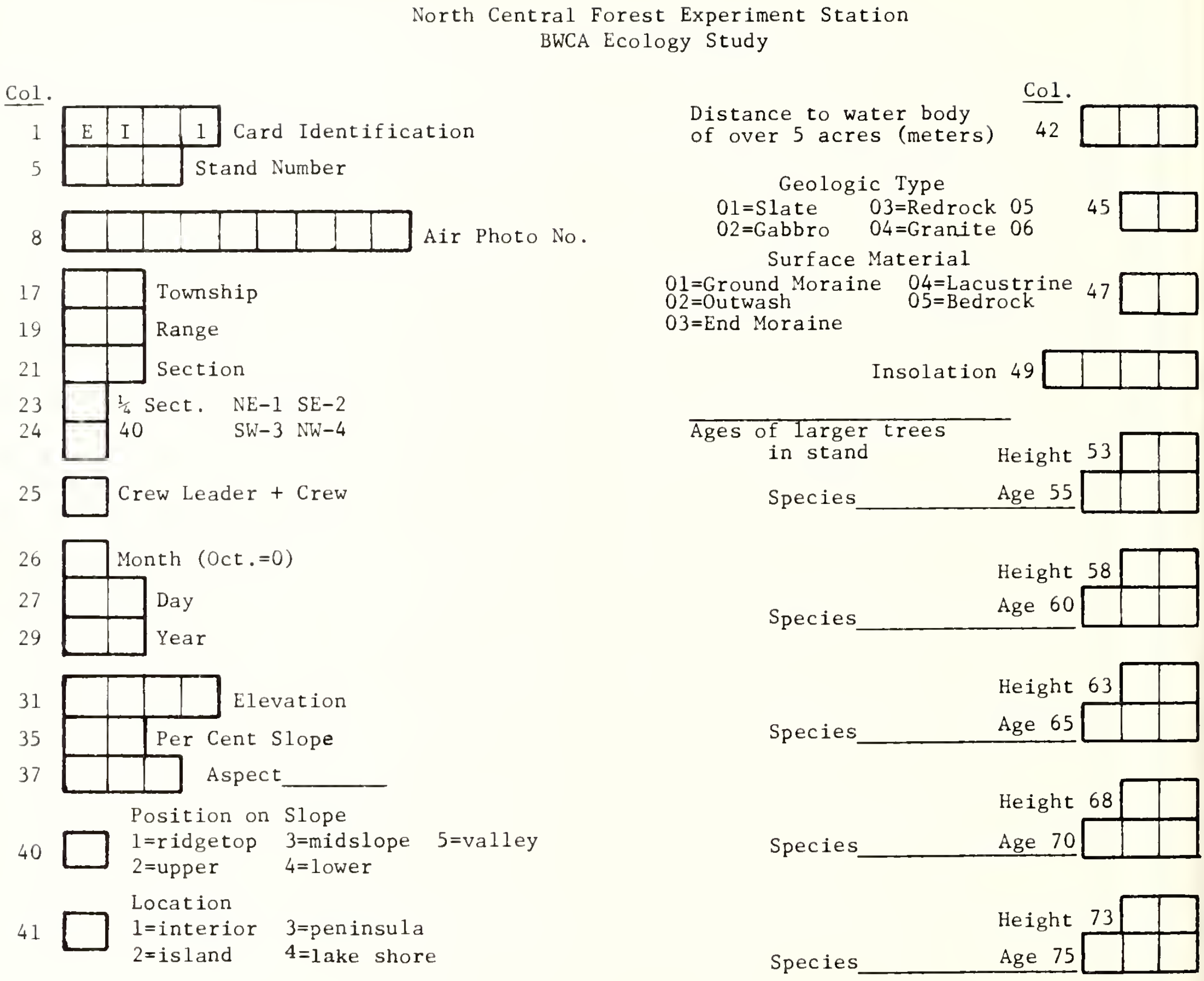
[4] *The number at beginning of this and subsequent paragraphs refers to the initial column where the field data is to be punched on ADP cards. It also identifies location on the data form.*

[5] *02 and 01 as defined by the Seventh Approximation.*



North Central Forest Experiment Station
BWCA Ecology Study

Col.
1 E I 1 Card Identification
5 Stand Number
8 Air Photo No.
17 Township
19 Range
21 Section
23 ¼ Sect. NE-1 SE-2
24 40 SW-3 NW-4
25 Crew Leader + Crew
26 Month (Oct.=0)
27 Day
29 Year
31 Elevation
35 Per Cent Slope
37 Aspect________
Position on Slope
40 1=ridgetop 3=midslope 5=valley
2=upper 4=lower
Location
41 1=interior 3=peninsula
2=island 4=lake shore

Col.
Distance to water body of over 5 acres (meters) 42
Geologic Type
01=Slate 03=Redrock 05
02=Gabbro 04=Granite 06 45
Surface Material
01=Ground Moraine 04=Lacustrine
02=Outwash 05=Bedrock 47
03=End Moraine
Insolation 49
Ages of larger trees in stand
Height 53
Species____________ Age 55
Height 58
Species____________ Age 60
Height 63
Species____________ Age 65
Height 68
Species____________ Age 70
Height 73
Species____________ Age 75

Figure 1. — Form for recording general environment data within each sample stand.

Vegetative Data

A discussion of sampling efficiency versus accuracy is beyond the scope of this paper. A variety of views can be found in the ecological literature (Greig-Smith 1964, Cottam and Curtis 1956, Curtis 1959, Lindsey *et al.* 1958, Lindsey 1956, Shanks 1954, Rice and Penfound 1955). We chose 20 points as a compromise between effort in obtaining the data and the accuracy of the results needed for an extensive type survey. This choice was also influenced by the previous use of similar methods in comparable vegetation (Curtis 1959, Maycock and Curtis 1960). Use of these methods in more complex vegetation might require an increase in the number of sample points taken in each stand, depending on manpower and field ti available, and extensiveness of the survey balanced against the accuracy.

The first sample point was selected ra domly and the remaining points were spac equidistant from one another. This genera was 20 steps or 60 feet apart. We tried to spa the points in block design of five rows of fo points or four rows of five points, but this d pended on the size of the area and its topograp (fig. 2). In non-forested stands the distance tween the sample points was much smaller conform to the smaller area occupied by t vegetation.

Permanent plots in the strict sense w not established as part of this survey; the of carrying suitable materials for perman

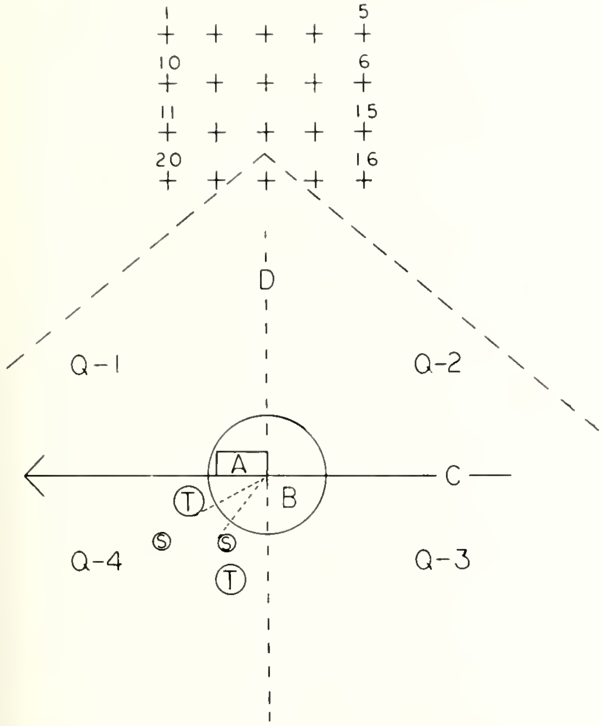


Figure 2. — *The upper portion of the figure shows the distribution of sample points within a stand. The lower portion is a diagrammatic representation of the sampling system at one sample point. A is a 1 by 2-foot plot for sampling herbs and low shrubs. B is a milacre plot for sampling tall shrubs and tree seedlings. C is the line of travel and D is an imaginary line perpendicular to it, forming the four quadrants (Q) of the point-centered quarter method. S is a sapling. T is a tree.*

plot marking into the Wilderness Area would have been prohibitive. The sample areas (stands) are permanently recorded on both topographic maps and on aerial photographs and can thus be resampled.

Tree and Sapling Size Classes — Data Form EI-2 (Fig. 3)

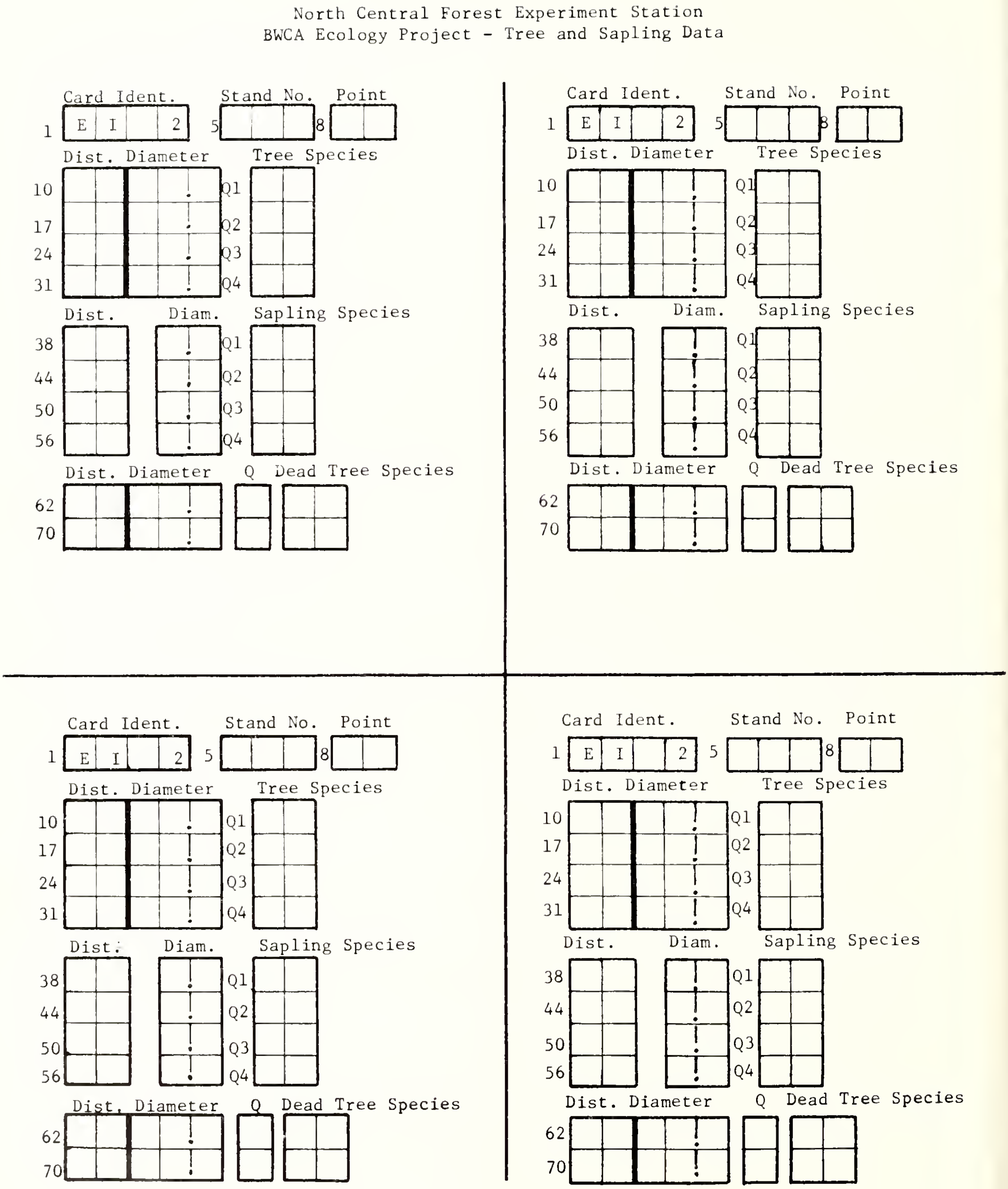
Trees [more than 4 inches diameter at breast height (d.b.h.)] and saplings (1 to 4 inches d.b.h.) were sampled by the point-centered quarter method (Cottam and Curtis 1956). The direction of travel from point to point constituted one line and an imaginary line was drawn perpendicular to it, forming four quadrants around the sample point (fig. 2). The distance to the nearest tree and sapling in each quadrant was measured and the species and d.b.h. of each was recorded. Each sheet of this data form was used to record the data from four points, so five sheets were needed per sample area. Species were recorded by a two-digit code number. We found it helpful to add the species initials to the right of the code number blanks to check against errors in recording code numbers. The notations Q1, Q2, etc. to the right of the diameter columns refer to the quadrant. Quadrants were numbered as: forward right (Q1), rear right (Q2), rear left (Q3), forward left (Q4), with regard to the direction of travel (fig. 2). This is not essential, but might be used in statistical analysis later. Distances were recorded to the nearest foot, and d.b.h. to the nearest one-tenth inch. Distances were measured with either a tape or range finder. D.b.h. was measured with a standard diameter tape. Columns 62 to 77 of the ADP card were reserved for recording dead tree species. To provide insight into the dynamics of these forest stands, we recorded any identifiable tree with a stem present at breast height or the stump of a beaver logged tree if it would have been included in the sample if still alive.

Tree Seedling Class — Data Form EI-4 (Fig. 4)

Seedlings of each tree species were counted and their projected ground cover estimated in a milacre circular plot located at each sample point used in the point-centered quarter method. Two ADP cards were punched for each species, one containing the cover data, the other the number of individuals; therefore, space is provided on the data form for a two-digit code for cover (example: *Abies balsamea* 151) and a two-digit code for number of individuals (*Abies balsamea* 152) for each species in each plot. The presence of deer, moose, and rabbit pellet groups were also noted in each milacre plot and recorded on this data form below seedlings.

Tall Shrubs — Data Form EI-5 (Fig. 5)

We arbitrarily divided the shrub species of this region into tall and low-growing shrubs for convenience of sampling. Tall shrubs were sampled in the same circular milacre plots used for tree seedlings. We recorded the number of stems for each shrub species by 1-centimeter size classes; stems were measured at 6 inches above the ground. Stem diameter was determined by use of a template. Where shrubs were



North Central Forest Experiment Station
BWCA Ecology Project - Tree and Sapling Data

Card Ident. Stand No. Point
1 E I 2 5 8
Dist. Diameter Tree Species
10 Q1
17 Q2
24 Q3
31 Q4
Dist. Diam. Sapling Species
38 Q1
44 Q2
50 Q3
56 Q4
Dist. Diameter Q Dead Tree Species
62
70

Card Ident. Stand No. Point
1 E I 2 5 8
Dist. Diameter Tree Species
10 Q1
17 Q2
24 Q3
31 Q4
Dist. Diam. Sapling Species
38 Q1
44 Q2
50 Q3
56 Q4
Dist. Diameter Q Dead Tree Species
62
70

Card Ident. Stand No. Point
1 E I 2 5 8
Dist. Diameter Tree Species
10 Q1
17 Q2
24 Q3
31 Q4
Dist. Diam. Sapling Species
38 Q1
44 Q2
50 Q3
56 Q4
Dist. Diameter Q Dead Tree Species
62
70

Card Ident. Stand No. Point
1 E I 2 5 8
Dist. Diameter Tree Species
10 Q1
17 Q2
24 Q3
31 Q4
Dist. Diam. Sapling Species
38 Q1
44 Q2
50 Q3
56 Q4
Dist. Diameter Q Dead Tree Species
62
70

Figure 3. — Form for recording tree and sapling size-class data as measured by use of the point-centered quarter method.

Card Identif. EI 4 Stand Number Tree Seedling Data
1--4 Columns 567

	col.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	8 9 10	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49
Abies bal c	151																				
Abies bal #	152																				
Acer rub c	153																				
Acer rub #	154																				
Bet pap c	155																				
Bet pap #	156																				
Pic glau c	157																				
Pic glau #	158																				
Pic mari c	159																				
Pic mari #	160																				
Pin bank c	161																				
Pin bank #	162																				
Pin res c	163																				
Pin res #	164																				
Pin str c	165																				
Pin str #	166																				
Pop tre c	167																				
Pop tre #	168																				
Thu occ c	169																				
Thu occ #	170																				
Deer Pell.	200																				

Figure 4. — Form for recording tree seedling size-class data as measured in the milacre plots.

CARD 1. D.
E 1 5
1-2-3-4

STAND NO.
5-6-7

SPECIES			QUADRAT NUMBER																			
	SP CODE	BA CLASS CODE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
IBM Column Number	8-9	10-11	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50

Figure 5. — Form for recording tall shrub data as measured in the milacre plots.

very abundant they were counted on only half of each milacre plot. Because each shrub species requires as many rows on the data form as there are diameter classes represented, no species names were preprinted. In column 8-9 a two-digit species code was later assigned. In column 10-11 a basal area (diameter class) code was recorded (01, 02, 03, 04, 05, etc.). In the rest of the columns the number of stems counted were recorded for each diameter class under the appropriate quadrate number.

In addition to diameter class and number of stems for each species, we estimated the amount of browsing on woody twigs. The following four categories were used: 0 = no browsing; 1 = 0 to ⅓ of all the tips available browsed; 2 = ⅓ to ⅔ browsed; 3 = more than ⅔ browsed. This was recorded directly under the last diameter class category for each species, using the appropriate species code and number 99 in the basal area code columns.

Ground Cover Characteristics and Vegetation — Data Form EI-3 (Fig. 6)

A 1- by 2-foot rectangular quadrat was established at each tree sample point; ground cover characteristics were recorded by percent of plot occupied. These characteristics were: bare rock, live wood, dead wood (over 1 inch diameter), bare ground, litter, lichens, and mosses. In addition, litter depth over the plot was estimated and averaged. Ground cover characteristics and most species names were preprinted on this data form, but spaces were provided for less common species.

Herbaceous and low shrub vegetation. — Low shrubs, ferns and fern allies, and herbs were recorded as the percent of projected ground cover they occupied within the 1- by 2-foot rectangular plot. This was an ocular estimate. Occasionally a species partially covered a plot but was not rooted in that plot. We recorded the plot number each time this occurred, because we wished to base frequency of occurrence values on actual rooting of a species within the sampling plot. These plot numbers were recorded in columns 51 through 62 of the data form. The summary program then subtracted those plots from the percent frequency calculation for the species. Some plants were treated as groups, because sampling was conducted throughout the growing season making some species difficult to identify during certain times of the season. For instance, because grasses and sedges are of minor importance in our vegetation, we treated all grass species as a unit and all sedge species as a unit. Identification can be as detailed or as general as the purpose of the survey indicates.

LABORATORY METHODS

Soils

Values were obtained for the combined fermentation and humus layers by ashing 1 gram of sample at 525 C. for 15 hours. The ash was brought into solution and analyzed on a Jarrell-Ash emission spectrometer model 66-000.[6] Values for phosphorus, potassium, calcium, aluminum, sodium, iron, magnesium, zinc, copper, molybdenum, manganese, and boron were determined. Samples of mineral soil (B horizon) were analyzed for exchangeable calcium, magnesium, potassium, and sodium by shaking 5 grams of soil in 20 ml. neutral normal ammonium acetate for one-half hour. This mixture was filtered and leached with an additional 80 ml. of ammonium acetate. Calcium and magnesium were analyzed on a Perkin Elmer atomic absorption spectrophotometer Model 303. Sodium and potassium were analyzed on a Perkin Elmer flame photometer. Exchangeable hydrogen was measured by the $BaC1_2$ -Triethanolamine, pH8.1 method. Extractable phosphorus was analyzed by shaking 5 grams of soil for 5 minutes in 50 ml. of Bray's No. 1 solution (0.025N HC1 — 0.03N NH_3F). Molybdophosphoric blue color was developed and read on a Klett Summerson Colorimeter. Acidity (pH) was determined on a 1:1 soil-water mixture by a Beckman Zeromatic pH meter.

Mineral soil samples were analyzed by the hydrometer method (Bouyoucos 1951) for percent sand, silt, and clay. Moisture retention capacity was determined by pressure membrane extraction at 15 and pressure plate extraction at ⅓ atmospheres (Richards 1954).

Vegetation

Information from the field data forms was punched directly onto computer cards. Four computer programs were written in Fortran I

---

[6] *Mention of trade names does not constitut endorsement by the USDA Forest Service.*

Card Identif. EI 3 Stand Number
1--4 Columns 567

North Central Forest Experiment Station
BWCA Ecology Study

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Cover– Not Rooted In Quadrats No.						
Col.	8-10	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39	41	43	45	47	49	51	53	55	57	59	61	
Bare Rock	001																											
Live Wood	002																											
Dead Wood	003																											
Bare Ground	004																											
Litter	005																											
Call. schr.	006																											
Clad. alpest	007																											
Clad.rangif	008																											
Clad. mitis	009																											
Clad. sylv.	010																											
Dicranum sp	011																											
Hylo splend	012																											
Hypnum cris	013																											
Polytri sp.	014																											
Lyco annot	020																											
Lyco clavat	021																											
Lyco compla	022																											
Lyco lucidu	023																											
Lyco obscu	024																											
Poly Virgin	025																											
Pterid aqui	026																											
Arcto uva	031																											
Chima umb	032																											
Gaulth hisp	033																											
Gaulth pro	034																											
Rosa acic	035																											
Rubus idae	036																											
Rubus pub	037																											
Vaccin Ang	038																											
Vaccin myr	039																											
Aralia nud	050																											
Aster mac	051																											
Clint bor	052																											
Coptis groe	053																											
Cornus can	054																											
Epilob ang	055																											
Frag vesca	056																											
Galium tri	057																											
Goodye rep	058																											
Goodye tes	059																											
Lathy ochro	060																											
Linn bore	061																											
Maianth can	062																											
Melamp lin	063																											

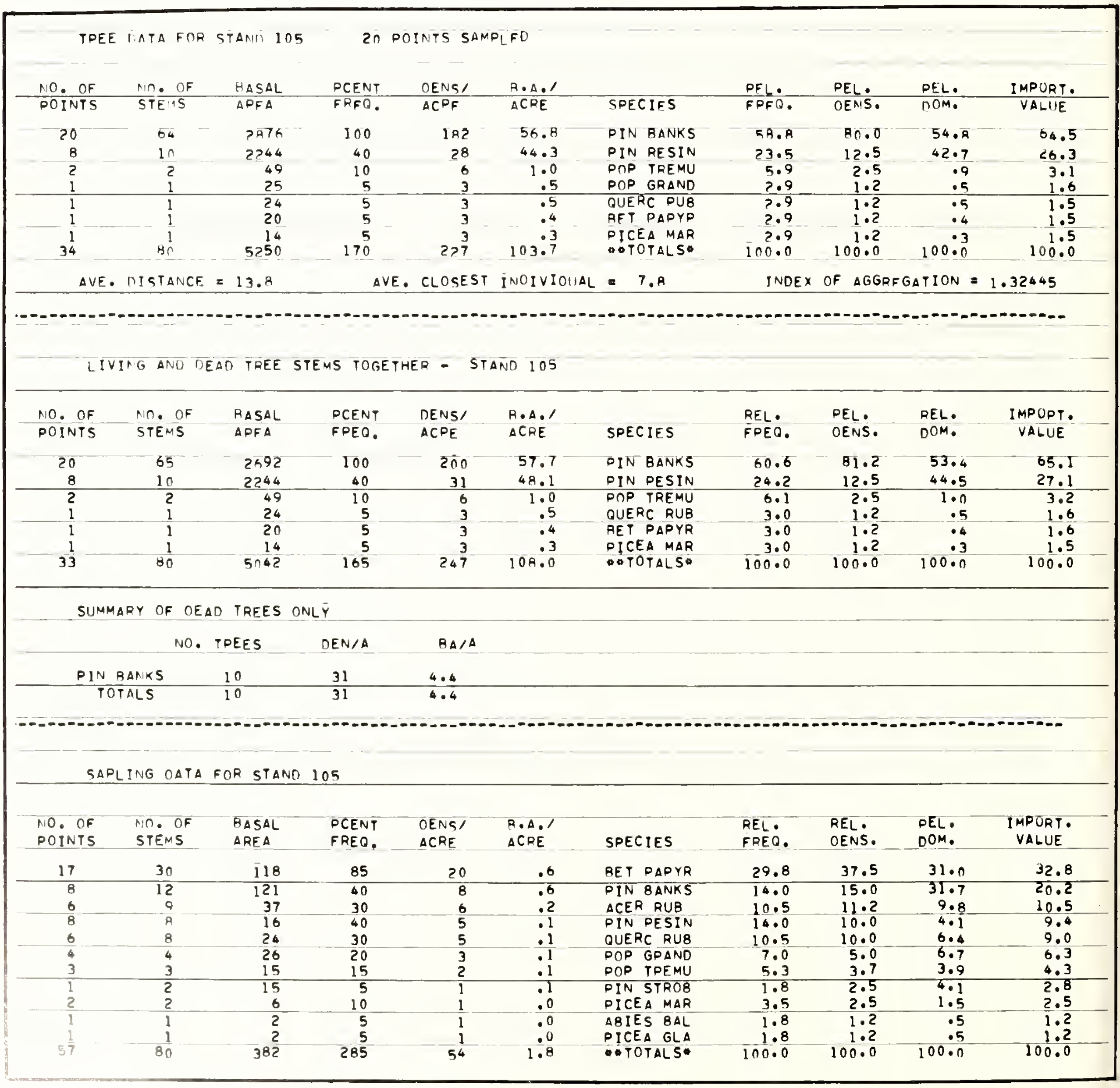
Figure 6. — Form for recording ground cover characteristics, low shrubs, and herbaceous species data as measured in the 1- by 2-foot plots.

language for use on a CDC-6600 computer. These programs summarize the vegetation data for each sample area. Each program is described and an example of the printout summaries included in the following section.

TREESUM

Previously prepared computer cards (EI-2, fig. 3) containing measurements for the trees and saplings are the input for this program. Program output (fig. 7) for each sample area consists of three units:

(1) *Living tree size class.* – The first three columns of figures summarize the actual number of points of occurrence, number of stems, and basal area for each species sampled within the stand. The computer uses these data plus the distance measures to calculate percent frequency of occurrence at the points, density in stems per acre, and basal area (at breast height) in square feet per acre for each species; these are totaled for the sample area. The computer also calculates relative frequency, density, dominance (basal area), and their summa-



```
     TPEE DATA FOR STAND 105       20 POINTS SAMPLED

 NO. OF      NO. OF      BASAL      PCENT      OENS/      B.A./                       PEL.      PEL.      PEL.      IMPORT.
 POINTS      STEMS       APFA       FRFQ.      ACPF       ACRE        SPECIES         FPFQ.     OENS.     DOM.       VALUE

   20          64        2876        100        182       56.8        PIN BANKS       58.8      80.0      54.8        64.5
    8          10        2244         40         28       44.3        PIN RESIN       23.5      12.5      42.7        26.3
    2           2          49         10          6        1.0        POP TREMU        5.9       2.5        .9         3.1
    1           1          25          5          3         .5        POP GRAND        2.9       1.2        .5         1.6
    1           1          24          5          3         .5        QUERC PUB        2.9       1.2        .5         1.5
    1           1          20          5          3         .4        BET PAPYP        2.9       1.2        .4         1.5
    1           1          14          5          3         .3        PICEA MAR        2.9       1.2        .3         1.5
   34          80        5250        170        227      103.7        **TOTALS*      100.0     100.0     100.0       100.0

     AVE. DISTANCE = 13.8           AVE. CLOSEST INOIVIOUAL =  7.8         INDEX OF AGGRFGATION = 1.32445

----------------------------------------------------------------------------------------------------------------------

      LIVING AND OEAD TREE STEMS TOGETHER -   STAND 105

 NO. OF      NO. OF      BASAL      PCENT      DENS/      B.A./                       REL.      PEL.      REL.      IMPOPT.
 POINTS      STEMS       APFA       FPEQ.      ACPE       ACRE        SPECIES         FPEQ.     OENS.     DOM.       VALUE

   20          65        2692        100        200       57.7        PIN BANKS       60.6      81.2      53.4        65.1
    8          10        2244         40         31       48.1        PIN PESIN       24.2      12.5      44.5        27.1
    2           2          49         10          6        1.0        POP TREMU        6.1       2.5       1.0         3.2
    1           1          24          5          3         .5        QUERC RUB        3.0       1.2        .5         1.6
    1           1          20          5          3         .4        BET PAPYR        3.0       1.2        .4         1.6
    1           1          14          5          3         .3        PICEA MAR        3.0       1.2        .3         1.5
   33          80        5042        165        247      108.0        **TOTALS*      100.0     100.0     100.0       100.0

     SUMMARY OF OEAD TREES ONLY

                 NO. TPEES        DEN/A         BA/A

     PIN BANKS      10              31           4.4
       TOTALS       10              31           4.4

----------------------------------------------------------------------------------------------------------------------

      SAPLING OATA FOR STAND 105

 NO. OF      NO. OF      BASAL      PCENT      OENS/      B.A./                       REL.      REL.      PEL.      IMPORT.
 POINTS      STEMS       AREA       FREQ.      ACRE       ACRE        SPECIES         FREQ.     OENS.     DOM.       VALUE

   17          30         118         85         20         .6        BET PAPYR       29.8      37.5      31.0        32.8
    8          12         121         40          8         .6        PIN BANKS       14.0      15.0      31.7        20.2
    6           9          37         30          6         .2        ACER RUB        10.5      11.2       9.8        10.5
    8           8          16         40          5         .1        PIN PESIN       14.0      10.0       4.1         9.4
    6           8          24         30          5         .1        QUERC RUB       10.5      10.0       6.4         9.0
    4           4          26         20          3         .1        POP GPAND        7.0       5.0       6.7         6.3
    3           3          15         15          2         .1        POP TPEMU        5.3       3.7       3.9         4.3
    1           2          15          5          1         .1        PIN STROB        1.8       2.5       4.1         2.8
    2           2           6         10          1         .0        PICEA MAR        3.5       2.5       1.5         2.5
    1           1           2          5          1         .0        ABIES BAL        1.8       1.2        .5         1.2
    1           1           2          5          1         .0        PICEA GLA        1.8       1.2        .5         1.2
   57          80         382        285         54        1.8        **TOTALS*      100.0     100.0     100.0       100.0
```

Figure 7. – Example of the printout of Computer Program TREESUM.

tion — the importance value (Cottam and Curtis 1956) — for each species in the sample area. The average distance from the 20 sample points to the 80 trees measured, the average distance to the nearest individual measured at each sample point, and an index of aggregation value (Eberhardt 1967) is also printed.

(2) *Living and dead stems.* — This portion of the program is the same as described above, except that dead trees which would have been included in the sample if they were still alive are substituted as input for the next nearest live tree that was measured. A summary of the dead trees by species including number measured, density in stems per acre, and basal area in square feet per acre is also printed.

(3) *Sapling size class.* — The sapling size-class field measurements are summarized as in (1) above.

SEEDSUM

This program uses field measurements of tree seedlings (computer cards EI-4, fig. 4) and pellet group counts for moose, deer, and rabbits on the milacre plots as input. Program output (fig. 8) includes stand number, number of plots measured, number of plots on which each species occurred, number of seedlings counted, and average percent cover of the milacre plot occupied by each species. Using these data the computer calculates density in stems per acre, relative values of frequency, density, cover, and their summation, the importance value. The computer summarizes pellet groups for each species by number of plots on which they occurred, percent frequency of occurrence, and total number counted.

SHRUBSUM

Tall shrub measurements from the milacre plots at each point within the stand (computer cards EI-5, fig. 5) are used as input for this program.

Output from this program for each species (fig. 9) consists of average browse index, percent frequency of occurrence, total number of shrubs counted in all size classes, total basal area measured in all size classes, density in stems per acre, and basal area (at 6 inches above the ground) in square feet per acre; also included are relative frequency, density, dominance (basal area), and their summation, the importance value.

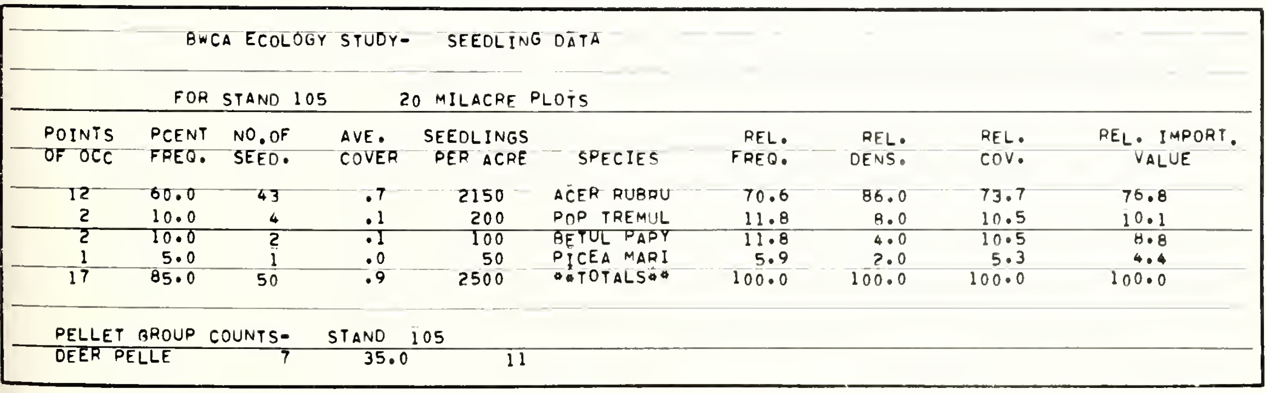
HERBSUM

This program uses as input all data collected in the 1- by 2-foot rectangular quadrats (computer cards EI-3, fig. 6). Program output (fig. 10) is divided into three parts: ground cover, low shrubs, and herbs.

(1) *Ground cover.* — Information on ground cover characteristics, lichens, and mosses is summarized. For each characteristic or species the following are computed: the number of points of occurrence, percent frequency of occurrence, total percent cover estimated in the 20 quadrats, and average percent cover per plot. Relative frequency, percent cover, and the importance value are also computed.

(2) *Low shrubs.* — Data are summarized for low shrubs as in (1) above.

(3) *Herbs, ferns, and fern allies.* — Data for this group are summarized as in (1) above. In

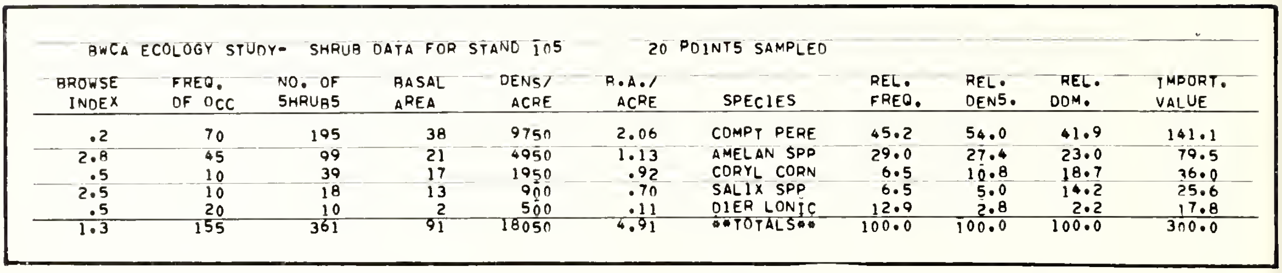


```
            BWCA ECOLOGY STUDY-   SEEDLING DATA

           FOR STAND 105        20 MILACRE PLOTS
  POINTS    PCENT   NO.OF    AVE.    SEEDLINGS                     REL.      REL.      REL.       REL. IMPORT.
  OF OCC    FREQ.   SEED.    COVER    PER ACRE     SPECIES         FREQ.     DENS.     COV.          VALUE
     12      60.0     43       .7       2150     ACER RUBRU         70.6      86.0      73.7          76.8
      2      10.0      4       .1        200     POP TREMUL         11.8       8.0      10.5          10.1
      2      10.0      2       .1        100     BETUL PAPY         11.8       4.0      10.5           8.8
      1       5.0      1       .0         50     PICEA MARI          5.9       2.0       5.3           4.4
     17      85.0     50       .9       2500     **TOTALS**        100.0     100.0     100.0         100.0

   PELLET GROUP COUNTS-    STAND  105
   DEER PELLE          7     35.0        11
```

Figure 8. — Example of the printout of Computer Program SEEDSUM.

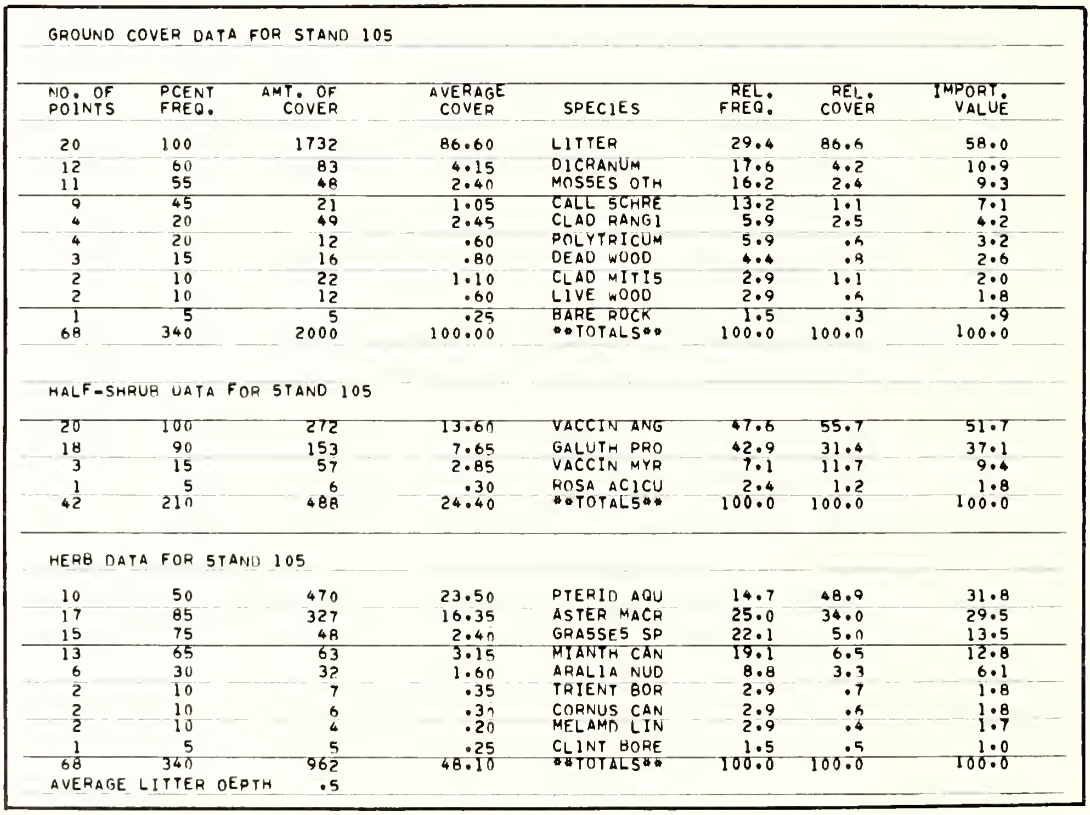


```
      BWCA ECOLOGY STUDY-  SHRUB DATA FOR STAND 105          20 POINTS SAMPLED

  BROWSE     FREQ.     NO. OF     BASAL      DENS/      B.A./                         REL.      REL.     REL.       IMPORT.
   INDEX     OF OCC    SHRUBS     AREA        ACRE       ACRE        SPECIES          FREQ.     DENS.    DOM.        VALUE

       .2       70        195        38       9750       2.06       COMPT PERE        45.2      54.0      41.9       141.1
      2.8       45         99        21       4950       1.13       AMELAN SPP        29.0      27.4      23.0        79.5
       .5       10         39        17       1950        .92       CORYL CORN         6.5      10.8      18.7        36.0
      2.5       10         18        13        900        .70       SALIX SPP          6.5       5.0      14.2        25.6
       .5       20         10         2        500        .11       DIER LONIC        12.9       2.8       2.2        17.8
      1.3      155        361        91      18050       4.91       **TOTALS**       100.0     100.0     100.0       300.0
```

Figure 9. — Example of the printout of Computer Program SHRUBSUM.



```
  GROUND COVER DATA FOR STAND 105


  NO. OF     PCENT      AMT. OF         AVERAGE                         REL.      REL.        IMPORT.
  POINTS     FREQ.       COVER           COVER        SPECIES          FREQ.     COVER         VALUE

     20       100         1732           86.60        LITTER            29.4      86.6          58.0
     12        60           83            4.15        DICRANUM          17.6       4.2          10.9
     11        55           48            2.40        MOSSES OTH        16.2       2.4           9.3
      9        45           21            1.05        CALL SCHRE        13.2       1.1           7.1
      4        20           49            2.45        CLAD RANGI         5.9       2.5           4.2
      4        20           12             .60        POLYTRICUM         5.9        .6           3.2
      3        15           16             .80        DEAD WOOD          4.4        .8           2.6
      2        10           22            1.10        CLAD MITIS         2.9       1.1           2.0
      2        10           12             .60        LIVE WOOD          2.9        .6           1.8
      1         5            5             .25        BARE ROCK          1.5        .3            .9
     68       340         2000          100.00        **TOTALS**       100.0     100.0         100.0


  HALF-SHRUB DATA FOR STAND 105

     20       100          272           13.60        VACCIN ANG        47.6      55.7          51.7
     18        90          153            7.65        GALUTH PRO        42.9      31.4          37.1
      3        15           57            2.85        VACCIN MYR         7.1      11.7           9.4
      1         5            6             .30        ROSA ACICU         2.4       1.2           1.8
     42       210          488           24.40        **TOTALS**       100.0     100.0         100.0


  HERB DATA FOR STAND 105

     10        50          470           23.50        PTERID AQU        14.7      48.9          31.8
     17        85          327           16.35        ASTER MACR        25.0      34.0          29.5
     15        75           48            2.40        GRASSES SP        22.1       5.0          13.5
     13        65           63            3.15        MIANTH CAN        19.1       6.5          12.8
      6        30           32            1.60        ARALIA NUD         8.8       3.3           6.1
      2        10            7             .35        TRIENT BOR         2.9        .7           1.8
      2        10            6             .30        CORNUS CAN         2.9        .6           1.8
      2        10            4             .20        MELAMP LIN         2.9        .4           1.7
      1         5            5             .25        CLINT BORE         1.5        .5           1.0
     68       340          962           48.10        **TOTALS**       100.0     100.0         100.0
  AVERAGE LITTER DEPTH      .5
```

Figure 10. — Example of the printout of Computer Program HERBSUM.

addition, the average litter depth for all rectangular plots within the stand is computed.

In addition to printouts the programs can provide instructions for punching summary data on ADP cards. These cards may serve as input for the further analysis of sample-area relationships, classification by community types, and community-environment relationships.

DISCUSSION

The methods described provide minimum data for plant community description. Summaries on individual stands permit the investigator to classify stands by community type using any method he chooses. We used quantitative methods (principle component and optimal agglomeration) for determining the similarity of the stands to one another (Ream and Ohmann 1970), and grouped the stands into community types on those bases. Preliminary testing of species composition of these groups showed that they differ statistically.

In keeping with the spirit of the Wilderness Act, sampling crews traveled within the BWCA by the same means available to the public. In nonmotor zones we paddled; in motor-use zones we used motors. It was apparent that our major cost would be in time spent traveling to sample areas. In an effort to reduce these costs our crews worked on 10-day schedules, spending eight or nine days in the field and camping out for six or seven nights. Travel time to sample areas was still significant but much reduced. In the kind of vegetation we were sampling, one stand was sampled per day under average conditions. In dense shrub conditions sampling often required more than one day and in sparse shrub conditions a sample was obtained in less than one day. This includes time spent traveling from camp site, locating the stand, and obtaining the sample. Each crew generally sampled six or seven stands on each trip.

These methods can be modified to better sample a particular type of vegetation. In uniform vegetation the number of sample points and plots and size of plots might be reduced, or in more complex vegetation they might be increased. Sample areas might be selected by a stratified sample based on topography, or by some other scheme. The types of data collected could be modified to fit the interests of the particular survey. In areas where soils are well described, the soil collection could be eliminated. Many modifications in the sampling method, however, will require changes in the computer summarization programs.

Office time required for such things as checking data sheets and entering species code numbers is less than 15 minutes per stand. Punching field data onto ADP cards constitutes the primary computing cost. Each stand requires 45 to 50 cards for the vegetation data, each of which costs about $0.08 to punch. If soil data and other environmental data are punched on cards, costs are slightly higher. However, the total number of cards per stand is well under 75. Other indirect computing costs are involved in writing computer programs for data summarization. While these programs may have to be modified for other areas or surveys, this would not be a major cost.

Cost of computer time itself is minor and represents a large saving over hand calculation, especially when one considers the increase in accuracy obtained.

Other costs incurred in the inventory are the chemical and mechanical analyses of soil samples. Actual costs depend on the availability of local personnel or organizations to conduct such analyses.

Materials required to conduct a vegetative survey using the methods described here are not extensive nor particularly expensive. A set of U.S. Geological Survey quadrangle maps of the survey area, diameter tapes, distance tapes, data sheets, a range finder, a shovel, an increment borer, a compass, and clipboards nearly complete the list of required items. Additional valuable items include plant keys, plant presses, and collection bags for both soils and plants.

These methods have been used for 3 years. Much of the field sampling has been handled by crews made up of senior botany or forestry undergraduates, or first-year graduate students assisted by first- or second-year botany or forestry undergraduate students. Training of the crews has been accomplished in the field by sampling up to four areas with experienced personnel.

The upland natural vegetation sampled included rock outcrops dominated by lichens, aspen-birch, maple-aspen-birch, jack pine, red pine, white pine, balsam fir, black spruce, and white-cedar communities (Ohmann and Ream 1969). The methods were about as readily applied in all these communites. The major factor in application seems to be the abundance of shrubs in each stand. Since designing these methods we have used them in three surveys. We are using the methods without modification in the areas logged in the 1890-1930 era. In an intensive survey of two 1936 wildfire areas we have had to modify the size-class categories for tree species, because trees 4 inches and larger are not frequent enough to efficiently sample as a separate class. We are using a single tree-size class of 1 inch d.b.h. and larger.

Few, if any, of the methods described above are original — most have been part of the ecological literature for some time. We have simply brought together a set of flexible, efficient methods to collect and summarize large quantities of data in a form suitable for automatic data processing and further quantitative analysis.

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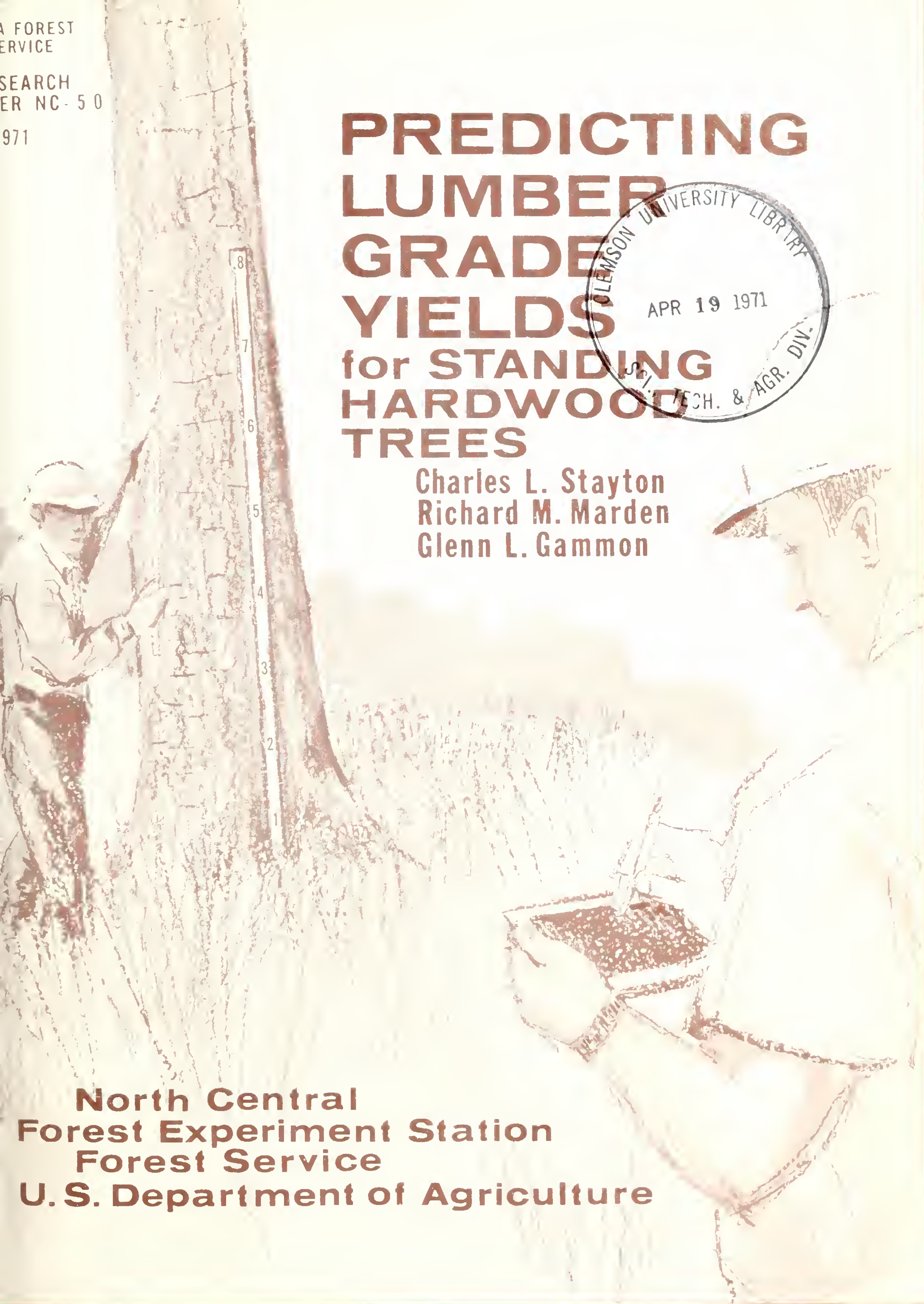

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PREDICTING LUMBER GRADE YIELDS for STANDING HARDWOOD TREES

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Predicting Lumber Grade Yields For Standing Hardwood Trees

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and Glenn L. Gammon

Methods of assessing product yields for standng timber are needed to determine the quantity and quality of the timber resource in many areas. This information will indicate where new industries can be located and how much timber is economically operable to existing industries. Accurate timber resource information will assist n development of forest management, timber production, manufacturing, and marketing techniques to meet the increased demand for forest products.

To meet the goals outlined above, we developed a method to assess the quality of standing hardwood sawtimber. Tree stem characteristics were used to predict lumber grade yields for standing sugar maple trees. This paper is an extension of earlier work (Marden 1965), which presented the methodology for developing continuous prediction equations for estimating product yields in standing trees.

SAMPLE DESCRIPTION

Data were collected from sugar maple trees n old-growth northern hardwood stands in Upper Michigan. The trees were selected on the basis of d.b.h. and the number of clear faces within the butt one-quarter of the merchantable[1] stem. Trees were separated into three d.b.h. classes, and three quality classes within each d.b.h. class. The d.b.h. classes were 11-15 inches, 16-20 inches, and 21-26 inches; the quality classes were 0-1 clear faces, 2 clear faces, and 3-4 clear faces. There were 10 trees in each d.b.h. quality class combination, for a total of 90 trees. Tree age ranged from 92 to 289 years, and tree growth rate from 6 to 23 rings per inch of diameter.

PREVIOUS WORK

Originally this study was designed to develop prediction equations for estimating clearcutting yields in standing sugar maple trees (Marden 1965). The 90 trees were bucked into 8-foot lengths, which were sawed through-and-through into 1-inch-thick flitches. The flitches were photographed, projected on a screen, and clearcutting yields measured. The clearcutting yields were then related to stem characteristics. Lumber grade yields were not obtained.

Recent publications (Dunmire and Englerth 1967, Englerth and Schumann 1969, Schumann and Englerth 1967) give yields of random-width and specific-width dimension from 4/4 hard maple lumber. Thus, it was obvious that equations were needed to predict lumber grade yields from standing trees. Therefore, we calculated lumber grade yields for each sample tree, using Research Paper FPL-63, "Hardwood Log Grades for Standard Lumber" (Vaughan *et al.* 1966), developed prediction equations, and tested their accuracy. The testing was done by comparing the predicted with the observed lumber grade yields for additional sugar maple trees cut from four different National Forests.

CALCULATING LUMBER GRADE YIELDS

Because our 90 trees had been bucked into 8-foot logs, we had to paper-diagram the merchantable stem length of each tree to permit simulated bucking and log grading according to Research Paper FPL-63. All four merchantable stem faces were drawn to scale on paper, showing defect locations and sizes. This task was relatively easy because all surface abnormalities found on each log had been identified and measured, and all log faces and ends photographed in color.

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[1] *The merchantable height was restricted by a 6-inch d.i.b. minimum top or separation of stem into two or more distinct branches.*

The board-foot volume for each paper-diagram-graded log of each tree was obtained using the Scribner Decimal C Log Rule. Sound and unsound cull volumes for each log had been measured for another study (Stayton and Marden 1970), and were deducted from the gross volumes. Overrun was accounted for as given in Research Paper FPL-63. Thus, the net mill tally (bd. ft.) was obtained for each log of all 90 trees. Tables 10, 11, and 12 from Research Paper FPL-63 were then used to calculate lumber yields by grade for each log, and log yields were summed to get total yields per tree. These yields, by grade, were used as dependent variables.

INDEPENDENT VARIABLE SELECTION

The independent variables related to clearcutting yields had been selected earlier (Marden 1965). These variables now had to be tested for predicting lumber grade yields. A defect analysis helped to improve two of the selected independent variables; to determine the stem section that should be used to measure them, the optimum grading section was studied and selected.

Defect Analysis

Tree surface abnormalities, such as knots, bumps, and seams, are related to product yields in standing trees. Therefore, we wanted to know more about relationships between these exterior defect indicators and their associated interior defects.

From a precise study (Stayton *et al.* 1970) we found how often defect indicators had underlying defects in the quality zone.[2] Also, we found that size of the defect indicator (except flutes) was not related to associated interior defect. Therefore, instead of using the size of defect indicators as an independent variable, such defects were now only counted. The percentages of these exterior defect indicators that had interior defects were then applied:

X_5 = Number of knots, bumps, and surfac
rises + 0.62 (number of bark distor
tions + number of adventitious bud
and/or epicormic branches) divide
by merchantable stem length.

Because the percentage of flutes with under lying defect increased with increasing flut length, this defect was measured as before. W also continued to measure length of open an overgrown seams, because these defect indicator are often long enough to affect several logs. Th percentages of flutes and seams that had asso ciated interior defect were then applied to thei total length measurements:

X_6 = Total length of open seams + 0.6
(total length of overgrown seams) -
0.49 (total length of flutes) divide
by merchantable stem length.

Optimum Grading Section

Counting or measuring defect indicators t calculate X_5 and X_6 should be restricted to portion of the stem (optimum grading sectio between 0 and 16 feet. Obviously, the mo desirable section is that nearest the groun level.

We had recorded defect indicators by 4-fo sections up the stem. Therefore, we calculate X_5 and X_6 for all combinations of 4-foot sectio within the first 16 feet of each tree. Each the different defect counts and measures we divided by the length of the section used count or measure instead of total merchantab stem length.

The R^2 values and residuals for the origin regression analyses, where X_5 and X_6 were ca culated using all defects per tree and total me chantable stem, were compared with tho obtained when each of the new values of X_5 a X_6 were used. The different 4-foot section co binations worked about equally well (R^2 valu ranged from 0.80 to 0.91), with R^2 values almo equal to those obtained using total defects a stem length (values ranged from 0.81 to 0.9 One possible explanation for this came from o defect analysis study. We found, on the avera that 89 percent of all defect types were fai uniformly distributed by 4-foot sections up t stem (Stayton *et al.* 1970). Therefore, beca

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[2] *The quality zone was the portion outside of a core that had a diameter equal to one-half the diameter of the tree at that point.*

the defects counted or measured for variables X_5 and X_6 are divided by the stem length used, these variables would remain almost constant regardless of the 4-foot section combination used.

Values of X_5 and X_6, calculated for all 90 trees using the optimum grading sections 0 to 4 feet and 0 to 8 feet, were combined with other independent variables to develop prediction equations for estimating lumber grade yields. These two sets of equations were tested on 66 additional trees to select the final grading section. The 0 to 8 foot section was selected on the basis of best performance.

Independent Variables

The independent variables used to develop equations for estimating lumber grade yields in standing sugar maple trees were:

X_1 = Diameter breast height, inches
X_2 = Merchantable stem length, feet
X_3 = Merchantable stem volume inside bark (Smalian formula), cubic feet[3]
X_4 = Stem taper, inches per foot[4]
X_5 = Number of knots, bumps, and surface rises + 0.62 (number of bark distortions + number of adventitious buds and/or epicormics) within first 8 feet of stem divided by 8 feet, number per foot
X_6 = Total length of open seams + 0.67 (total length of overgrown seams) + 0.49 (total length of flutes) within first 8 feet of stem divided by 8 feet, inches per foot
X_7 = Average tree diameter, inches
X_8 = Diameter breast height squared.

Average tree diameter (X_7) was calculated using the equation,

$$D = \left(\frac{4\ X_3}{\pi\ X_2}\right)^{1/2}$$

Because X_3 was obtained for each tree by summing log volumes, D is a good representative average diameter.

Although there is a total of eight independent variables, only a minimum number of tree measurements were required. All of the variables were generated from these measurements: (1) counts and measures of certain defect indicators within the first 8 feet of the stem, (2) d.b.h., (3) several d.o.b. measurements up the stem, and (4) stem length.

Equations

Prediction equations were calculated for each lumber grade — FAS (Y_1); SEL (Y_2); #1C (Y_3); #2C (Y_4); #3A (Y_5); and #3B (Y_6):

$$Y_1 = 48.778 - 4.430\ X_1 - 0.440\ X_2 + 1.115\ X_3 - 63.646\ X_4 - 7.376\ X_5 + 0.063\ X_6 - 0.122\ X_7 + 0.115\ X_8$$

$$Y_2 = 6.743 - 0.777\ X_1 - 0.024\ X_2 + 0.523\ X_3 - 33.844\ X_4 - 2.743\ X_5 + 0.049\ X_6 + 0.125\ X_7 + 0.016\ X_8$$

$$Y_3 = 29.242 - 1.586\ X_1 - 0.325\ X_2 + 1.826\ X_3 - 119.850\ X_4 - 1.683\ X_5 + 0.076\ X_6 + 0.176\ X_7 + 0.020\ X_8$$

$$Y_4 = -\ 12.301 + 3.308\ X_1 - 0.190\ X_2 + 1.105\ X_3 - 34.130\ X_4 + 3.930\ X_5 + 0.125\ X_6 - 0.378\ X_7 - 0.095\ X_8$$

$$Y_5 = -\ 32.159 + 4.227\ X_1 + 0.107\ X_2 + 0.236\ X_3 - 2.366\ X_4 + 1.726\ X_5 + 0.034\ X_6 - 0.232\ X_7 - 0.113\ X_8$$

$$Y_6 = -\ 69.126 + 10.101\ X_1 + 0.579\ X_2 + 0.907\ X_3 - 41.147\ X_4 + 6.114\ X_5 + 0.009\ X_6 - 0.421\ X_7 - 0.288\ X_8$$

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[3] *Bark volume can be accurately estimated (Stayton and Hoffman 1970) and used to calculate merchantable stem volume inside bark.*

[4] *Stem taper was calculated using bottom and top d.o.b. measurements of merchantable stem.*

The R^2 values for these equations are as follows:

Lumber Grade	R^2
FAS	0.71
SEL	.73
#1C	.89
#2C	.94
#3A	.81
#3B	.84

One reason high R^2 values were obtained is because we used lumber grade yields obtained from published tables as dependent variables. Because these tables give average yields by log size and grade, the variation about the means is eliminated from our regression analysis.

Testing the Equations

The regression analyses indicated that our proposed methodology could be used to develop equations for estimating lumber grade yields for standing hardwood trees. However, the real test of any grading system is whether it accurately predicts yields for trees other than those used to develop the equations. To test our equations, we used data collected for 199 sugar maple trees from four different National Forests – 66 trees from the Ottawa National Forest, 39 from the Monongahela National Forest, 45 from the Green Mountain National Forest, and 49 from the White Mountain National Forest.[5] Because our original 90 sample trees came from the Ottawa National Forest, we were able to test the equations on trees from the same general area and also on trees from the Northeast and West Virginia.

Ottawa National Forest Test

The prediction equations (page 3) gave good estimates of total lumber yield and dollar value for the 66 trees from the Ottawa National Forest. Total lumber yield and dollar value were underpredicted by 7 and 4.4 percent, respectively. The difference between observed and predicted values within lumber grades ranged from 3 to 61 percent. The largest differences occurred for the #3A and 3B grades. The percent differences for the other grades ranged from 3 to 24 percent. The observed and predicted combined yields of #1 Common and Better lumber were almost identical – 14,036 bd. ft. versus 14,078 bd. ft. respectively (table 1).

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[5] *The lumber grade yields and tree-stem measurements for these trees were provided by the USDA Forest Service, Northeastern Forest Experiment Station's Grade and Quality of Hardwood Timber Project, Columbus, Ohio.*

Table 1.–*Comparison between observed and predicted lumber yields and dollar values for 66 sugar maple trees from the Ottawa National Forest*

Grade	Observed		Predicted		Bd.-ft. difference	Dollar-value difference
	Yield	Value 1/	Yield	Value		
	(Bd.ft.)	(Dollars)	(Bd.ft.)	(Dollars)	(Percent)	(Percent)
FAS	2,705	708.71	3,295	863.29	-22.0	--
SEL	3,217	778.51	2,439	590.24	24.0	--
#1C	8,114	1,338.81	8,344	1,376.76	- 3.0	--
Subtotal	14,036	2,826.03	14,078	2,830.29	- 0.3	- 0.2
#2C	6,322	568.98	5,002	450.18	21.0	--
#3A	3,757	262.99	1,450	101.50	61.0	--
#3B	4,347	282.56	5,926	385.19	-36.0	--
Subtotal	14,426	1,114.53	12,378	936.87	14.0	16.0
Grand total	28,462	3,940.56	26,456	3,767.16	7.0	4.4

1/ Dollar values for tables 1 through 4 were taken from "Hardwood Market Report Weekly News Letter," Jan. 31, 1970, Memphis, Tennessee.

Other National Forest Tests

Total lumber yield and dollar value were underpredicted by 4 and 14 percent, respectively, for the 39 trees from the Monongahela National Forest. Observed and predicted lumber yields within grades, however, differed by 13 to 140 percent. The predicted yield of #1 Common and Better lumber was 14 percent lower than the observed — 4,478 bd. ft. versus 5,231 bd ft. (table 2).

Total lumber yields and dollar values were overestimated for the trees from the White and Green Mountain National Forests by about 24 and 30 percent. Within-grade yield predictions were considerably different from observed values — ranging from 7 to 64 percent (tables 3 and 4).

Table 2.—*Comparison between observed and predicted lumber yields and dollar values for 39 sugar maple trees from the Monongahela National Forest*

Grade	Observed		Predicted		Bd.-ft. difference	Dollar-value difference
	Yield	Value	Yield	Value		
	(Bd.ft.)	(Dollars)	(Bd.ft.)	(Dollars)	(Percent)	(Percent)
FAS	1,143	299.47	706	184.97	38.0	--
SEL	1,491	360.82	831	201.10	44.0	--
#1C	2,597	428.51	2,941	485.27	- 13.0	--
Subtotal	5,231	1,088.80	4,478	871.34	14.0	20.0
#2C	2,392	215.28	2,147	193.23	10.0	--
#3A	1,794	125.58	763	53.41	57.0	--
#3B	1,169	75.99	2,815	182.98	-140.0	--
Subtotal	5,355	416.85	5,725	429.62	- 7.0	- 3.0
Grand total	10,586	1,505.65	10,203	1,300.96	4.0	14.0

Table 3.—*Comparison between observed and predicted lumber yields and dollar values for 45 sugar maple trees from the Green Mountain National Forest*

Grade	Observed		Predicted		Bd.-ft. difference	Dollar-value difference
	Yield	Value	Yield	Value		
	(Bd.ft.)	(Dollars)	(Bd.ft.)	(Dollars)	(Percent)	(Percent)
FAS	679	177.90	476	124.71	30.0	--
SEL	1,977	478.43	703	170.13	64.0	--
#1C	2,732	450.78	2,547	420.26	7.0	--
Subtotal	5,388	1,107.11	3,726	715.10	31.0	35.0
#2C	2,816	253.44	1,867	168.03	34.0	--
#3A	1,335	93.45	693	48.51	48.0	--
#3B	2,054	133.51	2,617	170.11	-27.0	--
Subtotal	6,205	480.40	5,177	386.65	17.0	20.0
Grand total	11,593	1,587.51	8,903	1,101.75	23.0	31.0

Table 4.—*Comparison between observed and predicted lumber yields and dollar values for 49 sugar maple trees from the White Mountain National Forest*

Grade	Observed		Predicted		Bd.-ft. difference	Dollar-value difference
	Yield	Value	Yield	Value		
	(Bd.ft.)	(Dollars)	(Bd.ft.)	(Dollars)	(Percent)	(Percent)
FAS	304	79.65	254	66.55	16.0	--
SEL	1,188	287.50	627	151.73	47.0	--
#1C	3,272	539.88	2,296	378.84	30.0	--
Subtotal	4,764	907.03	3,177	597.12	33.0	34.0
#2C	2,613	235.17	1,859	167.31	29.0	--
#3A	1,483	103.81	711	49.77	52.0	--
#3B	2,202	143.13	2,565	166.73	-16.0	--
Subtotal	6,298	482.11	5,135	383.81	18.0	20.0
Grand total	11,062	1,389.14	8,312	980.93	25.0	29.0

DISCUSSION

The predictions of total lumber yield and dollar value and lumber yields by grade for the 66 trees from the Ottawa National Forest indicate the grading system has potential use. The tree-stem characteristic that apparently caused the prediction inaccuracies for the Monongahela National Forest and particularly the White and Green Mountain National Forests was stem length. The average merchantable stem length for the 90 sample trees was 42 feet. The 66 trees from the Ottawa had an average length of 41 feet, but the Monongahela trees were 36 feet, and the White and Green Mountain trees only about 30 feet. Average d.b.h. for the Ottawa and Monongahela trees was about 3 inches larger than for the sample trees, but the White and Green Mountain trees had average d.b.h. values almost identical to the sample trees. Using prediction equations calculated by d.b.h. classes for [illegible] sample trees (11.0 to 17.9 inches, 18.0 to [illegible] inches, and 11.0 to 26.0 inches) did not improve the accuracy of predicting lumber yields for trees from the four National Forests. Thus, best results will probably be obtained using different coefficients and perhaps different models for different areas. It may also be necessary to include new independent variables that adjust predicted yields for heavy insect damage or other defect factors peculiar to certain areas. Whether coefficients could be applied to large areas such as the Lake States or Northeast will have to be determined. If merchantable stem length is a critical variable, perhaps equations that apply to all areas or large areas such as the Lake States could be calculated by height classes. However, adequate sampling to obtain merchantable stem-length variation for important hardwoods would be necessary.

USE OF RESULTS

The real value of this research is the methodology for estimating lumber grade yields for hardwood timber. However, the prediction equations presented for sugar maple could have some immediate use. Hardwood species similar to sugar maple can possibly be evaluated using the same tree-stem characteristics. Of course, coefficients would have to be calculated for each species. Coefficients for all species should be calculated by area, using actual lumber grade yields from a larger sample size than 90 trees. Our equations are based on estimated yields obtained from published tables, which give average yields and have not been precisely tested for accuracy. In fact, one of the major difficulties in developing new systems for grading trees is that the accuracy of present systems is unknown. Therefore, there are no published results for comparison.

Equations developed using actual yields from a larger sample size could reduce the large differences between observed and predicted lumber grade yields we experienced for the 66 trees from the Ottawa. However, these differences may also have been reduced if we could have tested our equations on more than 66 trees. But even if the differences cannot be significantly reduced, definite over-or-underpredicting trends can possibly be established. Correction factors could then be applied to give accurate estimates of lumber grade yields. With such accurate estimates, dimension yields can then be calculated for standing timber using published dimension yield tables (Dunmire and Englerth 1967, Englerth and Schumann 1969, Schumann and Englerth 1967).

One such table published by Englerth and Schumann (1969) gives dimension yields for #1 Common and Better lumber where the total yield is 25 percent FAS, 25 percent Selects, and 50 percent #1 Common. Therefore, accurate predictions of #1 Common and Better lumber for standing trees can permit good dimension yield estimates. Our equations for sugar maple trees predicted #1 Common and Better lumber for the 66 trees from the Ottawa National Forest almost perfectly (table 1), and underpredicted this grouped lumber yield by only 14 percent for the 39 trees from the Monongahela National Forest (table 2). Thus, the proposed methodology offers great opportunity to develop prediction equations that accurately estimate total lumber yield and dollar value, individual or grouped lumber grade yields, and dimension yields for standing hardwood trees.

In addition, the system is simple to apply. Only a minimum number of tree measurements are required. The most difficult measurements, stem length and several d.o.b. measurements up the stem to determine stem taper and volume, can now be obtained using an optical dendrometer. In addition, a computer program is available for calculating stem volume directly from the dendrometer readings (Grosenbaugh 1963). Bark volume can be estimated (Stayton and Hoffman 1970) to obtain merchantable stem volume exluding bark. The grader will not have to determine which defect indicators are grade defects, only recognize and measure or count them; and photographic defect guides (Lockard *et al.* 1963, Marden and Stayton 1970) are available to help the grader recognize defect types. However, more precise information on the significance of defect types will have to be determined for important hardwood species in addition to sugar maple. The grader will not have to separate trees by grade classes because the proposed system is continuous. And, since the predicted values are in product yields rather than dollars, a change in product value will not require new equations.

This proposed grading system offers another possible significant breakthrough – development of a multiproduct predicting system for standing trees. Much of the credit for this possibility must go to those people who have shown that dimension yields can be accurately predicted from lumber grade yields. Combining the two systems permits estimation of lumber grade yields or dimension yields from standing trees. If additional relationships between lumber grade yields, dimension yields, and other products such as veneer can be established, these products can also be estimated from standing timber. A computer program could then be written that would combine these relationships to predict various product yields, and compare values to provide economic alternative decisions for timber and production managers. However, these alternate decisions would be based on trees yielding only one product. Segregation of tree-stem portions into best end-use classes is extremely difficult and would require additional information.

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EARCH PAPER NC-51
1971

Tables of Compound-Discount Interest Rate Multipliers for Evaluating Forestry Investments

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U.S. DEPARTMENT OF AGRICULTURE

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LIST OF TABLES

TABLES OF COMPOUND-DISCOUNT INTEREST RATE MULTIPLIERS FOR EVALUATING FORESTRY INVESTMENTS

Allen L. Lundgren

The following tables were prepared by computer for 10 selected compound-discount interest rate multipliers commonly used in financial analyses of forestry investments.[1] A few of these tables are readily available in other publications, but some cannot be easily obtained by those evaluating forestry investments. To provide a single source for interest-rate tables with an identical format, even the most common tables are included here. These tables previously were issued by the North Central Station in 1965 and 1967 as a multilithed report.

There are two sets of tables for each of the 10 compound-discount multipliers. The first set gives multipliers for each year from 1 to 40 years; the second set gives multipliers at 5-year intervals from 5 to 160 years. Multipliers are given for 24 interest rates.

.005	.010	.015	.020
.025	.030	.035	.040
.045	.050	.055	.060
.070	.080	.090	.100
.110	.120	.130	.140
.150	.200	.250	.300

Each table is briefly explained and an example of its use is given.

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[1] The computer program used in preparing these tables is a revised version of a program originally written by Dennis L. Schweitzer, formerly Associate Statistician, Lake States Forest Experiment Station. Dr. Schweitzer is now Principal Economist with the Pacific Northwest Forest and Range Experiment Station, USDA, Forest Service, Portland, Oregon. A listing of the revised program is available from the North Central Station.

Note

Multipliers equal to or greater than 1,000,000 exceed the column width available in the tables. These large multipliers are indicated by 999999.99999, and should be ignored.

Table 1.--Compounded Single Payment Multiplier

The value of a \$1 payment compounded for n years.

$$(1+i)^n$$

This multiplier is used to find the future value in n years (V_n) of a present payment or value (V_o), which may be a cost or an income, compounded annually for n years at the interest rate i.

To find the future value, multiply the present value by the multiplier for the rate i and years n desired:

$$V_n = V_o \left[(1+i)^n \right]$$

Example: Find the future value of a present investment of \$10 compounded for 20 years at an interest rate of 4 percent.

Present value: $V_o = \$10$

Interest rate: $i = .04$

Years: $n = 20$

Multiplier: = 2.19112 (page 6)

Future value: $V_n = \$10\ (2.1911) = \$21.91$

Notes

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED FOR N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
1	1.00500	1.01000	1.01500	1.02000
2	1.01002	1.02010	1.03022	1.04040
3	1.01508	1.03030	1.04568	1.06121
4	1.02015	1.04060	1.06136	1.08243
5	1.02525	1.05101	1.07728	1.10408
6	1.03038	1.06152	1.09344	1.12616
7	1.03553	1.07214	1.10984	1.14869
8	1.04071	1.08286	1.12649	1.17166
9	1.04591	1.09369	1.14339	1.19509
10	1.05114	1.10462	1.16054	1.21899
11	1.05640	1.11567	1.17795	1.24337
12	1.06168	1.12683	1.19562	1.26824
13	1.06699	1.13809	1.21355	1.29361
14	1.07232	1.14947	1.23176	1.31948
15	1.07768	1.16097	1.25023	1.34587
16	1.08307	1.17258	1.26899	1.37279
17	1.08849	1.18430	1.28802	1.40024
18	1.09393	1.19615	1.30734	1.42825
19	1.09940	1.20811	1.32695	1.45681
20	1.10490	1.22019	1.34686	1.48595
21	1.11042	1.23239	1.36706	1.51567
22	1.11597	1.24472	1.38756	1.54598
23	1.12155	1.25716	1.40838	1.57690
24	1.12716	1.26973	1.42950	1.60844
25	1.13280	1.28243	1.45095	1.64061
26	1.13846	1.29526	1.47271	1.67342
27	1.14415	1.30821	1.49480	1.70689
28	1.14987	1.32129	1.51722	1.74102
29	1.15562	1.33450	1.53998	1.77584
30	1.16140	1.34785	1.56308	1.81136
31	1.16721	1.36133	1.58653	1.84759
32	1.17304	1.37494	1.61032	1.88454
33	1.17891	1.38869	1.63448	1.92223
34	1.18480	1.40258	1.65900	1.96068
35	1.19073	1.41660	1.68388	1.99989
36	1.19668	1.43077	1.70914	2.03989
37	1.20266	1.44508	1.73478	2.08069
38	1.20868	1.45953	1.76080	2.12230
39	1.21472	1.47412	1.78721	2.16474
40	1.22079	1.48886	1.81402	2.20804

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED
FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
1	1.02500	1.03000	1.03500	1.04000
2	1.05062	1.06090	1.07122	1.08160
3	1.07689	1.09273	1.10872	1.12486
4	1.10381	1.12551	1.14752	1.16986
5	1.13141	1.15927	1.18769	1.21665
6	1.15969	1.19405	1.22926	1.26532
7	1.18869	1.22987	1.27228	1.31593
8	1.21840	1.26677	1.31681	1.36857
9	1.24886	1.30477	1.36290	1.42331
10	1.28008	1.34392	1.41060	1.48024
11	1.31209	1.38423	1.45997	1.53945
12	1.34489	1.42576	1.51107	1.60103
13	1.37851	1.46853	1.56396	1.66507
14	1.41297	1.51259	1.61869	1.73168
15	1.44830	1.55797	1.67535	1.80094
16	1.48451	1.60471	1.73399	1.87298
17	1.52162	1.65285	1.79468	1.94790
18	1.55966	1.70243	1.85749	2.02582
19	1.59865	1.75351	1.92250	2.10685
20	1.63862	1.80611	1.98979	2.19112
21	1.67958	1.86029	2.05943	2.27877
22	1.72157	1.91610	2.13151	2.36992
23	1.76461	1.97359	2.20611	2.46472
24	1.80873	2.03279	2.28333	2.56330
25	1.85394	2.09378	2.36324	2.66584
26	1.90029	2.15659	2.44596	2.77247
27	1.94780	2.22129	2.53157	2.88337
28	1.99650	2.28793	2.62017	2.99870
29	2.04641	2.35657	2.71188	3.11865
30	2.09757	2.42726	2.80679	3.24340
31	2.15001	2.50008	2.90503	3.37313
32	2.20376	2.57508	3.00671	3.50806
33	2.25885	2.65234	3.11194	3.64838
34	2.31532	2.73191	3.22086	3.79432
35	2.37321	2.81386	3.33359	3.94609
36	2.43254	2.89828	3.45027	4.10393
37	2.49335	2.98523	3.57103	4.26809
38	2.55568	3.07478	3.69601	4.43881
39	2.61957	3.16703	3.82537	4.61637
40	2.68506	3.26204	3.95926	4.80102

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED FOR N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
1	1.04500	1.05000	1.05500	1.06000
2	1.09202	1.10250	1.11303	1.12360
3	1.14117	1.15762	1.17424	1.19102
4	1.19252	1.21551	1.23882	1.26248
5	1.24618	1.27628	1.30696	1.33823
6	1.30226	1.34010	1.37884	1.41852
7	1.36086	1.40710	1.45468	1.50363
8	1.42210	1.47746	1.53469	1.59385
9	1.48610	1.55133	1.61909	1.68948
10	1.55297	1.62889	1.70814	1.79085
11	1.62285	1.71034	1.80209	1.89830
12	1.69588	1.79586	1.90121	2.01220
13	1.77220	1.88565	2.00577	2.13293
14	1.85194	1.97993	2.11609	2.26090
15	1.93528	2.07893	2.23248	2.39656
16	2.02237	2.18287	2.35526	2.54035
17	2.11338	2.29202	2.48480	2.69277
18	2.20848	2.40662	2.62147	2.85434
19	2.30786	2.52695	2.76565	3.02560
20	2.41171	2.65330	2.91776	3.20714
21	2.52024	2.78596	3.07823	3.39956
22	2.63365	2.92526	3.24754	3.60354
23	2.75217	3.07152	3.42615	3.81975
24	2.87601	3.22510	3.61459	4.04893
25	3.00543	3.38635	3.81339	4.29187
26	3.14068	3.55567	4.02313	4.54938
27	3.28201	3.73346	4.24440	4.82235
28	3.42970	3.92013	4.47784	5.11169
29	3.58404	4.11614	4.72412	5.41839
30	3.74532	4.32194	4.98395	5.74349
31	3.91386	4.53804	5.25807	6.08810
32	4.08998	4.76494	5.54726	6.45339
33	4.27403	5.00319	5.85236	6.84059
34	4.46636	5.25335	6.17424	7.25103
35	4.66735	5.51602	6.51383	7.68609
36	4.87738	5.79182	6.87209	8.14725
37	5.09686	6.08141	7.25005	8.63609
38	5.32622	6.38548	7.64880	9.15425
39	5.56590	6.70475	8.06949	9.70351
40	5.81636	7.03999	8.51331	10.28572

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED
FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
1	1.07000	1.08000	1.09000	1.10000
2	1.14490	1.16640	1.18810	1.21000
3	1.22504	1.25971	1.29503	1.33100
4	1.31080	1.36049	1.41158	1.46410
5	1.40255	1.46933	1.53862	1.61051
6	1.50073	1.58687	1.67710	1.77156
7	1.60578	1.71382	1.82804	1.94872
8	1.71819	1.85093	1.99256	2.14359
9	1.83846	1.99900	2.17189	2.35795
10	1.96715	2.15892	2.36736	2.59374
11	2.10485	2.33164	2.58043	2.85312
12	2.25219	2.51817	2.81266	3.13843
13	2.40985	2.71962	3.06580	3.45227
14	2.57853	2.93719	3.34173	3.79750
15	2.75903	3.17217	3.64248	4.17725
16	2.95216	3.42594	3.97031	4.59497
17	3.15882	3.70002	4.32763	5.05447
18	3.37993	3.99602	4.71712	5.55992
19	3.61653	4.31570	5.14166	6.11591
20	3.86968	4.66096	5.60441	6.72750
21	4.14056	5.03383	6.10881	7.40025
22	4.43040	5.43654	6.65860	8.14027
23	4.74053	5.87146	7.25787	8.95430
24	5.07237	6.34118	7.91108	9.84973
25	5.42743	6.84848	8.62308	10.83471
26	5.80735	7.39635	9.39916	11.91818
27	6.21387	7.98806	10.24508	13.10999
28	6.64884	8.62711	11.16714	14.42099
29	7.11426	9.31727	12.17218	15.86309
30	7.61226	10.06266	13.26768	17.44940
31	8.14511	10.86767	14.46177	19.19434
32	8.71527	11.73708	15.76333	21.11378
33	9.32534	12.67605	17.18203	23.22515
34	9.97811	13.69013	18.72841	25.54767
35	10.67658	14.78534	20.41397	28.10244
36	11.42394	15.96817	22.25123	30.91268
37	12.22362	17.24563	24.25384	34.00395
38	13.07927	18.62528	26.43668	37.40434
39	13.99482	20.11530	28.81598	41.14478
40	14.97446	21.72452	31.40942	45.25926

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED
FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
1	1.11000	1.12000	1.13000	1.14000
2	1.23210	1.25440	1.27690	1.29960
3	1.36763	1.40493	1.44290	1.48154
4	1.51807	1.57352	1.63047	1.68896
5	1.68506	1.76234	1.84244	1.92541
6	1.87041	1.97382	2.08195	2.19497
7	2.07616	2.21068	2.35261	2.50227
8	2.30454	2.47596	2.65844	2.85259
9	2.55804	2.77308	3.00404	3.25195
10	2.83942	3.10585	3.39457	3.70722
11	3.15176	3.47855	3.83586	4.22623
12	3.49845	3.89598	4.33452	4.81790
13	3.88328	4.36349	4.89801	5.49241
14	4.31044	4.88711	5.53475	6.26135
15	4.78459	5.47357	6.25427	7.13794
16	5.31089	6.13039	7.06733	8.13725
17	5.89509	6.86604	7.98608	9.27646
18	6.54355	7.68997	9.02427	10.57517
19	7.26334	8.61276	10.19742	12.05569
20	8.06231	9.64629	11.52309	13.74349
21	8.94917	10.80385	13.02109	15.66758
22	9.93357	12.10031	14.71383	17.86104
23	11.02627	13.55235	16.62663	20.36158
24	12.23916	15.17863	18.78809	23.21221
25	13.58546	17.00006	21.23054	26.46192
26	15.07986	19.04007	23.99051	30.16658
27	16.73865	21.32488	27.10928	34.38991
28	18.57990	23.88387	30.63349	39.20449
29	20.62369	26.74993	34.61584	44.69312
30	22.89230	29.95992	39.11590	50.95016
31	25.41045	33.55511	44.20096	58.08318
32	28.20560	37.58173	49.94709	66.21483
33	31.30821	42.09153	56.44021	75.48490
34	34.75212	47.14252	63.77744	86.05279
35	38.57485	52.79962	72.06851	98.10018
36	42.81808	59.13557	81.43741	111.83420
37	47.52807	66.23184	92.02428	127.49099
38	52.75616	74.17966	103.98743	145.33973
39	58.55934	83.08122	117.50580	165.68729
40	65.00087	93.05097	132.78155	188.88351

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
1	1.15000	1.20000	1.25000	1.30000
2	1.32250	1.44000	1.56250	1.69000
3	1.52087	1.72800	1.95313	2.19700
4	1.74901	2.07360	2.44141	2.85610
5	2.01136	2.48832	3.05176	3.71293
6	2.31306	2.98598	3.81470	4.82681
7	2.66002	3.58318	4.76837	6.27485
8	3.05902	4.29982	5.96046	8.15731
9	3.51788	5.15978	7.45058	10.60450
10	4.04556	6.19174	9.31323	13.78585
11	4.65239	7.43008	11.64153	17.92160
12	5.35025	8.91610	14.55192	23.29809
13	6.15279	10.69932	18.18989	30.28751
14	7.07571	12.83918	22.73737	39.37376
15	8.13706	15.40702	28.42171	51.18589
16	9.35762	18.48843	35.52714	66.54166
17	10.76126	22.18611	44.40892	86.50416
18	12.37545	26.62333	55.51115	112.45541
19	14.23177	31.94800	69.38894	146.19203
20	16.36654	38.33760	86.73617	190.04964
21	18.82152	46.00512	108.42022	247.06453
22	21.64475	55.20614	135.52527	321.18389
23	24.89146	66.24737	169.40659	417.53905
24	28.62518	79.49685	211.75824	542.80077
25	32.91895	95.39622	264.69780	705.64100
26	37.85680	114.47546	330.87225	917.33330
27	43.53531	137.37055	413.59031	1192.53329
28	50.06561	164.84466	516.98788	1550.29328
29	57.57545	197.81359	646.23485	2015.38126
30	66.21177	237.37631	807.79357	2619.99564
31	76.14354	284.85158	1009.74196	3405.99434
32	87.56507	341.82189	1262.17745	4427.79264
33	100.69983	410.18627	1577.72181	5756.13043
34	115.80480	492.22352	1972.15226	7482.96956
35	133.17552	590.66823	2465.19033	9727.86043
36	153.15185	708.80187	3081.48791	12646.21855
37	176.12463	850.56225	3851.85989	16440.08412
38	202.54332	1020.67470	4814.82486	21372.10935
39	232.92482	1224.80964	6018.53108	27783.74216
40	267.86355	1469.77157	7523.16385	36118.86481

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED FOR N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
5	1.02525	1.05101	1.07728	1.10408
10	1.05114	1.10462	1.16054	1.21899
15	1.07768	1.16097	1.25023	1.34587
20	1.10490	1.22019	1.34686	1.48595
25	1.13280	1.28243	1.45095	1.64061
30	1.16140	1.34785	1.56308	1.81136
35	1.19073	1.41660	1.68388	1.99989
40	1.22079	1.48886	1.81402	2.20804
45	1.25162	1.56481	1.95421	2.43785
50	1.28323	1.64463	2.10524	2.69159
55	1.31563	1.72852	2.26794	2.97173
60	1.34885	1.81670	2.44322	3.28103
65	1.38291	1.90937	2.63204	3.62252
70	1.41783	2.00676	2.83546	3.99956
75	1.45363	2.10913	3.05459	4.41584
80	1.49034	2.21672	3.29066	4.87544
85	1.52797	2.32979	3.54498	5.38288
90	1.56655	2.44863	3.81895	5.94313
95	1.60611	2.57354	4.11409	6.56170
100	1.64667	2.70481	4.43205	7.24465
105	1.68825	2.84279	4.77457	7.99867
110	1.73088	2.98780	5.14357	8.83118
115	1.77459	3.14020	5.54109	9.75034
120	1.81940	3.30039	5.96932	10.76516
125	1.86534	3.46874	6.43066	11.88561
130	1.91244	3.64568	6.92764	13.12267
135	1.96073	3.83165	7.46304	14.48849
140	2.01024	4.02710	8.03981	15.99647
145	2.06100	4.23252	8.66116	17.66139
150	2.11305	4.44842	9.33053	19.49960
155	2.16640	4.67534	10.05163	21.52914
160	2.22111	4.91383	10.82846	23.76991

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
5	1.13141	1.15927	1.18769	1.21665
10	1.28008	1.34392	1.41060	1.48024
15	1.44830	1.55797	1.67535	1.80094
20	1.63862	1.80611	1.98979	2.19112
25	1.85394	2.09378	2.36324	2.66584
30	2.09757	2.42726	2.80679	3.24340
35	2.37321	2.81386	3.33359	3.94609
40	2.68506	3.26204	3.95926	4.80102
45	3.03790	3.78160	4.70236	5.84118
50	3.43711	4.38391	5.58493	7.10668
55	3.88877	5.08215	6.63314	8.64637
60	4.39979	5.89160	7.87809	10.51963
65	4.97796	6.82998	9.35670	12.79874
70	5.63210	7.91782	11.11283	15.57162
75	6.37221	9.17893	13.19855	18.94525
80	7.20957	10.64089	15.67574	23.04980
85	8.15696	12.33571	18.61786	28.04360
90	9.22886	14.30047	22.11218	34.11933
95	10.44160	16.57816	26.26233	41.51139
100	11.81372	19.21863	31.19141	50.50495
105	13.36614	22.27966	37.04561	61.44699
110	15.12256	25.82823	43.99856	74.75966
115	17.10978	29.94200	52.25649	90.95656
120	19.35815	34.71099	62.06432	110.66256
125	21.90197	40.23955	73.71294	134.63793
130	24.78007	46.64866	87.54785	163.80762
135	28.03637	54.07859	103.97938	199.29702
140	31.72058	62.69190	123.49489	242.47530
145	35.88893	72.67710	146.67318	295.00828
150	40.60503	84.25268	174.20173	358.92267
155	45.94086	97.67194	206.89701	436.68431
160	51.97787	113.22855	245.72875	531.29324

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED FOR N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
5	1.24618	1.27628	1.30696	1.33823
10	1.55297	1.62889	1.70814	1.79085
15	1.93528	2.07893	2.23248	2.39656
20	2.41171	2.65330	2.91776	3.20714
25	3.00543	3.38635	3.81339	4.29187
30	3.74532	4.32194	4.98395	5.74349
35	4.66735	5.51602	6.51383	7.68609
40	5.81636	7.03999	8.51331	10.28572
45	7.24825	8.98501	11.12655	13.76461
50	9.03264	11.46740	14.54196	18.42015
55	11.25631	14.63563	19.00576	24.65032
60	14.02741	18.67919	24.83977	32.98769
65	17.48070	23.83990	32.46459	44.14497
70	21.78414	30.42643	42.42992	59.07593
75	27.14700	38.83269	55.45420	79.05692
80	33.83010	49.56144	72.47643	105.79599
85	42.15846	63.25435	94.72379	141.57890
90	52.53711	80.73037	123.80021	189.46451
95	65.47079	103.03468	161.80192	253.54625
100	81.58852	131.50126	211.46864	339.30208
105	101.67414	167.83263	276.38105	454.06273
110	126.70447	214.20169	361.21898	607.63835
115	157.89683	273.38167	472.09876	813.15719
120	196.76817	348.91199	617.01420	1088.18775
125	245.20894	445.30993	806.41288	1456.24068
130	305.57496	568.34086	1053.94938	1948.77852
135	380.80199	725.36296	1377.46969	2607.90526
140	474.54856	925.76737	1800.29779	3489.96553
145	591.37385	1181.53983	2352.91721	4670.36113
150	736.95941	1507.97750	3075.16870	6249.99672
155	918.38550	1924.60387	4019.12250	8363.90548
160	1144.47542	2456.33644	5252.83237	11192.79224

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
5	1.40255	1.46933	1.53862	1.61051
10	1.96715	2.15892	2.36736	2.59374
15	2.75903	3.17217	3.64248	4.17725
20	3.86968	4.66096	5.60441	6.72750
25	5.42743	6.84848	8.62308	10.83471
30	7.61226	10.06266	13.26768	17.44940
35	10.67658	14.78534	20.41397	28.10244
40	14.97446	21.72452	31.40942	45.25926
45	21.00245	31.92045	48.32729	72.89048
50	29.45703	46.90161	74.35752	117.39085
55	41.31500	68.91386	114.40826	189.05914
60	57.94643	101.25706	176.03129	304.48164
65	81.27286	148.77985	270.84596	490.37073
70	113.98939	218.60641	416.73009	789.74696
75	159.87602	321.20453	641.19089	1271.89537
80	224.23439	471.95483	986.55167	2048.40021
85	314.50033	693.45649	1517.93203	3298.96903
90	441.10298	1018.91509	2335.52658	5313.02261
95	618.66975	1497.12055	3593.49715	8556.67605
100	867.71633	2199.76126	5529.04079	13780.61234
105	1217.01703	3232.17098	8507.11461	22193.81398
110	1706.92935	4749.11956	13089.25033	35743.35935
115	2394.05671	6978.01472	20139.43410	57565.03767
120	3357.78838	10252.99294	30987.01575	92709.06882
125	4709.47191	15065.01040	47677.36472	149308.88242
130	6605.27797	22135.44276	73357.53547	240463.44823
135	9264.24406	32524.22754	112869.66134	387268.78801
140	12993.58153	47788.76070	173663.96473	623700.25577
145	18224.17027	70217.36785	267203.53623	999999.99999
150	25560.34155	103172.35007	411125.76168	999999.99999
155	35849.70128	151594.03070	632567.94539	999999.99999
160	50281.06057	222741.36559	973284.19389	999999.99999

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER

THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
5	1.68506	1.76234	1.84244	1.92541
10	2.83942	3.10585	3.39457	3.70722
15	4.78459	5.47357	6.25427	7.13794
20	8.06231	9.64629	11.52309	13.74349
25	13.58546	17.00006	21.23054	26.46192
30	22.89230	29.95992	39.11590	50.95016
35	38.57485	52.79962	72.06851	98.10018
40	65.00087	93.05097	132.78155	188.88351
45	109.53024	163.98760	244.64140	363.67907
50	184.56483	289.00219	450.73593	700.23299
55	311.00247	509.32061	830.45173	1348.23881
60	524.05724	897.59693	1530.05347	2595.91866
65	883.06693	1581.87249	2819.02434	4998.21964
70	1488.01913	2787.79983	5193.86962	9623.64498
75	2507.39877	4913.05584	9569.36811	18529.50639
80	4225.11275	8658.48310	17630.94045	35676.98181
85	7119.56070	15259.20568	32483.86494	68692.98103
90	11996.87381	26891.93422	59849.41552	132262.46738
95	20215.43005	47392.77662	110268.66861	254660.08340
100	34064.17527	83522.26573	203162.87423	490326.23813
105	57400.11633	147194.77037	374314.42661	944081.28902
110	96722.53413	259407.47936	689650.06770	999999.99999
115	162983.09492	457164.61382	999999.99999	999999.99999
120	274635.99325	805680.25501	999999.99999	999999.99999
125	462777.62010	999999.99999	999999.99999	999999.99999
130	779807.20275	999999.99999	999999.99999	999999.99999
135	999999.99999	999999.99999	999999.99999	999999.99999
140	999999.99999	999999.99999	999999.99999	999999.99999
145	999999.99999	999999.99999	999999.99999	999999.99999
150	999999.99999	999999.99999	999999.99999	999999.99999
155	999999.99999	999999.99999	999999.99999	999999.99999
160	999999.99999	999999.99999	999999.99999	999999.99999

TABLE 1. COMPOUNDED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT COMPOUNDED
FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
5	2.01136	2.48832	3.05176	3.71293
10	4.04556	6.19174	9.31323	13.78585
15	8.13706	15.40702	28.42171	51.18589
20	16.36654	38.33760	86.73617	190.04964
25	32.91895	95.39622	264.69780	705.64100
30	66.21177	237.37631	807.79357	2619.99564
35	133.17552	590.66823	2465.19033	9727.86043
40	267.86355	1469.77157	7523.16385	36118.86481
45	538.76927	3657.26199	22958.87404	134106.81671
50	1083.65744	9100.43815	70064.92322	497929.22298
55	2179.62218	22644.80226	213821.17681	999999.99999
60	4383.99875	56347.51435	652530.44680	999999.99999
65	8817.78739	140210.64692	999999.99999	999999.99999
70	17735.72004	348888.95693	999999.99999	999999.99999
75	35672.86798	868147.36931	999999.99999	999999.99999
80	71750.87940	999999.99999	999999.99999	999999.99999
85	144316.64699	999999.99999	999999.99999	999999.99999
90	290272.32521	999999.99999	999999.99999	999999.99999
95	583841.32764	999999.99999	999999.99999	999999.99999
100	999999.99999	999999.99999	999999.99999	999999.99999
105	999999.99999	999999.99999	999999.99999	999999.99999
110	999999.99999	999999.99999	999999.99999	999999.99999
115	999999.99999	999999.99999	999999.99999	999999.99999
120	999999.99999	999999.99999	999999.99999	999999.99999
125	999999.99999	999999.99999	999999.99999	999999.99999
130	999999.99999	999999.99999	999999.99999	999999.99999
135	999999.99999	999999.99999	999999.99999	999999.99999
140	999999.99999	999999.99999	999999.99999	999999.99999
145	999999.99999	999999.99999	999999.99999	999999.99999
150	999999.99999	999999.99999	999999.99999	999999.99999
155	999999.99999	999999.99999	999999.99999	999999.99999
160	999999.99999	999999.99999	999999.99999	999999.99999

U.S. DEPT. AGR., FOREST SERVICE. 1970.

Table 2.--Discounted Single Payment Multiplier

The value of a \$1 payment discounted for n years.

$$\frac{1}{(1+i)^n}$$

This multiplier is used to find the present value now (V_o) of a future payment or value in n years (V_n), which may be a cost or an income, discounted annually for n years at the interest rate i.

To find the present value, multiply the future value by the multiplier for the desired rate i and years n:

$$V_o = V_n \left[\frac{1}{(1+i)^n} \right]$$

Example: Find the present value of a future sale of timber for \$100 to be made 20 years from now, using a discount rate of 4 percent.

Future value: V_n = \$100

Interest rate: i = .04

Years: n = 20

Multiplier: = 0.45639 (page 20)

Present value: V_o = \$100 (0.4564) = \$45.64

Notes

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
1	.99502	.99010	.98522	.98039
2	.99007	.98030	.97066	.96117
3	.98515	.97059	.95632	.94232
4	.98025	.96098	.94218	.92385
5	.97537	.95147	.92826	.90573
6	.97052	.94205	.91454	.88797
7	.96569	.93272	.90103	.87056
8	.96089	.92348	.88771	.85349
9	.95610	.91434	.87459	.83676
10	.95135	.90529	.86167	.82035
11	.94661	.89632	.84893	.80426
12	.94191	.88745	.83639	.78849
13	.93722	.87866	.82403	.77303
14	.93256	.86996	.81185	.75788
15	.92792	.86135	.79985	.74301
16	.92330	.85282	.78803	.72845
17	.91871	.84438	.77639	.71416
18	.91414	.83602	.76491	.70016
19	.90959	.82774	.75361	.68643
20	.90506	.81954	.74247	.67297
21	.90056	.81143	.73150	.65978
22	.89608	.80340	.72069	.64684
23	.89162	.79544	.71004	.63416
24	.88719	.78757	.69954	.62172
25	.88277	.77977	.68921	.60953
26	.87838	.77205	.67902	.59758
27	.87401	.76440	.66899	.58586
28	.86966	.75684	.65910	.57437
29	.86533	.74934	.64936	.56311
30	.86103	.74192	.63976	.55207
31	.85675	.73458	.63031	.54125
32	.85248	.72730	.62099	.53063
33	.84824	.72010	.61182	.52023
34	.84402	.71297	.60277	.51003
35	.83982	.70591	.59387	.50003
36	.83564	.69892	.58509	.49022
37	.83149	.69200	.57644	.48061
38	.82735	.68515	.56792	.47119
39	.82323	.67837	.55953	.46195
40	.81914	.67165	.55126	.45289

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TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
1	.97561	.97087	.96618	.96154
2	.95181	.94260	.93351	.92456
3	.92860	.91514	.90194	.88900
4	.90595	.88849	.87144	.85480
5	.88385	.86261	.84197	.82193
6	.86230	.83748	.81350	.79031
7	.84127	.81309	.78599	.75992
8	.82075	.78941	.75941	.73069
9	.80073	.76642	.73373	.70259
10	.78120	.74409	.70892	.67556
11	.76214	.72242	.68495	.64958
12	.74356	.70138	.66178	.62460
13	.72542	.68095	.63940	.60057
14	.70773	.66112	.61778	.57748
15	.69047	.64186	.59689	.55526
16	.67362	.62317	.57671	.53391
17	.65720	.60502	.55720	.51337
18	.64117	.58739	.53836	.49363
19	.62553	.57029	.52016	.47464
20	.61027	.55368	.50257	.45639
21	.59539	.53755	.48557	.43883
22	.58086	.52189	.46915	.42196
23	.56670	.50669	.45329	.40573
24	.55288	.49193	.43796	.39012
25	.53939	.47761	.42315	.37512
26	.52623	.46369	.40884	.36069
27	.51340	.45019	.39501	.34682
28	.50088	.43708	.38165	.33348
29	.48866	.42435	.36875	.32065
30	.47674	.41199	.35628	.30832
31	.46511	.39999	.34423	.29646
32	.45377	.38834	.33259	.28506
33	.44270	.37703	.32134	.27409
34	.43191	.36604	.31048	.26355
35	.42137	.35538	.29998	.25342
36	.41109	.34503	.28983	.24367
37	.40107	.33498	.28003	.23430
38	.39128	.32523	.27056	.22529
39	.38174	.31575	.26141	.21662
40	.37243	.30656	.25257	.20829

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
1	.95694	.95238	.94787	.94340
2	.91573	.90703	.89845	.89000
3	.87630	.86384	.85161	.83962
4	.83856	.82270	.80722	.79209
5	.80245	.78353	.76513	.74726
6	.76790	.74622	.72525	.70496
7	.73483	.71068	.68744	.66506
8	.70319	.67684	.65160	.62741
9	.67290	.64461	.61763	.59190
10	.64393	.61391	.58543	.55839
11	.61620	.58468	.55491	.52679
12	.58966	.55684	.52598	.49697
13	.56427	.53032	.49856	.46884
14	.53997	.50507	.47257	.44230
15	.51672	.48102	.44793	.41727
16	.49447	.45811	.42458	.39365
17	.47318	.43630	.40245	.37136
18	.45280	.41552	.38147	.35034
19	.43330	.39573	.36158	.33051
20	.41464	.37689	.34273	.31180
21	.39679	.35894	.32486	.29416
22	.37970	.34185	.30793	.27751
23	.36335	.32557	.29187	.26180
24	.34770	.31007	.27666	.24698
25	.33273	.29530	.26223	.23300
26	.31840	.28124	.24856	.21981
27	.30469	.26785	.23560	.20737
28	.29157	.25509	.22332	.19563
29	.27902	.24295	.21168	.18456
30	.26700	.23138	.20064	.17411
31	.25550	.22036	.19018	.16425
32	.24450	.20987	.18027	.15496
33	.23397	.19987	.17087	.14619
34	.22390	.19035	.16196	.13791
35	.21425	.18129	.15352	.13011
36	.20503	.17266	.14552	.12274
37	.19620	.16444	.13793	.11579
38	.18775	.15661	.13074	.10924
39	.17967	.14915	.12392	.10306
40	.17193	.14205	.11746	.09722

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
1	.93458	.92593	.91743	.90909
2	.87344	.85734	.84168	.82645
3	.81630	.79383	.77218	.75131
4	.76290	.73503	.70843	.68301
5	.71299	.68058	.64993	.62092
6	.66634	.63017	.59627	.56447
7	.62275	.58349	.54703	.51316
8	.58201	.54027	.50187	.46651
9	.54393	.50025	.46043	.42410
10	.50835	.46319	.42241	.38554
11	.47509	.42888	.38753	.35049
12	.44401	.39711	.35553	.31863
13	.41496	.36770	.32618	.28966
14	.38782	.34046	.29925	.26333
15	.36245	.31524	.27454	.23939
16	.33873	.29189	.25187	.21763
17	.31657	.27027	.23107	.19784
18	.29586	.25025	.21199	.17986
19	.27651	.23171	.19449	.16351
20	.25842	.21455	.17843	.14864
21	.24151	.19866	.16370	.13513
22	.22571	.18394	.15018	.12285
23	.21095	.17032	.13778	.11168
24	.19715	.15770	.12640	.10153
25	.18425	.14602	.11597	.09230
26	.17220	.13520	.10639	.08391
27	.16093	.12519	.09761	.07628
28	.15040	.11591	.08955	.06934
29	.14056	.10733	.08215	.06304
30	.13137	.09938	.07537	.05731
31	.12277	.09202	.06915	.05210
32	.11474	.08520	.06344	.04736
33	.10723	.07889	.05820	.04306
34	.10022	.07305	.05339	.03914
35	.09366	.06763	.04899	.03558
36	.08754	.06262	.04494	.03235
37	.08181	.05799	.04123	.02941
38	.07646	.05369	.03783	.02673
39	.07146	.04971	.03470	.02430
40	.06678	.04603	.03184	.02209

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
1	.90090	.89286	.88496	.87719
2	.81162	.79719	.78315	.76947
3	.73119	.71178	.69305	.67497
4	.65873	.63552	.61332	.59208
5	.59345	.56743	.54276	.51937
6	.53464	.50663	.48032	.45559
7	.48166	.45235	.42506	.39964
8	.43393	.40388	.37616	.35056
9	.39092	.36061	.33288	.30751
10	.35218	.32197	.29459	.26974
11	.31728	.28748	.26070	.23662
12	.28584	.25668	.23071	.20756
13	.25751	.22917	.20416	.18207
14	.23199	.20462	.18068	.15971
15	.20900	.18270	.15989	.14010
16	.18829	.16312	.14150	.12289
17	.16963	.14564	.12522	.10780
18	.15282	.13004	.11081	.09456
19	.13768	.11611	.09806	.08295
20	.12403	.10367	.08678	.07276
21	.11174	.09256	.07680	.06383
22	.10067	.08264	.06796	.05599
23	.09069	.07379	.06014	.04911
24	.08170	.06588	.05323	.04308
25	.07361	.05882	.04710	.03779
26	.06631	.05252	.04168	.03315
27	.05974	.04689	.03689	.02908
28	.05382	.04187	.03264	.02551
29	.04849	.03738	.02889	.02237
30	.04368	.03338	.02557	.01963
31	.03935	.02980	.02262	.01722
32	.03545	.02661	.02002	.01510
33	.03194	.02376	.01772	.01325
34	.02878	.02121	.01568	.01162
35	.02592	.01894	.01388	.01019
36	.02335	.01691	.01228	.00894
37	.02104	.01510	.01087	.00784
38	.01896	.01348	.00962	.00688
39	.01708	.01204	.00851	.00604
40	.01538	.01075	.00753	.00529

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
1	.86957	.83333	.80000	.76923
2	.75614	.69444	.64000	.59172
3	.65752	.57870	.51200	.45517
4	.57175	.48225	.40960	.35013
5	.49718	.40188	.32768	.26933
6	.43233	.33490	.26214	.20718
7	.37594	.27908	.20972	.15937
8	.32690	.23257	.16777	.12259
9	.28426	.19381	.13422	.09430
10	.24718	.16151	.10737	.07254
11	.21494	.13459	.08590	.05580
12	.18691	.11216	.06872	.04292
13	.16253	.09346	.05498	.03302
14	.14133	.07789	.04398	.02540
15	.12289	.06491	.03518	.01954
16	.10686	.05409	.02815	.01503
17	.09293	.04507	.02252	.01156
18	.08081	.03756	.01801	.00889
19	.07027	.03130	.01441	.00684
20	.06110	.02608	.01153	.00526
21	.05313	.02174	.00922	.00405
22	.04620	.01811	.00738	.00311
23	.04017	.01509	.00590	.00239
24	.03493	.01258	.00472	.00184
25	.03038	.01048	.00378	.00142
26	.02642	.00874	.00302	.00109
27	.02297	.00728	.00242	.00084
28	.01997	.00607	.00193	.00065
29	.01737	.00506	.00155	.00050
30	.01510	.00421	.00124	.00038
31	.01313	.00351	.00099	.00029
32	.01142	.00293	.00079	.00023
33	.00993	.00244	.00063	.00017
34	.00864	.00203	.00051	.00013
35	.00751	.00169	.00041	.00010
36	.00653	.00141	.00032	.00008
37	.00568	.00118	.00026	.00006
38	.00494	.00098	.00021	.00005
39	.00429	.00082	.00017	.00004
40	.00373	.00068	.00013	.00003

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR
N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
5	.97537	.95147	.92826	.90573
10	.95135	.90529	.86167	.82035
15	.92792	.86135	.79985	.74301
20	.90506	.81954	.74247	.67297
25	.88277	.77977	.68921	.60953
30	.86103	.74192	.63976	.55207
35	.83982	.70591	.59387	.50003
40	.81914	.67165	.55126	.45289
45	.79896	.63905	.51171	.41020
50	.77929	.60804	.47500	.37153
55	.76009	.57853	.44093	.33650
60	.74137	.55045	.40930	.30478
65	.72311	.52373	.37993	.27605
70	.70530	.49831	.35268	.25003
75	.68793	.47413	.32738	.22646
80	.67099	.45112	.30389	.20511
85	.65446	.42922	.28209	.18577
90	.63834	.40839	.26185	.16826
95	.62262	.38857	.24307	.15240
100	.60729	.36971	.22563	.13803
105	.59233	.35177	.20944	.12502
110	.57774	.33469	.19442	.11324
115	.56351	.31845	.18047	.10256
120	.54963	.30299	.16752	.09289
125	.53610	.28829	.15551	.08414
130	.52289	.27430	.14435	.07620
135	.51001	.26098	.13399	.06902
140	.49745	.24832	.12438	.06251
145	.48520	.23627	.11546	.05662
150	.47325	.22480	.10718	.05128
155	.46159	.21389	.09949	.04645
160	.45023	.20351	.09235	.04207

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR
N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
5	.88385	.86261	.84197	.82193
10	.78120	.74409	.70892	.67556
15	.69047	.64186	.59689	.55526
20	.61027	.55368	.50257	.45639
25	.53939	.47761	.42315	.37512
30	.47674	.41199	.35628	.30832
35	.42137	.35538	.29998	.25342
40	.37243	.30656	.25257	.20829
45	.32917	.26444	.21266	.17120
50	.29094	.22811	.17905	.14071
55	.25715	.19677	.15076	.11566
60	.22728	.16973	.12693	.09506
65	.20089	.14641	.10688	.07813
70	.17755	.12630	.08999	.06422
75	.15693	.10895	.07577	.05278
80	.13870	.09398	.06379	.04338
85	.12259	.08107	.05371	.03566
90	.10836	.06993	.04522	.02931
95	.09577	.06032	.03808	.02409
100	.08465	.05203	.03206	.01980
105	.07482	.04488	.02699	.01627
110	.06613	.03872	.02273	.01338
115	.05845	.03340	.01914	.01099
120	.05166	.02881	.01611	.00904
125	.04566	.02485	.01357	.00743
130	.04036	.02144	.01142	.00610
135	.03567	.01849	.00962	.00502
140	.03153	.01595	.00810	.00412
145	.02786	.01376	.00682	.00339
150	.02463	.01187	.00574	.00279
155	.02177	.01024	.00483	.00229
160	.01924	.00883	.00407	.00188

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
5	.80245	.78353	.76513	.74726
10	.64393	.61391	.58543	.55839
15	.51672	.48102	.44793	.41727
20	.41464	.37689	.34273	.31180
25	.33273	.29530	.26223	.23300
30	.26700	.23138	.20064	.17411
35	.21425	.18129	.15352	.13011
40	.17193	.14205	.11746	.09722
45	.13796	.11130	.08988	.07265
50	.11071	.08720	.06877	.05429
55	.08884	.06833	.05262	.04057
60	.07129	.05354	.04026	.03031
65	.05721	.04195	.03080	.02265
70	.04590	.03287	.02357	.01693
75	.03684	.02575	.01803	.01265
80	.02956	.02018	.01380	.00945
85	.02372	.01581	.01056	.00706
90	.01903	.01239	.00808	.00528
95	.01527	.00971	.00618	.00394
100	.01226	.00760	.00473	.00295
105	.00984	.00596	.00362	.00220
110	.00789	.00467	.00277	.00165
115	.00633	.00366	.00212	.00123
120	.00508	.00287	.00162	.00092
125	.00408	.00225	.00124	.00069
130	.00327	.00176	.00095	.00051
135	.00263	.00138	.00073	.00038
140	.00211	.00108	.00056	.00029
145	.00169	.00085	.00043	.00021
150	.00136	.00066	.00033	.00016
155	.00109	.00052	.00025	.00012
160	.00087	.00041	.00019	.00009

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
5	.71299	.68058	.64993	.62092
10	.50835	.46319	.42241	.38554
15	.36245	.31524	.27454	.23939
20	.25842	.21455	.17843	.14864
25	.18425	.14602	.11597	.09230
30	.13137	.09938	.07537	.05731
35	.09366	.06763	.04899	.03558
40	.06678	.04603	.03184	.02209
45	.04761	.03133	.02069	.01372
50	.03395	.02132	.01345	.00852
55	.02420	.01451	.00874	.00529
60	.01726	.00988	.00568	.00328
65	.01230	.00672	.00369	.00204
70	.00877	.00457	.00240	.00127
75	.00625	.00311	.00156	.00079
80	.00446	.00212	.00101	.00049
85	.00318	.00144	.00066	.00030
90	.00227	.00098	.00043	.00019
95	.00162	.00067	.00028	.00012
100	.00115	.00045	.00018	.00007
105	.00082	.00031	.00012	.00005
110	.00059	.00021	.00008	.00003
115	.00042	.00014	.00005	.00002
120	.00030	.00010	.00003	.00001
125	.00021	.00007	.00002	.00001
130	.00015	.00005	.00001	.00000
135	.00011	.00003	.00001	.00000
140	.00008	.00002	.00001	.00000
145	.00005	.00001	.00000	.00000
150	.00004	.00001	.00000	.00000
155	.00003	.00001	.00000	.00000
160	.00002	.00000	.00000	.00000

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
5	.59345	.56743	.54276	.51937
10	.35218	.32197	.29459	.26974
15	.20900	.18270	.15989	.14010
20	.12403	.10367	.08678	.07276
25	.07361	.05882	.04710	.03779
30	.04368	.03338	.02557	.01963
35	.02592	.01894	.01388	.01019
40	.01538	.01075	.00753	.00529
45	.00913	.00610	.00409	.00275
50	.00542	.00346	.00222	.00143
55	.00322	.00196	.00120	.00074
60	.00191	.00111	.00065	.00039
65	.00113	.00063	.00035	.00020
70	.00067	.00036	.00019	.00010
75	.00040	.00020	.00010	.00005
80	.00024	.00012	.00006	.00003
85	.00014	.00007	.00003	.00001
90	.00008	.00004	.00002	.00001
95	.00005	.00002	.00001	.00000
100	.00003	.00001	.00000	.00000
105	.00002	.00001	.00000	.00000
110	.00001	.00000	.00000	.00000
115	.00001	.00000	.00000	.00000
120	.00000	.00000	.00000	.00000
125	.00000	.00000	.00000	.00000
130	.00000	.00000	.00000	.00000
135	.00000	.00000	.00000	.00000
140	.00000	.00000	.00000	.00000
145	.00000	.00000	.00000	.00000
150	.00000	.00000	.00000	.00000
155	.00000	.00000	.00000	.00000
160	.00000	.00000	.00000	.00000

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 2. DISCOUNTED SINGLE PAYMENT MULTIPLIER
THE VALUE OF A ONE DOLLAR PAYMENT DISCOUNTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
5	.49718	.40188	.32768	.26933
10	.24718	.16151	.10737	.07254
15	.12289	.06491	.03518	.01954
20	.06110	.02608	.01153	.00526
25	.03038	.01048	.00378	.00142
30	.01510	.00421	.00124	.00038
35	.00751	.00169	.00041	.00010
40	.00373	.00068	.00013	.00003
45	.00186	.00027	.00004	.00001
50	.00092	.00011	.00001	.00000
55	.00046	.00004	.00000	.00000
60	.00023	.00002	.00000	.00000
65	.00011	.00001	.00000	.00000
70	.00006	.00000	.00000	.00000
75	.00003	.00000	.00000	.00000
80	.00001	.00000	.00000	.00000
85	.00001	.00000	.00000	.00000
90	.00000	.00000	.00000	.00000
95	.00000	.00000	.00000	.00000
100	.00000	.00000	.00000	.00000
105	.00000	.00000	.00000	.00000
110	.00000	.00000	.00000	.00000
115	.00000	.00000	.00000	.00000
120	.00000	.00000	.00000	.00000
125	.00000	.00000	.00000	.00000
130	.00000	.00000	.00000	.00000
135	.00000	.00000	.00000	.00000
140	.00000	.00000	.00000	.00000
145	.00000	.00000	.00000	.00000
150	.00000	.00000	.00000	.00000
155	.00000	.00000	.00000	.00000
160	.00000	.00000	.00000	.00000

Table 3.--Compounded Periodic Payment Multiplier
(Also called the Land Expectation Value Multiplier)

The present value of a $1 payment now and every n years thereafter.

$$\frac{(1+i)^n}{(1+i)^n - 1}$$

This multiplier is used to find the value now (V_o) of a periodic payment or value (V), either a cost or an income, made now, n years from now, and every n years thereafter. It is commonly used to determine the value of an unending series of rotations (the land expectation value V_o) when the present net value for one rotation (V) is known.

To find the present value of an unending sequence of payments or values beginning now, multiply the periodic payment or value by the multiplier for the appropriate interest rate i and years n:

$$V_o = V\left[\frac{(1+i)^n}{(1+i)^n - 1}\right]$$

Example: Find the present net value of a timber-growing investment for an unending sequence of rotations (the land expectation value), when the present net value for one 50-year rotation is $20 per acre, with an interest rate of 6 percent.

Periodic value: $V = \$20$ per acre

Interest rate: $i = .06$

Years: $n = 50$

Multiplier: $= 1.05740$ (page 41)

Land expectation value: $V_o = \$20\ (1.0574) = \$21.15$ per acre

Notes

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER. THE PRESENT VALUE OF A ONE DOLLAR PAYMENT NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
1	201.00000	101.00000	67.66667	51.00000
2	100.75062	50.75124	34.08519	25.75248
3	67.33444	34.00221	22.89220	17.33773
4	50.62656	25.62811	17.29632	13.13119
5	40.60199	20.60398	13.93929	10.60792
6	33.91909	17.25484	11.70168	8.92629
7	29.14571	14.86283	10.10374	7.72560
8	25.56577	13.06903	8.90560	6.82549
9	22.78147	11.67404	7.97399	6.12577
10	20.55411	10.55821	7.22895	5.56633
11	18.73181	9.64541	6.61959	5.10890
12	17.21329	8.88488	6.11200	4.72798
13	15.92845	8.24148	5.68269	4.40592
14	14.82722	7.69012	5.31489	4.13010
15	13.87287	7.21238	4.99629	3.89127
16	13.03787	6.79446	4.71767	3.68251
17	12.30116	6.42581	4.47198	3.49849
18	11.64635	6.09820	4.25372	3.33511
19	11.06051	5.80518	4.05856	3.18909
20	10.53329	5.54153	3.88305	3.05784
21	10.05633	5.30308	3.72437	2.93924
22	9.62276	5.08637	3.58022	2.83157
23	9.22693	4.88858	3.44872	2.73340
24	8.86412	4.70735	3.32827	2.64355
25	8.53037	4.54068	3.21756	2.56102
26	8.22233	4.38689	3.11546	2.48496
27	7.93713	4.24455	3.02102	2.41465
28	7.67233	4.11244	2.93341	2.34948
29	7.42583	3.98950	2.85192	2.28892
30	7.19578	3.87481	2.77595	2.23250
31	6.98061	3.76757	2.70495	2.17982
32	6.77891	3.66709	2.63847	2.13053
33	6.58945	3.57274	2.57610	2.08433
34	6.41117	3.48400	2.51746	2.04093
35	6.24310	3.40037	2.46224	2.00011
36	6.08439	3.32143	2.41016	1.96164
37	5.93428	3.24680	2.36096	1.92534
38	5.79209	3.17615	2.31441	1.89103
39	5.65721	3.10916	2.27031	1.85856
40	5.52910	3.04556	2.22847	1.82779

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER.
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT
NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
1	41.00000	34.33333	29.57143	26.00000
2	20.75309	17.42036	15.04001	13.25490
3	14.00549	11.78435	10.19812	9.00871
4	10.63272	8.96757	7.77860	6.88725
5	8.60987	7.27849	6.32804	5.61568
6	7.26200	6.15325	5.36195	4.76905
7	6.29982	5.35021	4.67270	4.16524
8	5.57869	4.74855	4.15648	3.71320
9	5.01828	4.28113	3.75560	3.36232
10	4.57035	3.90768	3.43547	3.08227
11	4.20424	3.60258	3.17406	2.85373
12	3.89949	3.34874	2.95668	2.66380
13	3.64193	3.13432	2.77319	2.50359
14	3.42146	2.95088	2.61631	2.36672
15	3.23066	2.79222	2.48072	2.24853
16	3.06396	2.65369	2.36242	2.14550
17	2.91711	2.53175	2.25838	2.05496
18	2.78680	2.42362	2.16620	1.97483
19	2.67042	2.32713	2.08401	1.90347
20	2.56589	2.24052	2.01032	1.83954
21	2.47149	2.16239	1.94390	1.78200
22	2.38586	2.09158	1.88377	1.72997
23	2.30786	2.02713	1.82911	1.68273
24	2.23651	1.96825	1.77922	1.63967
25	2.17104	1.91426	1.73354	1.60030
26	2.11075	1.86461	1.69158	1.56418
27	2.05507	1.81881	1.65293	1.53096
28	2.00352	1.77644	1.61722	1.50032
29	1.95565	1.73716	1.58415	1.47200
30	1.91111	1.70064	1.55347	1.44575
31	1.86956	1.66663	1.52493	1.42138
32	1.83073	1.63489	1.49833	1.39871
33	1.79438	1.60520	1.47350	1.37759
34	1.76027	1.57740	1.45028	1.35787
35	1.72822	1.55131	1.42852	1.33943
36	1.69806	1.52679	1.40812	1.32217
37	1.66964	1.50372	1.38895	1.30599
38	1.64280	1.48198	1.37092	1.29080
39	1.61745	1.46146	1.35394	1.27652
40	1.59345	1.44208	1.33792	1.26309

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER.
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT
NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
1	23.22222	21.00000	19.18182	17.66667
2	11.86661	10.75610	9.84760	9.09061
3	8.08385	7.34417	6.73916	6.23516
4	6.19430	5.64024	5.18717	4.80986
5	5.06204	4.61950	4.25775	3.95661
6	4.30841	3.94035	3.63962	3.38938
7	3.77114	3.45640	3.19935	2.98558
8	3.36910	3.09444	2.87025	2.68393
9	3.05721	2.81380	2.61526	2.45037
10	2.80842	2.59009	2.41214	2.26447
11	2.60552	2.40778	2.24674	2.11322
12	2.43703	2.25651	2.10962	1.98795
13	2.29501	2.12912	1.99426	1.88267
14	2.17378	2.02048	1.89598	1.79308
15	2.06920	1.92685	1.81137	1.71605
16	1.97812	1.84540	1.73786	1.64920
17	1.89817	1.77398	1.67349	1.59075
18	1.82749	1.71092	1.61673	1.53928
19	1.76461	1.65490	1.56636	1.49368
20	1.70836	1.60485	1.52144	1.45308
21	1.65779	1.55992	1.48118	1.41674
22	1.61213	1.51941	1.44493	1.38409
23	1.57072	1.48274	1.41218	1.35464
24	1.53305	1.44942	1.38247	1.32798
25	1.49865	1.41905	1.35544	1.30378
26	1.46714	1.39129	1.33078	1.28174
27	1.43821	1.36584	1.30822	1.26162
28	1.41157	1.34245	1.28753	1.24321
29	1.38699	1.32091	1.26852	1.22633
30	1.36426	1.30103	1.25101	1.21082
31	1.34319	1.28264	1.23485	1.19654
32	1.32363	1.26561	1.21991	1.18337
33	1.30543	1.24980	1.20609	1.17122
34	1.28849	1.23511	1.19327	1.15997
35	1.27268	1.22143	1.18136	1.14956
36	1.25791	1.20869	1.17030	1.13991
37	1.24409	1.19680	1.16000	1.13096
38	1.23115	1.18568	1.15040	1.12264
39	1.21901	1.17529	1.14145	1.11490
40	1.20763	1.16556	1.13310	1.10769

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER.
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
1	15.28571	13.50000	12.11111	11.00000
2	7.90131	7.00962	6.31632	5.76190
3	5.44360	4.85042	4.38950	4.02115
4	4.21754	3.77401	3.42965	3.15471
5	3.48415	3.13071	2.85658	2.63797
6	2.99708	2.70394	2.47689	2.29607
7	2.65076	2.40091	2.20767	2.05405
8	2.39240	2.17518	2.00749	1.87444
9	2.19266	2.00100	1.85332	1.73641
10	2.03396	1.86287	1.73133	1.62745
11	1.90510	1.75095	1.63274	1.53963
12	1.79860	1.65869	1.55167	1.46763
13	1.70930	1.58152	1.48407	1.40779
14	1.63350	1.51621	1.42704	1.35746
15	1.56849	1.46037	1.37843	1.31474
16	1.51225	1.41221	1.33667	1.27817
17	1.46322	1.37037	1.30051	1.24664
18	1.42018	1.33378	1.26903	1.21930
19	1.38219	1.30160	1.24145	1.19547
20	1.34847	1.27315	1.21718	1.17460
21	1.31841	1.24790	1.19574	1.15624
22	1.29151	1.22540	1.17672	1.14005
23	1.26734	1.20528	1.15980	1.12572
24	1.24556	1.18722	1.14470	1.11300
25	1.22586	1.17098	1.13118	1.10168
26	1.20801	1.15634	1.11906	1.09159
27	1.19180	1.14310	1.10817	1.08258
28	1.17703	1.13111	1.09836	1.07451
29	1.16355	1.12023	1.08951	1.06728
30	1.15123	1.11034	1.08152	1.06079
31	1.13996	1.10134	1.07428	1.05496
32	1.12961	1.09314	1.06774	1.04972
33	1.12012	1.08565	1.06180	1.04499
34	1.11138	1.07880	1.05641	1.04074
35	1.10334	1.07254	1.05151	1.03690
36	1.09593	1.06681	1.04706	1.03343
37	1.08910	1.06156	1.04300	1.03030
38	1.08279	1.05674	1.03931	1.02747
39	1.07695	1.05231	1.03595	1.02491
40	1.07156	1.04825	1.03288	1.02259

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER.
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
1	10.09091	9.33333	8.69231	8.14286
2	5.30849	4.93082	4.61141	4.33778
3	3.72012	3.46957	3.25786	3.07665
4	2.93024	2.74362	2.58611	2.45146
5	2.45973	2.31175	2.18703	2.08060
6	2.14888	2.02688	1.92426	1.83684
7	1.92923	1.82598	1.73931	1.66566
8	1.76656	1.67752	1.60297	1.53979
9	1.64183	1.56399	1.49899	1.44406
10	1.54365	1.47487	1.41761	1.36938
11	1.46474	1.40346	1.35263	1.30996
12	1.40025	1.34531	1.29989	1.26192
13	1.34683	1.29731	1.25654	1.22260
14	1.30207	1.25726	1.22052	1.19007
15	1.26423	1.22354	1.19032	1.16292
16	1.23197	1.19492	1.16482	1.14011
17	1.20429	1.17047	1.14314	1.12082
18	1.18039	1.14948	1.12462	1.10444
19	1.15966	1.13136	1.10873	1.09045
20	1.14160	1.11566	1.09503	1.07847
21	1.12580	1.10200	1.08319	1.06818
22	1.11194	1.09009	1.07292	1.05931
23	1.09974	1.07967	1.06399	1.05165
24	1.08897	1.07053	1.05622	1.04502
25	1.07946	1.06250	1.04943	1.03927
26	1.07102	1.05543	1.04350	1.03429
27	1.06354	1.04920	1.03830	1.02995
28	1.05688	1.04370	1.03375	1.02617
29	1.05096	1.03884	1.02975	1.02289
30	1.04568	1.03453	1.02624	1.02002
31	1.04097	1.03072	1.02315	1.01752
32	1.03676	1.02734	1.02043	1.01533
33	1.03299	1.02434	1.01804	1.01343
34	1.02963	1.02167	1.01593	1.01176
35	1.02661	1.01931	1.01407	1.01030
36	1.02391	1.01720	1.01243	1.00902
37	1.02149	1.01533	1.01099	1.00791
38	1.01932	1.01366	1.00971	1.00693
39	1.01737	1.01218	1.00858	1.00607
40	1.01562	1.01086	1.00759	1.00532

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER.
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT
NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
1	7.66667	6.00000	5.00000	4.33333
2	4.10078	3.27273	2.77778	2.44928
3	2.91985	2.37363	2.04918	1.83542
4	2.33510	1.93145	1.69377	1.53876
5	1.98877	1.67190	1.48739	1.36861
6	1.76158	1.50353	1.35528	1.26131
7	1.60240	1.38712	1.26537	1.18958
8	1.48567	1.30305	1.20159	1.13972
9	1.39716	1.24040	1.15502	1.10412
10	1.32835	1.19261	1.12029	1.07821
11	1.27379	1.15552	1.09397	1.05910
12	1.22987	1.12632	1.07379	1.04485
13	1.19407	1.10310	1.05817	1.03414
14	1.16459	1.08447	1.04600	1.02606
15	1.14011	1.06941	1.03647	1.01993
16	1.11965	1.05718	1.02896	1.01526
17	1.10245	1.04720	1.02304	1.01170
18	1.08791	1.03903	1.01834	1.00897
19	1.07558	1.03231	1.01462	1.00689
20	1.06508	1.02678	1.01166	1.00529
21	1.05611	1.02222	1.00931	1.00406
22	1.04844	1.01845	1.00743	1.00312
23	1.04186	1.01533	1.00594	1.00240
24	1.03620	1.01274	1.00474	1.00185
25	1.03133	1.01059	1.00379	1.00142
26	1.02713	1.00881	1.00303	1.00109
27	1.02351	1.00733	1.00242	1.00084
28	1.02038	1.00610	1.00194	1.00065
29	1.01768	1.00508	1.00155	1.00050
30	1.01533	1.00423	1.00124	1.00038
31	1.01331	1.00352	1.00099	1.00029
32	1.01155	1.00293	1.00079	1.00023
33	1.01003	1.00244	1.00063	1.00017
34	1.00871	1.00204	1.00051	1.00013
35	1.00757	1.00170	1.00041	1.00010
36	1.00657	1.00141	1.00032	1.00008
37	1.00571	1.00118	1.00026	1.00006
38	1.00496	1.00098	1.00021	1.00005
39	1.00431	1.00082	1.00017	1.00004
40	1.00375	1.00068	1.00013	1.00003

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER.
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT
NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
5	40.60199	20.60398	13.93929	10.60792
10	20.55411	10.55821	7.22895	5.56633
15	13.87287	7.21238	4.99629	3.89127
20	10.53329	5.54153	3.88305	3.05784
25	8.53037	4.54068	3.21756	2.56102
30	7.19578	3.87481	2.77595	2.23250
35	6.24310	3.40037	2.46224	2.00011
40	5.52910	3.04556	2.22847	1.82779
45	4.97423	2.77050	2.04798	1.69548
50	4.53075	2.55127	1.90478	1.59116
55	4.16828	2.37264	1.78868	1.50717
60	3.86656	2.22444	1.69290	1.43840
65	3.61158	2.09967	1.61273	1.38131
70	3.39331	1.99328	1.54482	1.33338
75	3.20443	1.90161	1.48671	1.29275
80	3.03941	1.82189	1.43655	1.25804
85	2.89404	1.75200	1.39293	1.22816
90	2.76505	1.69031	1.35474	1.20230
95	2.64986	1.63551	1.32112	1.17980
100	2.54639	1.58657	1.29137	1.16014
105	2.45296	1.54266	1.26493	1.14288
110	2.36821	1.50307	1.24134	1.12769
115	2.29101	1.46724	1.22021	1.11428
120	2.22041	1.43471	1.20123	1.10240
125	2.15562	1.40506	1.18414	1.09186
130	2.09596	1.37797	1.16870	1.08249
135	2.04087	1.35315	1.15473	1.07414
140	1.98986	1.33035	1.14205	1.06668
145	1.94250	1.30936	1.13053	1.06002
150	1.89843	1.28999	1.12004	1.05406
155	1.85734	1.27208	1.11048	1.04871
160	1.81893	1.25550	1.10175	1.04392

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER. THE PRESENT VALUE OF A ONE DOLLAR PAYMENT NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
5	8.60987	7.27849	6.32804	5.61568
10	4.57035	3.90768	3.43547	3.08227
15	3.23066	2.79222	2.48072	2.24853
20	2.56589	2.24052	2.01032	1.83954
25	2.17104	1.91426	1.73354	1.60030
30	1.91111	1.70064	1.55347	1.44575
35	1.72822	1.55131	1.42852	1.33943
40	1.59345	1.44208	1.33792	1.26309
45	1.49070	1.35951	1.27010	1.20656
50	1.41032	1.29552	1.21811	1.16376
55	1.34617	1.24497	1.17752	1.13078
60	1.29414	1.20443	1.14539	1.10505
65	1.25139	1.17153	1.11966	1.08475
70	1.21588	1.14455	1.09888	1.06863
75	1.18614	1.12227	1.08198	1.05573
80	1.16104	1.10372	1.06814	1.04535
85	1.13972	1.08822	1.05676	1.03698
90	1.12152	1.07519	1.04737	1.03019
95	1.10591	1.06419	1.03958	1.02468
100	1.09248	1.05489	1.03312	1.02020
105	1.08087	1.04699	1.02774	1.01654
110	1.07081	1.04028	1.02326	1.01356
115	1.06207	1.03455	1.01951	1.01112
120	1.05447	1.02966	1.01638	1.00912
125	1.04784	1.02548	1.01375	1.00748
130	1.04205	1.02191	1.01155	1.00614
135	1.03699	1.01884	1.00971	1.00504
140	1.03255	1.01621	1.00816	1.00414
145	1.02866	1.01395	1.00686	1.00340
150	1.02525	1.01201	1.00577	1.00279
155	1.02225	1.01034	1.00486	1.00230
160	1.01962	1.00891	1.00409	1.00189

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER.
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
5	5.06204	4.61950	4.25775	3.95661
10	2.80842	2.59009	2.41214	2.26447
15	2.06920	1.92685	1.81137	1.71605
20	1.70836	1.60485	1.52144	1.45308
25	1.49865	1.41905	1.35544	1.30378
30	1.36426	1.30103	1.25101	1.21082
35	1.27268	1.22143	1.18136	1.14956
40	1.20763	1.16556	1.13310	1.10769
45	1.16004	1.12523	1.09875	1.07834
50	1.12449	1.09553	1.07384	1.05740
55	1.09750	1.07334	1.05554	1.04228
60	1.07676	1.05656	1.04195	1.03126
65	1.06068	1.04378	1.03178	1.02318
70	1.04811	1.03398	1.02414	1.01722
75	1.03825	1.02643	1.01836	1.01281
80	1.03046	1.02059	1.01399	1.00954
85	1.02430	1.01606	1.01067	1.00711
90	1.01940	1.01254	1.00814	1.00531
95	1.01551	1.00980	1.00622	1.00396
100	1.01241	1.00766	1.00475	1.00296
105	1.00993	1.00599	1.00363	1.00221
110	1.00796	1.00469	1.00278	1.00165
115	1.00637	1.00367	1.00212	1.00123
120	1.00511	1.00287	1.00162	1.00092
125	1.00409	1.00225	1.00124	1.00069
130	1.00328	1.00176	1.00095	1.00051
135	1.00263	1.00138	1.00073	1.00038
140	1.00211	1.00108	1.00056	1.00029
145	1.00169	1.00085	1.00043	1.00021
150	1.00136	1.00066	1.00033	1.00016
155	1.00109	1.00052	1.00025	1.00012
160	1.00087	1.00041	1.00019	1.00009

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER. THE PRESENT VALUE OF A ONE DOLLAR PAYMENT NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
5	3.48415	3.13071	2.85658	2.63797
10	2.03396	1.86287	1.73133	1.62745
15	1.56849	1.46037	1.37843	1.31474
20	1.34847	1.27315	1.21718	1.17460
25	1.22586	1.17098	1.13118	1.10168
30	1.15123	1.11034	1.08152	1.06079
35	1.10334	1.07254	1.05151	1.03690
40	1.07156	1.04825	1.03288	1.02259
45	1.04999	1.03234	1.02113	1.01391
50	1.03514	1.02179	1.01363	1.00859
55	1.02480	1.01472	1.00882	1.00532
60	1.01756	1.00997	1.00571	1.00330
65	1.01246	1.00677	1.00371	1.00204
70	1.00885	1.00460	1.00241	1.00127
75	1.00629	1.00312	1.00156	1.00079
80	1.00448	1.00212	1.00101	1.00049
85	1.00319	1.00144	1.00066	1.00030
90	1.00227	1.00098	1.00043	1.00019
95	1.00162	1.00067	1.00028	1.00012
100	1.00115	1.00045	1.00018	1.00007
105	1.00082	1.00031	1.00012	1.00005
110	1.00059	1.00021	1.00008	1.00003
115	1.00042	1.00014	1.00005	1.00002
120	1.00030	1.00010	1.00003	1.00001
125	1.00021	1.00007	1.00002	1.00001
130	1.00015	1.00005	1.00001	1.00000
135	1.00011	1.00003	1.00001	1.00000
140	1.00008	1.00002	1.00001	1.00000
145	1.00005	1.00001	1.00000	1.00000
150	1.00004	1.00001	1.00000	1.00000
155	1.00003	1.00001	1.00000	1.00000
160	1.00002	1.00000	1.00000	1.00000

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TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER.
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
5	2.45973	2.31175	2.18703	2.08060
10	1.54365	1.47487	1.41761	1.36938
15	1.26423	1.22354	1.19032	1.16292
20	1.14160	1.11566	1.09503	1.07847
25	1.07946	1.06250	1.04943	1.03927
30	1.04568	1.03453	1.02624	1.02002
35	1.02661	1.01931	1.01407	1.01030
40	1.01562	1.01086	1.00759	1.00532
45	1.00921	1.00614	1.00410	1.00276
50	1.00545	1.00347	1.00222	1.00143
55	1.00323	1.00197	1.00121	1.00074
60	1.00191	1.00112	1.00065	1.00039
65	1.00113	1.00063	1.00035	1.00020
70	1.00067	1.00036	1.00019	1.00010
75	1.00040	1.00020	1.00010	1.00005
80	1.00024	1.00012	1.00006	1.00003
85	1.00014	1.00007	1.00003	1.00001
90	1.00008	1.00004	1.00002	1.00001
95	1.00005	1.00002	1.00001	1.00000
100	1.00003	1.00001	1.00000	1.00000
105	1.00002	1.00001	1.00000	1.00000
110	1.00001	1.00000	1.00000	1.00000
115	1.00001	1.00000	1.00000	1.00000
120	1.00000	1.00000	1.00000	1.00000
125	1.00000	1.00000	1.00000	1.00000
130	1.00000	1.00000	1.00000	1.00000
135	1.00000	1.00000	1.00000	1.00000
140	1.00000	1.00000	1.00000	1.00000
145	1.00000	1.00000	1.00000	1.00000
150	1.00000	1.00000	1.00000	1.00000
155	1.00000	1.00000	1.00000	1.00000
160	1.00000	1.00000	1.00000	1.00000

TABLE 3. COMPOUNDED PERIODIC PAYMENT MULTIPLIER. THE PRESENT VALUE OF A ONE DOLLAR PAYMENT NOW AND EVERY N YEARS THEREAFTER.

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
5	1.98877	1.67190	1.48739	1.36861
10	1.32835	1.19261	1.12029	1.07821
15	1.14011	1.06941	1.03647	1.01993
20	1.06508	1.02678	1.01166	1.00529
25	1.03133	1.01059	1.00379	1.00142
30	1.01533	1.00423	1.00124	1.00038
35	1.00757	1.00170	1.00041	1.00010
40	1.00375	1.00068	1.00013	1.00003
45	1.00186	1.00027	1.00004	1.00001
50	1.00092	1.00011	1.00001	1.00000
55	1.00046	1.00004	1.00000	1.00000
60	1.00023	1.00002	1.00000	1.00000
65	1.00011	1.00001	1.00000	1.00000
70	1.00006	1.00000	1.00000	1.00000
75	1.00003	1.00000	1.00000	1.00000
80	1.00001	1.00000	1.00000	1.00000
85	1.00001	1.00000	1.00000	1.00000
90	1.00000	1.00000	1.00000	1.00000
95	1.00000	1.00000	1.00000	1.00000
100	1.00000	1.00000	1.00000	1.00000
105	1.00000	1.00000	1.00000	1.00000
110	1.00000	1.00000	1.00000	1.00000
115	1.00000	1.00000	1.00000	1.00000
120	1.00000	1.00000	1.00000	1.00000
125	1.00000	1.00000	1.00000	1.00000
130	1.00000	1.00000	1.00000	1.00000
135	1.00000	1.00000	1.00000	1.00000
140	1.00000	1.00000	1.00000	1.00000
145	1.00000	1.00000	1.00000	1.00000
150	1.00000	1.00000	1.00000	1.00000
155	1.00000	1.00000	1.00000	1.00000
160	1.00000	1.00000	1.00000	1.00000

Table 4.--Discounted Periodic Payment Multiplier

The present value of a \$1 payment n years
from now and every n years thereafter.

$$\frac{1}{(1+i)^n - 1}$$

This multiplier is used to find the value now (V_o) of a periodic payment or value (V_n) n years from now and every n years thereafter.

To find the present value of an unending sequence of payments or values beginning n years from now, multiply the periodic payment or value by the multiplier for the appropriate interest rate i and years n:

$$V_o = V_n \left[\frac{1}{(1+i)^n - 1} \right]$$

Example: Find the present value of future incomes from an unending series of timber sales of \$100 per acre at the end of 50-year rotations, the first income to be obtained 50 years from now, with a 5-percent interest rate.

Future income: V_n = \$100 per acre

Interest rate: $i = .05$

Years: $n = 50$

Multiplier: = 0.09553 (page 55)

Present value: V_o = \$100 (0.09553) = \$9.55 per acre

Notes

TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEARS
FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
EARS	.005	.010	.015	.020
1	200.00000	100.00000	66.66667	50.00000
2	99.75062	49.75124	33.08519	24.75248
3	66.33444	33.00221	21.89220	16.33773
4	49.62656	24.62811	16.29632	12.13119
5	39.60199	19.60398	12.93929	9.60792
6	32.91909	16.25484	10.70168	7.92629
7	28.14571	13.86283	9.10374	6.72560
8	24.56577	12.06903	7.90560	5.82549
9	21.78147	10.67404	6.97399	5.12577
10	19.55411	9.55821	6.22895	4.56633
11	17.73181	8.64541	5.61959	4.10890
12	16.21329	7.88488	5.11200	3.72798
13	14.92845	7.24148	4.68269	3.40592
14	13.82722	6.69012	4.31489	3.13010
15	12.87287	6.21238	3.99629	2.89127
16	12.03787	5.79446	3.71767	2.68251
17	11.30116	5.42581	3.47198	2.49849
18	10.64635	5.09820	3.25372	2.33511
19	10.06051	4.80518	3.05856	2.18909
20	9.53329	4.54153	2.88305	2.05784
21	9.05633	4.30308	2.72437	1.93924
22	8.62276	4.08637	2.58022	1.83157
23	8.22693	3.88858	2.44872	1.73340
24	7.86412	3.70735	2.32827	1.64355
25	7.53037	3.54068	2.21756	1.56102
26	7.22233	3.38689	2.11546	1.48496
27	6.93713	3.24455	2.02102	1.41465
28	6.67233	3.11244	1.93341	1.34948
29	6.42583	2.98950	1.85192	1.28892
30	6.19578	2.87481	1.77595	1.23250
31	5.98061	2.76757	1.70495	1.17982
32	5.77891	2.66709	1.63847	1.13053
33	5.58945	2.57274	1.57610	1.08433
34	5.41117	2.48400	1.51746	1.04093
35	5.24310	2.40037	1.46224	1.00011
36	5.08439	2.32143	1.41016	.96164
37	4.93428	2.24680	1.36096	.92534
38	4.79209	2.17615	1.31441	.89103
39	4.65721	2.10916	1.27031	.85856
40	4.52910	2.04556	1.22847	.82779

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TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEAR FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
1	40.00000	33.33333	28.57143	25.00000
2	19.75309	16.42036	14.04001	12.25490
3	13.00549	10.78435	9.19812	8.00871
4	9.63272	7.96757	6.77860	5.88725
5	7.60987	6.27849	5.32804	4.61566
6	6.26200	5.15325	4.36195	3.76905
7	5.29982	4.35021	3.67270	3.16524
8	4.57869	3.74855	3.15648	2.71320
9	4.01828	3.28113	2.75560	2.36232
10	3.57035	2.90768	2.43547	2.08227
11	3.20424	2.60258	2.17406	1.85373
12	2.89949	2.34874	1.95668	1.66380
13	2.64193	2.13432	1.77319	1.50359
14	2.42146	1.95088	1.61631	1.36672
15	2.23066	1.79222	1.48072	1.24853
16	2.06396	1.65369	1.36242	1.14550
17	1.91711	1.53175	1.25838	1.05496
18	1.78680	1.42362	1.16620	.97483
19	1.67042	1.32713	1.08401	.90347
20	1.56589	1.24052	1.01032	.83954
21	1.47149	1.16239	.94390	.78200
22	1.38586	1.09158	.88377	.72997
23	1.30786	1.02713	.82911	.68273
24	1.23651	.96825	.77922	.63967
25	1.17104	.91426	.73354	.60030
26	1.11075	.86461	.69158	.56416
27	1.05507	.81881	.65293	.53096
28	1.00352	.77644	.61722	.50032
29	.95565	.73716	.58415	.47200
30	.91111	.70064	.55347	.44575
31	.86956	.66663	.52493	.42138
32	.83073	.63489	.49833	.39871
33	.79438	.60520	.47350	.37759
34	.76027	.57740	.45028	.35787
35	.72822	.55131	.42852	.33943
36	.69806	.52679	.40812	.32217
37	.66964	.50372	.38895	.30599
38	.64280	.48198	.37092	.29080
39	.61745	.46146	.35394	.27652
40	.59345	.44208	.33792	.26309

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TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEARS FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
EARS	.045	.050	.055	.060
1	22.22222	20.00000	18.18182	16.66667
2	10.86661	9.75610	8.84760	8.09061
3	7.08385	6.34417	5.73916	5.23516
4	5.19430	4.64024	4.18717	3.80986
5	4.06204	3.61950	3.25775	2.95661
6	3.30841	2.94035	2.63962	2.38938
7	2.77114	2.45640	2.19935	1.98558
8	2.36910	2.09444	1.87025	1.68393
9	2.05721	1.81380	1.61526	1.45037
10	1.80842	1.59009	1.41214	1.26447
11	1.60552	1.40778	1.24674	1.11322
12	1.43703	1.25651	1.10962	.98795
13	1.29501	1.12912	.99426	.88267
14	1.17378	1.02048	.89598	.79308
15	1.06920	.92685	.81137	.71605
16	.97812	.84540	.73786	.64920
17	.89817	.77398	.67349	.59075
18	.82749	.71092	.61673	.53928
19	.76461	.65490	.56636	.49368
20	.70836	.60485	.52144	.45308
21	.65779	.55992	.48118	.41674
22	.61213	.51941	.44493	.38409
23	.57072	.48274	.41218	.35464
24	.53305	.44942	.38247	.32798
25	.49865	.41905	.35544	.30378
26	.46714	.39129	.33078	.28174
27	.43821	.36584	.30822	.26162
28	.41157	.34245	.28753	.24321
29	.38699	.32091	.26852	.22633
30	.36426	.30103	.25101	.21082
31	.34319	.28264	.23485	.19654
32	.32363	.26561	.21991	.18337
33	.30543	.24980	.20609	.17122
34	.28849	.23511	.19327	.15997
35	.27268	.22143	.18136	.14956
36	.25791	.20869	.17030	.13991
37	.24409	.19680	.16000	.13096
38	.23115	.18568	.15040	.12264
39	.21901	.17529	.14145	.11490
40	.20763	.16556	.13310	.10769

TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEAR
FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
1	14.28571	12.50000	11.11111	10.00000
2	6.90131	6.00962	5.31632	4.76190
3	4.44360	3.85042	3.38950	3.02115
4	3.21754	2.77401	2.42965	2.15471
5	2.48415	2.13071	1.85658	1.63797
6	1.99708	1.70394	1.47689	1.29607
7	1.65076	1.40091	1.20767	1.05405
8	1.39240	1.17518	1.00749	.87444
9	1.19266	1.00100	.85332	.73641
10	1.03396	.86287	.73133	.62745
11	.90510	.75095	.63274	.53963
12	.79860	.65869	.55167	.46763
13	.70930	.58152	.48407	.40779
14	.63350	.51621	.42704	.35746
15	.56849	.46037	.37843	.31474
16	.51225	.41221	.33667	.27817
17	.46322	.37037	.30051	.24664
18	.42018	.33378	.26903	.21930
19	.38219	.30160	.24145	.19547
20	.34847	.27315	.21718	.17460
21	.31841	.24790	.19574	.15624
22	.29151	.22540	.17672	.14005
23	.26734	.20528	.15980	.12572
24	.24556	.18722	.14470	.11300
25	.22586	.17098	.13118	.10168
26	.20801	.15634	.11906	.09159
27	.19180	.14310	.10817	.08258
28	.17703	.13111	.09836	.07451
29	.16355	.12023	.08951	.06728
30	.15123	.11034	.08152	.06079
31	.13996	.10134	.07428	.05496
32	.12961	.09314	.06774	.04972
33	.12012	.08565	.06180	.04499
34	.11138	.07880	.05641	.04074
35	.10334	.07254	.05151	.03690
36	.09593	.06681	.04706	.03343
37	.08910	.06156	.04300	.03030
38	.08279	.05674	.03931	.02747
39	.07695	.05231	.03595	.02491
40	.07156	.04825	.03288	.02259

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TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEARS
FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
EARS	.110	.120	.130	.140
1	9.09091	8.33333	7.69231	7.14286
2	4.30849	3.93082	3.61141	3.33778
3	2.72012	2.46957	2.25786	2.07665
4	1.93024	1.74362	1.58611	1.45146
5	1.45973	1.31175	1.18703	1.08060
6	1.14888	1.02688	.92426	.83684
7	.92923	.82598	.73931	.66566
8	.76656	.67752	.60297	.53979
9	.64183	.56399	.49899	.44406
10	.54365	.47487	.41761	.36938
11	.46474	.40346	.35263	.30996
12	.40025	.34531	.29989	.26192
13	.34683	.29731	.25654	.22260
14	.30207	.25726	.22052	.19007
15	.26423	.22354	.19032	.16292
16	.23197	.19492	.16482	.14011
17	.20429	.17047	.14314	.12082
18	.18039	.14948	.12462	.10444
19	.15966	.13136	.10873	.09045
20	.14160	.11566	.09503	.07847
21	.12580	.10200	.08319	.06818
22	.11194	.09009	.07292	.05931
23	.09974	.07967	.06399	.05165
24	.08897	.07053	.05622	.04502
25	.07946	.06250	.04943	.03927
26	.07102	.05543	.04350	.03429
27	.06354	.04920	.03830	.02995
28	.05688	.04370	.03375	.02617
29	.05096	.03884	.02975	.02289
30	.04568	.03453	.02624	.02002
31	.04097	.03072	.02315	.01752
32	.03676	.02734	.02043	.01533
33	.03299	.02434	.01804	.01343
34	.02963	.02167	.01593	.01176
35	.02661	.01931	.01407	.01030
36	.02391	.01720	.01243	.00902
37	.02149	.01533	.01099	.00791
38	.01932	.01366	.00971	.00693
39	.01737	.01218	.00858	.00607
40	.01562	.01086	.00759	.00532

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TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEARS
FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
1	6.66667	5.00000	4.00000	3.33333
2	3.10078	2.27273	1.77778	1.44928
3	1.91985	1.37363	1.04918	.83542
4	1.33510	.93145	.69377	.53876
5	.98877	.67190	.48739	.36861
6	.76158	.50353	.35528	.26131
7	.60240	.38712	.26537	.18958
8	.48567	.30305	.20159	.13972
9	.39716	.24040	.15502	.10412
10	.32835	.19261	.12029	.07821
11	.27379	.15552	.09397	.05910
12	.22987	.12632	.07379	.04485
13	.19407	.10310	.05817	.03414
14	.16459	.08447	.04600	.02606
15	.14011	.06941	.03647	.01993
16	.11965	.05718	.02896	.01526
17	.10245	.04720	.02304	.01170
18	.08791	.03903	.01834	.00897
19	.07558	.03231	.01462	.00689
20	.06508	.02678	.01166	.00529
21	.05611	.02222	.00931	.00406
22	.04844	.01845	.00743	.00312
23	.04186	.01533	.00594	.00240
24	.03620	.01274	.00474	.00185
25	.03133	.01059	.00379	.00142
26	.02713	.00881	.00303	.00109
27	.02351	.00733	.00242	.00084
28	.02038	.00610	.00194	.00065
29	.01768	.00508	.00155	.00050
30	.01533	.00423	.00124	.00038
31	.01331	.00352	.00099	.00029
32	.01155	.00293	.00079	.00023
33	.01003	.00244	.00063	.00017
34	.00871	.00204	.00051	.00013
35	.00757	.00170	.00041	.00010
36	.00657	.00141	.00032	.00008
37	.00571	.00118	.00026	.00006
38	.00496	.00098	.00021	.00005
39	.00431	.00082	.00017	.00004
40	.00375	.00068	.00013	.00003

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TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEARS FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
ARS	.005	.010	.015	.020
5	39.60199	19.60398	12.93929	9.60792
10	19.55411	9.55821	6.22895	4.56633
15	12.87287	6.21238	3.99629	2.89127
20	9.53329	4.54153	2.88305	2.05784
25	7.53037	3.54068	2.21756	1.56102
30	6.19578	2.87481	1.77595	1.23250
35	5.24310	2.40037	1.46224	1.00011
40	4.52910	2.04556	1.22847	.82779
45	3.97423	1.77050	1.04798	.69548
50	3.53075	1.55127	.90478	.59116
55	3.16828	1.37264	.78868	.50717
60	2.86656	1.22444	.69290	.43840
65	2.61158	1.09967	.61273	.38131
70	2.39331	.99328	.54482	.33338
75	2.20443	.90161	.48671	.29275
80	2.03941	.82189	.43655	.25804
85	1.89404	.75200	.39293	.22816
90	1.76505	.69031	.35474	.20230
95	1.64986	.63551	.32112	.17980
00	1.54639	.58657	.29137	.16014
05	1.45296	.54266	.26493	.14288
10	1.36821	.50307	.24134	.12769
15	1.29101	.46724	.22021	.11428
20	1.22041	.43471	.20123	.10240
25	1.15562	.40506	.18414	.09186
30	1.09596	.37797	.16870	.08249
35	1.04087	.35315	.15473	.07414
40	.98986	.33035	.14205	.06668
45	.94250	.30936	.13053	.06002
50	.89843	.28999	.12004	.05406
55	.85734	.27208	.11048	.04871
60	.81893	.25550	.10175	.04392

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TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEARS FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
5	7.60987	6.27849	5.32804	4.61568
10	3.57035	2.90768	2.43547	2.08227
15	2.23066	1.79222	1.48072	1.24853
20	1.56589	1.24052	1.01032	.83954
25	1.17104	.91426	.73354	.60030
30	.91111	.70064	.55347	.44575
35	.72822	.55131	.42852	.33943
40	.59345	.44208	.33792	.26309
45	.49070	.35951	.27010	.20656
50	.41032	.29552	.21811	.16376
55	.34617	.24497	.17752	.13078
60	.29414	.20443	.14539	.10505
65	.25139	.17153	.11966	.08475
70	.21588	.14455	.09888	.06863
75	.18614	.12227	.08198	.05573
80	.16104	.10372	.06814	.04535
85	.13972	.08822	.05676	.03698
90	.12152	.07519	.04737	.03019
95	.10591	.06419	.03958	.02468
100	.09248	.05489	.03312	.02020
105	.08087	.04699	.02774	.01654
110	.07081	.04028	.02326	.01356
115	.06207	.03455	.01951	.01112
120	.05447	.02966	.01638	.00912
125	.04784	.02548	.01375	.00748
130	.04205	.02191	.01155	.00614
135	.03699	.01884	.00971	.00504
140	.03255	.01621	.00816	.00414
145	.02866	.01395	.00686	.00340
150	.02525	.01201	.00577	.00279
155	.02225	.01034	.00486	.00230
160	.01962	.00891	.00409	.00189

TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEARS
FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
5	4.06204	3.61950	3.25775	2.95661
10	1.80842	1.59009	1.41214	1.26447
15	1.06920	.92685	.81137	.71605
20	.70836	.60485	.52144	.45308
25	.49865	.41905	.35544	.30378
30	.36426	.30103	.25101	.21082
35	.27268	.22143	.18136	.14956
40	.20763	.16556	.13310	.10769
45	.16004	.12523	.09875	.07834
50	.12449	.09553	.07384	.05740
55	.09750	.07334	.05554	.04228
60	.07676	.05656	.04195	.03126
65	.06068	.04378	.03178	.02318
70	.04811	.03398	.02414	.01722
75	.03825	.02643	.01836	.01281
80	.03046	.02059	.01399	.00954
85	.02430	.01606	.01067	.00711
90	.01940	.01254	.00814	.00531
95	.01551	.00980	.00622	.00396
100	.01241	.00766	.00475	.00296
105	.00993	.00599	.00363	.00221
110	.00796	.00469	.00278	.00165
115	.00637	.00367	.00212	.00123
120	.00511	.00287	.00162	.00092
125	.00409	.00225	.00124	.00069
130	.00328	.00176	.00095	.00051
135	.00263	.00138	.00073	.00038
140	.00211	.00108	.00056	.00029
145	.00169	.00085	.00043	.00021
150	.00136	.00066	.00033	.00016
155	.00109	.00052	.00025	.00012
160	.00087	.00041	.00019	.00009

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TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEARS FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
5	2.48415	2.13071	1.85658	1.63797
10	1.03396	.86287	.73133	.62745
15	.56849	.46037	.37843	.31474
20	.34847	.27315	.21718	.17460
25	.22586	.17098	.13118	.10168
30	.15123	.11034	.08152	.06079
35	.10334	.07254	.05151	.03690
40	.07156	.04825	.03288	.02259
45	.04999	.03234	.02113	.01391
50	.03514	.02179	.01363	.00859
55	.02480	.01472	.00882	.00532
60	.01756	.00997	.00571	.00330
65	.01246	.00677	.00371	.00204
70	.00885	.00460	.00241	.00127
75	.00629	.00312	.00156	.00079
80	.00448	.00212	.00101	.00049
85	.00319	.00144	.00066	.00030
90	.00227	.00098	.00043	.00019
95	.00162	.00067	.00028	.00012
100	.00115	.00045	.00018	.00007
105	.00082	.00031	.00012	.00005
110	.00059	.00021	.00008	.00003
115	.00042	.00014	.00005	.00002
120	.00030	.00010	.00003	.00001
125	.00021	.00007	.00002	.00001
130	.00015	.00005	.00001	.00000
135	.00011	.00003	.00001	.00000
140	.00008	.00002	.00001	.00000
145	.00005	.00001	.00000	.00000
150	.00004	.00001	.00000	.00000
155	.00003	.00001	.00000	.00000
160	.00002	.00000	.00000	.00000

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TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEARS FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
5	1.45973	1.31175	1.18703	1.08060
10	.54365	.47487	.41761	.36938
15	.26423	.22354	.19032	.16292
20	.14160	.11566	.09503	.07847
25	.07946	.06250	.04943	.03927
30	.04568	.03453	.02624	.02002
35	.02661	.01931	.01407	.01030
40	.01562	.01086	.00759	.00532
45	.00921	.00614	.00410	.00276
50	.00545	.00347	.00222	.00143
55	.00323	.00197	.00121	.00074
60	.00191	.00112	.00065	.00039
65	.00113	.00063	.00035	.00020
70	.00067	.00036	.00019	.00010
75	.00040	.00020	.00010	.00005
80	.00024	.00012	.00006	.00003
85	.00014	.00007	.00003	.00001
90	.00008	.00004	.00002	.00001
95	.00005	.00002	.00001	.00000
100	.00003	.00001	.00000	.00000
105	.00002	.00001	.00000	.00000
110	.00001	.00000	.00000	.00000
115	.00001	.00000	.00000	.00000
120	.00000	.00000	.00000	.00000
125	.00000	.00000	.00000	.00000
130	.00000	.00000	.00000	.00000
135	.00000	.00000	.00000	.00000
140	.00000	.00000	.00000	.00000
145	.00000	.00000	.00000	.00000
150	.00000	.00000	.00000	.00000
155	.00000	.00000	.00000	.00000
160	.00000	.00000	.00000	.00000

TABLE 4. DISCOUNTED PERIODIC PAYMENT MULTIPLIER
THE PRESENT VALUE OF A ONE DOLLAR PAYMENT N YEARS FROM NOW AND EVERY N YEARS THEREAFTER

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
5	.98877	.67190	.48739	.36861
10	.32835	.19261	.12029	.07821
15	.14011	.06941	.03647	.01993
20	.06508	.02678	.01166	.00529
25	.03133	.01059	.00379	.00142
30	.01533	.00423	.00124	.00038
35	.00757	.00170	.00041	.00010
40	.00375	.00068	.00013	.00003
45	.00186	.00027	.00004	.00001
50	.00092	.00011	.00001	.00000
55	.00046	.00004	.00000	.00000
60	.00023	.00002	.00000	.00000
65	.00011	.00001	.00000	.00000
70	.00006	.00000	.00000	.00000
75	.00003	.00000	.00000	.00000
80	.00001	.00000	.00000	.00000
85	.00001	.00000	.00000	.00000
90	.00000	.00000	.00000	.00000
95	.00000	.00000	.00000	.00000
100	.00000	.00000	.00000	.00000
105	.00000	.00000	.00000	.00000
110	.00000	.00000	.00000	.00000
115	.00000	.00000	.00000	.00000
120	.00000	.00000	.00000	.00000
125	.00000	.00000	.00000	.00000
130	.00000	.00000	.00000	.00000
135	.00000	.00000	.00000	.00000
140	.00000	.00000	.00000	.00000
145	.00000	.00000	.00000	.00000
150	.00000	.00000	.00000	.00000
155	.00000	.00000	.00000	.00000
160	.00000	.00000	.00000	.00000

Table 5.--Compounded Annual Payment Multiplier

The future value in n years of an annual payment of $1 for n years.

$$\frac{(1+i)^n - 1}{i}$$

This multiplier is used to find the future value (V_n) in n years of an annual payment (a) for n years.

To find the future value, multiply the annual payment by the multiplier for the desired interest rate i and years n:

$$V_n = a\left[\frac{(1+i)^n - 1}{i}\right]$$

Example: Determine the accumulated per-acre cost at the end of a 60-year rotation of an annual payment of $0.50 per acre for property taxes and administrative expenses, with an interest rate of 5 percent.

Annual cost:	$a = \$.50$ per acre
Interest rate:	$i = .05$
Years:	$n = 60$
Multiplier:	= 353.58372 (page 69)
Future value:	$V_n = \$.50\ (353.58) = \$176.79$ per acre

Notes

TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT
OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
1	1.00000	1.00000	1.00000	1.00000
2	2.00500	2.01000	2.01500	2.02000
3	3.01502	3.03010	3.04522	3.06040
4	4.03010	4.06040	4.09090	4.12161
5	5.05025	5.10101	5.15227	5.20404
6	6.07550	6.15202	6.22955	6.30812
7	7.10588	7.21354	7.32299	7.43428
8	8.14141	8.28567	8.43284	8.58297
9	9.18212	9.36853	9.55933	9.75463
10	10.22803	10.46221	10.70272	10.94972
11	11.27917	11.56683	11.86326	12.16872
12	12.33556	12.68250	13.04121	13.41209
13	13.39724	13.80933	14.23683	14.68033
14	14.46423	14.94742	15.45038	15.97394
15	15.53655	16.09690	16.68214	17.29342
16	16.61423	17.25786	17.93237	18.63929
17	17.69730	18.43044	19.20136	20.01207
18	18.78579	19.61475	20.48938	21.41231
19	19.87972	20.81090	21.79672	22.84056
20	20.97912	22.01900	23.12367	24.29737
21	22.08401	23.23919	24.47052	25.78332
22	23.19443	24.47159	25.83758	27.29898
23	24.31040	25.71630	27.22514	28.84496
24	25.43196	26.97346	28.63352	30.42186
25	26.55912	28.24320	30.06302	32.03030
26	27.69191	29.52563	31.51397	33.67091
27	28.83037	30.82089	32.98668	35.34432
28	29.97452	32.12910	34.48148	37.05121
29	31.12439	33.45039	35.99870	38.79223
30	32.28002	34.78489	37.53868	40.56808
31	33.44142	36.13274	39.10176	42.37944
32	34.60862	37.49407	40.68829	44.22703
33	35.78167	38.86901	42.29861	46.11157
34	36.96058	40.25770	43.93309	48.03380
35	38.14538	41.66028	45.59209	49.99448
36	39.33610	43.07688	47.27597	51.99437
37	40.53279	44.50765	48.98511	54.03425
38	41.73545	45.95272	50.71989	56.11494
39	42.94413	47.41225	52.48068	58.23724
40	44.15885	48.88637	54.26789	60.40198

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TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
1	1.00000	1.00000	1.00000	1.00000
2	2.02500	2.03000	2.03500	2.04000
3	3.07562	3.09090	3.10622	3.12160
4	4.15252	4.18363	4.21494	4.24646
5	5.25633	5.30914	5.36247	5.41632
6	6.38774	6.46841	6.55015	6.63298
7	7.54743	7.66246	7.77941	7.89829
8	8.73612	8.89234	9.05169	9.21423
9	9.95452	10.15911	10.36850	10.58280
10	11.20338	11.46388	11.73139	12.00611
11	12.48347	12.80780	13.14199	13.48635
12	13.79555	14.19203	14.60196	15.02581
13	15.14044	15.61779	16.11303	16.62684
14	16.51895	17.08632	17.67699	18.29191
15	17.93193	18.59891	19.29568	20.02359
16	19.38022	20.15688	20.97103	21.82453
17	20.86473	21.76159	22.70502	23.69751
18	22.38635	23.41444	24.49969	25.64541
19	23.94601	25.11687	26.35718	27.67123
20	25.54466	26.87037	28.27968	29.77808
21	27.18327	28.67649	30.26947	31.96920
22	28.86286	30.53678	32.32890	34.24797
23	30.58443	32.45288	34.46041	36.61789
24	32.34904	34.42647	36.66653	39.08260
25	34.15776	36.45926	38.94986	41.64591
26	36.01171	38.55304	41.31310	44.31174
27	37.91200	40.70963	43.75906	47.08421
28	39.85980	42.93092	46.29063	49.96758
29	41.85630	45.21885	48.91080	52.96629
30	43.90270	47.57542	51.62268	56.08494
31	46.00027	50.00268	54.42947	59.32834
32	48.15028	52.50276	57.33450	62.70147
33	50.35403	55.07784	60.34121	66.20953
34	52.61289	57.73018	63.45315	69.85791
35	54.92821	60.46208	66.67401	73.65222
36	57.30141	63.27594	70.00760	77.59831
37	59.73395	66.17422	73.45787	81.70225
38	62.22730	69.15945	77.02889	85.97034
39	64.78298	72.23423	80.72491	90.40915
40	67.40255	75.40126	84.55028	95.02552

TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
EARS	.045	.050	.055	.060
1	1.00000	1.00000	1.00000	1.00000
2	2.04500	2.05000	2.05500	2.06000
3	3.13702	3.15250	3.16803	3.18360
4	4.27819	4.31012	4.34227	4.37462
5	5.47071	5.52563	5.58109	5.63709
6	6.71689	6.80191	6.88805	6.97532
7	8.01915	8.14201	8.26689	8.39384
8	9.38001	9.54911	9.72157	9.89747
9	10.80211	11.02656	11.25626	11.49132
10	12.28821	12.57789	12.87535	13.18079
11	13.84118	14.20679	14.58350	14.97164
12	15.46403	15.91713	16.38559	16.86994
13	17.15991	17.71298	18.28680	18.88214
14	18.93211	19.59863	20.29257	21.01507
15	20.78405	21.57856	22.40866	23.27597
16	22.71934	23.65749	24.64114	25.67253
17	24.74171	25.84037	26.99640	28.21288
18	26.85508	28.13238	29.48120	30.90565
19	29.06356	30.53900	32.10267	33.75999
20	31.37142	33.06595	34.86832	36.78559
21	33.78314	35.71925	37.78608	39.99273
22	36.30338	38.50521	40.86431	43.39229
23	38.93703	41.43048	44.11185	46.99583
24	41.68920	44.50200	47.53800	50.81558
25	44.56521	47.72710	51.15259	54.86451
26	47.57064	51.11345	54.96598	59.15638
27	50.71132	54.66913	58.98911	63.70577
28	53.99333	58.40258	63.23351	68.52811
29	57.42303	62.32271	67.71135	73.63980
30	61.00707	66.43885	72.43548	79.05819
31	64.75239	70.76079	77.41943	84.80168
32	68.66625	75.29883	82.67750	90.88978
33	72.75623	80.06377	88.22476	97.34316
34	77.03026	85.06696	94.07712	104.18375
35	81.49662	90.32031	100.25136	111.43478
36	86.16397	95.83632	106.76519	119.12087
37	91.04134	101.62814	113.63727	127.26812
38	96.13820	107.70955	120.88732	135.90421
39	101.46442	114.09502	128.53613	145.05846
40	107.03032	120.79977	136.60561	154.76197

TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT
OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
1	1.00000	1.00000	1.00000	1.00000
2	2.07000	2.08000	2.09000	2.10000
3	3.21490	3.24640	3.27810	3.31000
4	4.43994	4.50611	4.57313	4.64100
5	5.75074	5.86660	5.98471	6.10510
6	7.15329	7.33593	7.52333	7.71561
7	8.65402	8.92280	9.20043	9.48717
8	10.25980	10.63663	11.02847	11.43589
9	11.97799	12.48756	13.02104	13.57948
10	13.81645	14.48656	15.19293	15.93742
11	15.78360	16.64549	17.56029	18.53117
12	17.88845	18.97713	20.14072	21.38428
13	20.14064	21.49530	22.95338	24.52271
14	22.55049	24.21492	26.01919	27.97498
15	25.12902	27.15211	29.36092	31.77248
16	27.88805	30.32428	33.00340	35.94973
17	30.84022	33.75023	36.97370	40.54470
18	33.99903	37.45024	41.30134	45.59917
19	37.37896	41.44626	46.01846	51.15909
20	40.99549	45.76196	51.16012	57.27500
21	44.86518	50.42292	56.76453	64.00250
22	49.00574	55.45676	62.87334	71.40275
23	53.43614	60.89330	69.53194	79.54302
24	58.17667	66.76476	76.78981	88.49733
25	63.24904	73.10594	84.70090	98.34706
26	68.67647	79.95442	93.32398	109.18177
27	74.48382	87.35077	102.72313	121.09994
28	80.69769	95.33883	112.96822	134.20994
29	87.34653	103.96594	124.13536	148.63093
30	94.46079	113.28321	136.30754	164.49402
31	102.07304	123.34587	149.57522	181.94342
32	110.21815	134.21354	164.03699	201.13777
33	118.93343	145.95062	179.80032	222.25154
34	128.25876	158.62667	196.98234	245.47670
35	138.23688	172.31680	215.71075	271.02437
36	148.91346	187.10215	236.12472	299.12681
37	160.33740	203.07032	258.37595	330.03949
38	172.56102	220.31595	282.62978	364.04343
39	185.64029	238.94122	309.06646	401.44778
40	199.63511	259.05652	337.88245	442.59256

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT
OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
1	1.00000	1.00000	1.00000	1.00000
2	2.11000	2.12000	2.13000	2.14000
3	3.34210	3.37440	3.40690	3.43960
4	4.70973	4.77933	4.84980	4.92114
5	6.22780	6.35285	6.48027	6.61010
6	7.91286	8.11519	8.32271	8.53552
7	9.78327	10.08901	10.40466	10.73049
8	11.85943	12.29969	12.75726	13.23276
9	14.16397	14.77566	15.41571	16.08535
10	16.72201	17.54874	18.41975	19.33730
11	19.56143	20.65458	21.81432	23.04452
12	22.71319	24.13313	25.65018	27.27075
13	26.21164	28.02911	29.98470	32.08865
14	30.09492	32.39260	34.88271	37.58107
15	34.40536	37.27971	40.41746	43.84241
16	39.18995	42.75328	46.67173	50.98035
17	44.50084	48.88367	53.73906	59.11760
18	50.39594	55.74971	61.72514	68.39407
19	56.93949	63.43968	70.74941	78.96923
20	64.20283	72.05244	80.94683	91.02493
21	72.26514	81.69874	92.46992	104.76842
22	81.21431	92.50258	105.49101	120.43600
23	91.14788	104.60289	120.20484	138.29704
24	102.17415	118.15524	136.83147	158.65862
25	114.41331	133.33387	155.61956	181.87083
26	127.99877	150.33393	176.85010	208.33274
27	143.07864	169.37401	200.84061	238.49933
28	159.81729	190.69889	227.94989	272.88923
29	178.39719	214.58275	258.58338	312.09373
30	199.02088	241.33268	293.19922	356.78685
31	221.91317	271.29261	332.31511	407.73701
32	247.32362	304.84772	376.51608	465.82019
33	275.52922	342.42945	426.46317	532.03501
34	306.83744	384.52098	482.90338	607.51991
35	341.58955	431.66350	546.68082	693.57270
36	380.16441	484.46312	618.74933	791.67288
37	422.98249	543.59869	700.18674	903.50708
38	470.51056	609.83053	792.21101	1030.99808
39	523.26673	684.01020	896.19845	1176.33781
40	581.82607	767.09142	1013.70424	1342.02510

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 3. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
1	1.00000	1.00000	1.00000	1.00000
2	2.15000	2.20000	2.25000	2.30000
3	3.47250	3.64000	3.81250	3.99000
4	4.99337	5.36800	5.76563	6.18700
5	6.74238	7.44160	8.20703	9.04310
6	8.75374	9.92992	11.25879	12.75603
7	11.06680	12.91590	15.07349	17.58284
8	13.72682	16.49908	19.84186	23.85769
9	16.78584	20.79890	25.80232	32.01500
10	20.30372	25.95868	33.25290	42.61950
11	24.34928	32.15042	42.56613	56.40535
12	29.00167	39.58050	54.20766	74.32695
13	34.35192	48.49660	68.75958	97.62504
14	40.50471	59.19592	86.94947	127.91255
15	47.58041	72.03511	109.68684	167.28631
16	55.71747	87.44213	138.10855	218.47220
17	65.07509	105.93056	173.63568	285.01386
18	75.83636	128.11667	218.04460	371.51802
19	88.21181	154.74000	273.55576	483.97343
20	102.44358	186.68800	342.94470	630.16546
21	118.81012	225.02560	429.68087	820.21510
22	137.63164	271.03072	538.10109	1067.27963
23	159.27638	326.23686	673.62636	1388.46351
24	184.16784	392.48424	843.03295	1806.00257
25	212.79302	471.98108	1054.79118	2348.80334
26	245.71197	567.37730	1319.48898	3054.44434
27	283.56877	681.85276	1650.36123	3971.77764
28	327.10408	819.22331	2063.95153	5164.31093
29	377.16969	984.06797	2580.93941	6714.60421
30	434.74515	1181.88157	3227.17427	8729.98548
31	500.95692	1419.25788	4034.96783	11349.98112
32	577.10046	1704.10946	5044.70979	14755.97546
33	664.66552	2045.93135	6306.88724	19183.76810
34	765.36535	2456.11762	7884.60905	24939.89853
35	881.17016	2948.34115	9856.76132	32422.86808
36	1014.34568	3539.00937	12321.95164	42150.72851
37	1167.49753	4247.81125	15403.43956	54796.94706
38	1343.62216	5098.37350	19255.29944	71237.03118
39	1546.16549	6119.04820	24070.12430	92609.14053
40	1779.09031	7343.85784	30088.65538	120392.88269

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TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
EARS	.005	.010	.015	.020
5	5.05025	5.10101	5.15227	5.20404
10	10.22803	10.46221	10.70272	10.94972
15	15.53655	16.09690	16.68214	17.29342
20	20.97912	22.01900	23.12367	24.29737
25	26.55912	28.24320	30.06302	32.03030
30	32.28002	34.78489	37.53868	40.56808
35	38.14538	41.66028	45.59209	49.99448
40	44.15885	48.88637	54.26789	60.40198
45	50.32416	56.48107	63.61420	71.89271
50	56.64516	64.46318	73.68283	84.57940
55	63.12577	72.85246	84.52960	98.58653
60	69.77003	81.66967	96.21465	114.05154
65	76.58206	90.93665	108.80277	131.12616
70	83.56611	100.67634	122.36375	149.97791
75	90.72650	110.91285	136.97278	170.79177
80	98.06771	121.67152	152.71085	193.77196
85	105.59430	132.97900	169.66523	219.14394
90	113.31094	144.86327	187.92990	247.15666
95	121.22243	157.35376	207.60614	278.08496
100	129.33370	170.48138	228.80304	312.23231
105	137.64979	184.27865	251.63813	349.93374
110	146.17587	198.77972	276.23799	391.55916
115	154.91724	214.02049	302.73904	437.51699
120	163.87935	230.03869	331.28819	488.25815
125	173.06776	246.87398	362.04374	544.28049
130	182.48818	264.56804	395.17619	606.13368
135	192.14649	283.16467	430.86926	674.42460
140	202.04868	302.70992	469.32083	749.82330
145	212.20091	323.25217	510.74409	833.06955
150	222.60950	344.84229	555.36870	924.98014
155	233.28092	367.53372	603.44208	1026.45685
160	244.22181	391.38264	655.23077	1138.49535

TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
5	5.25633	5.30914	5.36247	5.41632
10	11.20338	11.46388	11.73139	12.00611
15	17.93193	18.59891	19.29568	20.02359
20	25.54466	26.87037	28.27968	29.77808
25	34.15776	36.45926	38.94986	41.64591
30	43.90270	47.57542	51.62268	56.08494
35	54.92821	60.46208	66.67401	73.65222
40	67.40255	75.40126	84.55028	95.02552
45	81.51613	92.71986	105.78167	121.02939
50	97.48435	112.79687	130.99791	152.66708
55	115.55092	136.07162	160.94689	191.15917
60	135.99159	163.05344	196.51688	237.99069
65	159.11833	194.33276	238.76288	294.96838
70	185.28411	230.59406	288.93786	364.29046
75	214.88830	272.63086	348.53001	448.63137
80	248.38271	321.36302	419.30679	551.24498
85	286.27857	377.85695	503.36739	676.09012
90	329.15425	443.34890	603.20503	827.98333
95	377.66415	519.27203	721.78082	1012.78465
100	432.54865	607.28773	862.61166	1237.62370
105	494.64543	709.32206	1029.87452	1511.17479
110	564.90223	827.60781	1228.53033	1843.99152
115	644.39135	964.73341	1464.47111	2248.91396
120	734.32599	1123.69957	1744.69475	2741.56402
125	836.07879	1307.98492	2077.51253	3340.94814
130	951.20274	1521.62214	2472.79564	4070.19058
135	1081.45492	1769.28623	2942.26799	4957.42550
140	1228.82330	2056.39679	3499.85386	6036.88245
145	1395.55711	2389.23663	4162.09097	7350.20688
150	1584.20110	2775.08921	4948.62092	8948.06685
155	1797.63446	3222.39812	5882.77176	10892.10783
160	2039.11472	3740.95173	6992.24993	13257.33093

TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT
OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
EARS	.045	.050	.055	.060
5	5.47071	5.52563	5.58109	5.63709
10	12.28821	12.57789	12.87535	13.18079
15	20.78405	21.57856	22.40866	23.27597
20	31.37142	33.06595	34.86832	36.78559
25	44.56521	47.72710	51.15259	54.86451
30	61.00707	66.43885	72.43548	79.05819
35	81.49662	90.32031	100.25136	111.43478
40	107.03032	120.79977	136.60561	154.76197
45	138.84997	159.70016	184.11917	212.74351
50	178.50303	209.34800	246.21748	290.33590
55	227.91796	272.71262	327.37749	394.17203
60	289.49795	353.58372	433.45037	533.12818
65	366.23783	456.79801	572.08339	719.08286
70	461.86968	588.52851	753.27120	967.93217
75	581.04436	756.65372	990.07643	1300.94868
80	729.55770	971.22882	1299.57139	1746.59989
85	914.63234	1245.08707	1704.06892	2342.98174
90	1145.26901	1594.60730	2232.73102	3141.07519
95	1432.68426	2040.69353	2923.67123	4209.10425
100	1790.85596	2610.02516	3826.70247	5638.36806
105	2237.20306	3336.65262	5006.92817	7551.04544
110	2793.43275	4264.03385	6549.43597	10110.63924
115	3486.59614	5447.63341	8565.43196	13535.95314
120	4350.40385	6958.23971	11200.25810	18119.79580
125	5426.86541	8886.19868	14643.87050	24254.01129
130	6768.33236	11346.81717	19144.53417	32462.97536
135	8440.04424	14487.25918	25026.72159	43448.42105
140	10523.30140	18495.34742	32714.50530	58149.42544
145	13119.41884	23610.79653	42762.13115	77822.68554
150	16354.65350	30139.54992	55893.97629	104149.94540
155	20386.34450	38472.07750	73056.77270	139381.75794
160	25410.56499	49106.72881	95487.86121	186529.87061

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TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT
OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
5	5.75074	5.86660	5.98471	6.10510
10	13.81645	14.48656	15.19293	15.93742
15	25.12902	27.15211	29.36092	31.77248
20	40.99549	45.76196	51.16012	57.27500
25	63.24904	73.10594	84.70090	98.34706
30	94.46079	113.28321	136.30754	164.49402
35	138.23688	172.31680	215.71075	271.02437
40	199.63511	259.05652	337.88245	442.59256
45	285.74931	386.50562	525.85873	718.90484
50	406.52893	573.77016	815.08356	1163.90853
55	575.92859	848.92320	1260.09180	1880.59142
60	813.52038	1253.21330	1944.79213	3034.81640
65	1146.75516	1847.24808	2998.28847	4893.70725
70	1614.13417	2720.08007	4619.22318	7887.46957
75	2269.65742	4002.55662	7113.23215	12708.95371
80	3189.06268	5886.93543	10950.57409	20474.00215
85	4478.57612	8655.70611	16854.80033	32979.69030
90	6287.18543	12723.93862	25939.18425	53120.22612
95	8823.85354	18701.50686	39916.63496	85556.76047
100	12381.66179	27484.51570	61422.67546	137796.12340
105	17371.67192	40389.63720	94512.38455	221928.13979
110	24370.41925	59351.49455	145425.00362	357423.59352
115	34186.52444	87212.68395	223760.37892	575640.37670
120	47954.11976	128149.91178	344289.06388	927080.68818
125	67263.88440	188300.13002	529737.38581	999999.99999
130	94346.82821	276680.53451	815072.61632	999999.99999
135	132332.05793	406540.34425	999999.99999	999999.99999
140	185608.30762	597347.00877	999999.99999	999999.99999
145	260331.00383	877704.59817	999999.99999	999999.99999
150	365133.45071	999999.99999	999999.99999	999999.99999
155	512124.30397	999999.99999	999999.99999	999999.99999
160	718286.57961	999999.99999	999999.99999	999999.99999

TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
5	6.22780	6.35285	6.48027	6.61010
10	16.72201	17.54874	18.41975	19.33730
15	34.40536	37.27971	40.41746	43.84241
20	64.20283	72.05244	80.94683	91.02493
25	114.41331	133.33387	155.61956	181.87083
30	199.02088	241.33268	293.19922	356.78685
35	341.58955	431.66350	546.68082	693.57270
40	581.82607	767.09142	1013.70424	1342.02510
45	986.63856	1358.23003	1874.16463	2590.56480
50	1668.77115	2400.01825	3459.50712	4994.52135
55	2818.20424	4236.00505	6380.39789	9623.13434
60	4755.06584	7471.64111	11761.94979	18535.13328
65	8018.79027	13173.93742	21677.11035	35694.42601
70	13518.35574	23223.33190	39945.15096	68733.17846
75	22785.44339	40933.79867	73602.83163	132346.47421
80	38401.02500	72145.69250	135614.92657	254828.44148
85	64714.18815	127151.71400	249868.19182	490657.00734
90	109053.39829	224091.11853	460372.42707	944724.76699
95	183767.54594	394931.47186	848212.83549	999999.99999
100	309665.22972	696010.54772	999999.99999	999999.99999
105	521810.14850	999999.99999	999999.99999	999999.99999
110	879286.67394	999999.99999	999999.99999	999999.99999
115	999999.99999	999999.99999	999999.99999	999999.99999
120	999999.99999	999999.99999	999999.99999	999999.99999
125	999999.99999	999999.99999	999999.99999	999999.99999
130	999999.99999	999999.99999	999999.99999	999999.99999
135	999999.99999	999999.99999	999999.99999	999999.99999
140	999999.99999	999999.99999	999999.99999	999999.99999
145	999999.99999	999999.99999	999999.99999	999999.99999
150	999999.99999	999999.99999	999999.99999	999999.99999
155	999999.99999	999999.99999	999999.99999	999999.99999
160	999999.99999	999999.99999	999999.99999	999999.99999

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TABLE 5. COMPOUNDED ANNUAL PAYMENT MULTIPLIER
THE FUTURE VALUE IN N YEARS OF AN ANNUAL PAYMENT
OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
5	6.74238	7.44160	8.20703	9.04310
10	20.30372	25.95868	33.25290	42.61950
15	47.58041	72.03511	109.68684	167.28631
20	102.44358	186.68800	342.94470	630.16546
25	212.79302	471.98108	1054.79118	2348.80334
30	434.74515	1181.88157	3227.17427	8729.98548
35	881.17016	2948.34115	9856.76132	32422.86808
40	1779.09031	7343.85784	30088.65538	120392.88269
45	3585.12846	18281.30994	91831.49616	447019.38904
50	7217.71628	45497.19075	280255.69286	999999.99999
55	14524.14789	113219.01129	855280.70723	999999.99999
60	29219.99164	281732.57177	999999.99999	999999.99999
65	58778.58258	701048.23458	999999.99999	999999.99999
70	118231.46693	999999.99999	999999.99999	999999.99999
75	237812.45317	999999.99999	999999.99999	999999.99999
80	478332.52934	999999.99999	999999.99999	999999.99999
85	962104.31329	999999.99999	999999.99999	999999.99999
90	999999.99999	999999.99999	999999.99999	999999.99999
95	999999.99999	999999.99999	999999.99999	999999.99999
100	999999.99999	999999.99999	999999.99999	999999.99999
105	999999.99999	999999.99999	999999.99999	999999.99999
110	999999.99999	999999.99999	999999.99999	999999.99999
115	999999.99999	999999.99999	999999.99999	999999.99999
120	999999.99999	999999.99999	999999.99999	999999.99999
125	999999.99999	999999.99999	999999.99999	999999.99999
130	999999.99999	999999.99999	999999.99999	999999.99999
135	999999.99999	999999.99999	999999.99999	999999.99999
140	999999.99999	999999.99999	999999.99999	999999.99999
145	999999.99999	999999.99999	999999.99999	999999.99999
150	999999.99999	999999.99999	999999.99999	999999.99999
155	999999.99999	999999.99999	999999.99999	999999.99999
160	999999.99999	999999.99999	999999.99999	999999.99999

Table 6.--Discounted Annual Payment Multiplier

The present value of an annual payment of \$1 for n years.

$$\frac{(1+i)^n - 1}{i(1+i)^n}$$

This multiplier is used to find the present value (V_o) of an annual payment (a) for n years.

To find the present value of this future series of annual payments, multiply the annual payment by the multiplier for the desired interest rate i and years n:

$$V_o = a \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

Example: Determine the value now of an anticipated series of annual payments for taxes and administration over the next 60 years of \$0.50 per acre, using a 5-percent interest rate.

Annual cost:	$a = \$.50$ per acre
Interest rate:	$i = .05$
Years:	$n = 60$
Multiplier:	= 18.92929 (page 83)
Present value:	$V_o = \$.50\ (18.929) = \$9.46$ per acre

Notes

TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
1	.99502	.99010	.98522	.98039
2	1.98510	1.97040	1.95588	1.94156
3	2.97025	2.94099	2.91220	2.88388
4	3.95050	3.90197	3.85438	3.80773
5	4.92587	4.85343	4.78264	4.71346
6	5.89638	5.79548	5.69719	5.60143
7	6.86207	6.72819	6.59821	6.47199
8	7.82296	7.65168	7.48593	7.32548
9	8.77906	8.56602	8.36052	8.16224
10	9.73041	9.47130	9.22218	8.98259
11	10.67703	10.36763	10.07112	9.78685
12	11.61893	11.25508	10.90751	10.57534
13	12.55615	12.13374	11.73153	11.34837
14	13.48871	13.00370	12.54338	12.10625
15	14.41662	13.86505	13.34323	12.84926
16	15.33993	14.71787	14.13126	13.57771
17	16.25863	15.56225	14.90765	14.29187
18	17.17277	16.39827	15.67256	14.99203
19	18.08236	17.22601	16.42617	15.67846
20	18.98742	18.04555	17.16864	16.35143
21	19.88798	18.85698	17.90014	17.01121
22	20.78406	19.66038	18.62082	17.65805
23	21.67568	20.45582	19.33086	18.29220
24	22.56287	21.24339	20.03041	18.91393
25	23.44564	22.02316	20.71961	19.52346
26	24.32402	22.79520	21.39863	20.12104
27	25.19803	23.55961	22.06762	20.70690
28	26.06769	24.31644	22.72672	21.28127
29	26.93302	25.06579	23.37608	21.84438
30	27.79405	25.80771	24.01584	22.39646
31	28.65080	26.54229	24.64615	22.93770
32	29.50328	27.26959	25.26714	23.46833
33	30.35153	27.98969	25.87895	23.98856
34	31.19555	28.70267	26.48173	24.49859
35	32.03537	29.40858	27.07559	24.99862
36	32.87102	30.10751	27.66068	25.48884
37	33.70250	30.79951	28.23713	25.96945
38	34.52985	31.48466	28.80505	26.44064
39	35.35309	32.16303	29.36458	26.90259
40	36.17223	32.83469	29.91585	27.35548

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
1	.97561	.97087	.96618	.96154
2	1.92742	1.91347	1.89969	1.88609
3	2.85602	2.82861	2.80164	2.77509
4	3.76197	3.71710	3.67308	3.62990
5	4.64583	4.57971	4.51505	4.45182
6	5.50813	5.41719	5.32855	5.24214
7	6.34939	6.23028	6.11454	6.00205
8	7.17014	7.01969	6.87396	6.73274
9	7.97087	7.78611	7.60769	7.43533
10	8.75206	8.53020	8.31661	8.11090
11	9.51421	9.25262	9.00155	8.76048
12	10.25776	9.95400	9.66333	9.38507
13	10.98318	10.63496	10.30274	9.98565
14	11.69091	11.29607	10.92052	10.56312
15	12.38138	11.93794	11.51741	11.11839
16	13.05500	12.56110	12.09412	11.65230
17	13.71220	13.16612	12.65132	12.16567
18	14.35336	13.75351	13.18968	12.65930
19	14.97889	14.32380	13.70984	13.13394
20	15.58916	14.87747	14.21240	13.59033
21	16.18455	15.41502	14.69797	14.02916
22	16.76541	15.93692	15.16712	14.45112
23	17.33211	16.44361	15.62041	14.85684
24	17.88499	16.93554	16.05837	15.24696
25	18.42438	17.41315	16.48151	15.62208
26	18.95061	17.87684	16.89035	15.98277
27	19.46401	18.32703	17.28536	16.32959
28	19.96489	18.76411	17.66702	16.66306
29	20.45355	19.18845	18.03577	16.98371
30	20.93029	19.60044	18.39205	17.29203
31	21.39541	20.00043	18.73628	17.58849
32	21.84918	20.38877	19.06887	17.87355
33	22.29188	20.76579	19.39021	18.14765
34	22.72379	21.13184	19.70068	18.41120
35	23.14516	21.48722	20.00066	18.66461
36	23.55625	21.83225	20.29049	18.90828
37	23.95732	22.16724	20.57053	19.14258
38	24.34860	22.49246	20.84109	19.36786
39	24.73034	22.80822	21.10250	19.58448
40	25.10278	23.11477	21.35507	19.79277

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TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
1	.95694	.95238	.94787	.94340
2	1.87267	1.85941	1.84632	1.83339
3	2.74896	2.72325	2.69793	2.67301
4	3.58753	3.54595	3.50515	3.46511
5	4.38998	4.32948	4.27028	4.21236
6	5.15787	5.07569	4.99553	4.91732
7	5.89270	5.78637	5.68297	5.58238
8	6.59589	6.46321	6.33457	6.20979
9	7.26879	7.10782	6.95220	6.80169
10	7.91272	7.72173	7.53763	7.36009
11	8.52892	8.30641	8.09254	7.88687
12	9.11858	8.86325	8.61852	8.38384
13	9.68285	9.39357	9.11708	8.85268
14	10.22283	9.89864	9.58965	9.29498
15	10.73955	10.37966	10.03758	9.71225
16	11.23402	10.83777	10.46216	10.10590
17	11.70719	11.27407	10.86461	10.47726
18	12.15999	11.68959	11.24607	10.82760
19	12.59329	12.08532	11.60765	11.15812
20	13.00794	12.46221	11.95038	11.46992
21	13.40472	12.82115	12.27524	11.76408
22	13.78442	13.16300	12.58317	12.04158
23	14.14777	13.48857	12.87504	12.30338
24	14.49548	13.79864	13.15170	12.55036
25	14.82821	14.09394	13.41393	12.78336
26	15.14661	14.37519	13.66250	13.00317
27	15.45130	14.64303	13.89810	13.21053
28	15.74287	14.89813	14.12142	13.40616
29	16.02189	15.14107	14.33310	13.59072
30	16.28889	15.37245	14.53375	13.76483
31	16.54439	15.59281	14.72393	13.92909
32	16.78889	15.80268	14.90420	14.08404
33	17.02286	16.00255	15.07507	14.23023
34	17.24676	16.19290	15.23703	14.36814
35	17.46101	16.37419	15.39055	14.49825
36	17.66604	16.54685	15.53607	14.62099
37	17.86224	16.71129	15.67400	14.73678
38	18.04999	16.86789	15.80474	14.84602
39	18.22966	17.01704	15.92866	14.94907
40	18.40158	17.15909	16.04612	15.04630

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
1	.93458	.92593	.91743	.90909
2	1.80802	1.78326	1.75911	1.73554
3	2.62432	2.57710	2.53129	2.48685
4	3.38721	3.31213	3.23972	3.16987
5	4.10020	3.99271	3.88965	3.79079
6	4.76654	4.62288	4.48592	4.35526
7	5.38929	5.20637	5.03295	4.86842
8	5.97130	5.74664	5.53482	5.33493
9	6.51523	6.24689	5.99525	5.75902
10	7.02358	6.71008	6.41766	6.14457
11	7.49867	7.13896	6.80519	6.49506
12	7.94269	7.53608	7.16073	6.81369
13	8.35765	7.90378	7.48690	7.10336
14	8.74547	8.24424	7.78615	7.36669
15	9.10791	8.55948	8.06069	7.60608
16	9.44665	8.85137	8.31256	7.82371
17	9.76322	9.12164	8.54363	8.02155
18	10.05909	9.37189	8.75563	8.20141
19	10.33560	9.60360	8.95011	8.36492
20	10.59401	9.81815	9.12855	8.51356
21	10.83553	10.01680	9.29224	8.64869
22	11.06124	10.20074	9.44243	8.77154
23	11.27219	10.37106	9.58021	8.88322
24	11.46933	10.52876	9.70661	8.98474
25	11.65358	10.67478	9.82258	9.07704
26	11.82578	10.80998	9.92897	9.16095
27	11.98671	10.93516	10.02658	9.23722
28	12.13711	11.05108	10.11613	9.30657
29	12.27767	11.15841	10.19828	9.36961
30	12.40904	11.25778	10.27365	9.42691
31	12.53181	11.34980	10.34280	9.47901
32	12.64656	11.43500	10.40624	9.52638
33	12.75379	11.51389	10.46444	9.56943
34	12.85401	11.58693	10.51784	9.60857
35	12.94767	11.65457	10.56682	9.64416
36	13.03521	11.71719	10.61176	9.67651
37	13.11702	11.77518	10.65299	9.70592
38	13.19347	11.82887	10.69082	9.73265
39	13.26493	11.87858	10.72552	9.75696
40	13.33171	11.92461	10.75736	9.77905

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TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
1	.90090	.89286	.88496	.87719
2	1.71252	1.69005	1.66810	1.64666
3	2.44371	2.40183	2.36115	2.32163
4	3.10245	3.03735	2.97447	2.91371
5	3.69590	3.60478	3.51723	3.43308
6	4.23054	4.11141	3.99755	3.88867
7	4.71220	4.56376	4.42261	4.28830
8	5.14612	4.96764	4.79877	4.63886
9	5.53705	5.32825	5.13166	4.94637
10	5.88923	5.65022	5.42624	5.21612
11	6.20652	5.93770	5.68694	5.45273
12	6.49236	6.19437	5.91765	5.66029
13	6.74987	6.42355	6.12181	5.84236
14	6.98187	6.62817	6.30249	6.00207
15	7.19087	6.81086	6.46238	6.14217
16	7.37916	6.97399	6.60388	6.26506
17	7.54879	7.11963	6.72909	6.37286
18	7.70162	7.24967	6.83991	6.46742
19	7.83929	7.36578	6.93797	6.55037
20	7.96333	7.46944	7.02475	6.62313
21	8.07507	7.56200	7.10155	6.68696
22	8.17574	7.64465	7.16951	6.74294
23	8.26643	7.71843	7.22966	6.79206
24	8.34814	7.78432	7.28288	6.83514
25	8.42174	7.84314	7.32998	6.87293
26	8.48806	7.89566	7.37167	6.90608
27	8.54780	7.94255	7.40856	6.93515
28	8.60162	7.98442	7.44120	6.96066
29	8.65011	8.02181	7.47009	6.98304
30	8.69379	8.05518	7.49565	7.00266
31	8.73315	8.08499	7.51828	7.01988
32	8.76860	8.11159	7.53830	7.03498
33	8.80054	8.13535	7.55602	7.04823
34	8.82932	8.15656	7.57170	7.05985
35	8.85524	8.17550	7.58557	7.07005
36	8.87859	8.19241	7.59785	7.07899
37	8.89963	8.20751	7.60872	7.08683
38	8.91859	8.22099	7.61833	7.09371
39	8.93567	8.23303	7.62684	7.09975
40	8.95105	8.24378	7.63438	7.10504

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TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
1	.86957	.83333	.80000	.76923
2	1.62571	1.52778	1.44000	1.36095
3	2.28323	2.10648	1.95200	1.81611
4	2.85498	2.58873	2.36160	2.16624
5	3.35216	2.99061	2.68928	2.43557
6	3.78448	3.32551	2.95142	2.64275
7	4.16042	3.60459	3.16114	2.80211
8	4.48732	3.83716	3.32891	2.92470
9	4.77158	4.03097	3.46313	3.01900
10	5.01877	4.19247	3.57050	3.09154
11	5.23371	4.32706	3.65640	3.14734
12	5.42062	4.43922	3.72512	3.19026
13	5.58315	4.53268	3.78010	3.22328
14	5.72448	4.61057	3.82408	3.24867
15	5.84737	4.67547	3.85926	3.26821
16	5.95423	4.72956	3.88741	3.28324
17	6.04716	4.77463	3.90993	3.29480
18	6.12797	4.81219	3.92794	3.30369
19	6.19823	4.84350	3.94235	3.31053
20	6.25933	4.86958	3.95388	3.31579
21	6.31246	4.89132	3.96311	3.31984
22	6.35866	4.90943	3.97049	3.32296
23	6.39884	4.92453	3.97639	3.32535
24	6.43377	4.93710	3.98111	3.32719
25	6.46415	4.94759	3.98489	3.32861
26	6.49056	4.95632	3.98791	3.32970
27	6.51353	4.96360	3.99033	3.33054
28	6.53351	4.96967	3.99226	3.33118
29	6.55088	4.97472	3.99381	3.33168
30	6.56598	4.97894	3.99505	3.33206
31	6.57911	4.98245	3.99604	3.33235
32	6.59053	4.98537	3.99683	3.33258
33	6.60046	4.98781	3.99746	3.33275
34	6.60910	4.98984	3.99797	3.33289
35	6.61661	4.99154	3.99838	3.33299
36	6.62314	4.99295	3.99870	3.33307
37	6.62881	4.99412	3.99896	3.33313
38	6.63375	4.99510	3.99917	3.33318
39	6.63805	4.99592	3.99934	3.33321
40	6.64178	4.99660	3.99947	3.33324

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
5	4.92587	4.85343	4.78264	4.71346
10	9.73041	9.47130	9.22218	8.98259
15	14.41662	13.86505	13.34323	12.84926
20	18.98742	18.04555	17.16864	16.35143
25	23.44564	22.02316	20.71961	19.52346
30	27.79405	25.80771	24.01584	22.39646
35	32.03537	29.40858	27.07559	24.99862
40	36.17223	32.83469	29.91585	27.35548
45	40.20720	36.09451	32.55234	29.49016
50	44.14279	39.19612	34.99969	31.42361
55	47.98145	42.14719	37.27147	33.17479
60	51.72556	44.95504	39.38027	34.76089
65	55.37746	47.62661	41.33779	36.19747
70	58.93942	50.16851	43.15487	37.49862
75	62.41365	52.58705	44.84160	38.67711
80	65.80231	54.88821	46.40732	39.74451
85	69.10750	57.07768	47.86072	40.71129
90	72.33130	59.16088	49.20985	41.58693
95	75.47569	61.14298	50.46220	42.38002
100	78.54264	63.02888	51.62470	43.09835
105	81.53406	64.82325	52.70381	43.74896
110	84.45180	66.53053	53.70550	44.33824
115	87.29767	68.15494	54.63533	44.87197
120	90.07345	69.70052	55.49845	45.35539
125	92.78087	71.17109	56.29966	45.79323
130	95.42161	72.57028	57.04338	46.18980
135	97.99730	73.90156	57.73376	46.54899
140	100.50956	75.16823	58.37460	46.87431
145	102.95994	76.37342	58.96947	47.16897
150	105.34998	77.52012	59.52166	47.43585
155	107.68114	78.61117	60.03424	47.67757
160	109.95489	79.64926	60.51005	47.89650

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE
DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
5	4.64583	4.57971	4.51505	4.45182
10	8.75206	8.53020	8.31661	8.11090
15	12.38138	11.93794	11.51741	11.11839
20	15.58916	14.87747	14.21240	13.59033
25	18.42438	17.41315	16.48151	15.62208
30	20.93029	19.60044	18.39205	17.29203
35	23.14516	21.48722	20.00066	18.66461
40	25.10278	23.11477	21.35507	19.79277
45	26.83302	24.51871	22.49545	20.72004
50	28.36231	25.72976	23.45562	21.48218
55	29.71398	26.77443	24.26405	22.10861
60	30.90866	27.67556	24.94473	22.62349
65	31.96458	28.45289	25.51785	23.04668
70	32.89786	29.12342	26.00040	23.39451
75	33.72274	29.70183	26.40669	23.68041
80	34.45182	30.20076	26.74878	23.91539
85	35.09621	30.63115	27.03680	24.10853
90	35.66577	31.00241	27.27932	24.26728
95	36.16917	31.32266	27.48350	24.39776
100	36.61411	31.59891	27.65543	24.50500
105	37.00736	31.83720	27.80018	24.59315
110	37.35494	32.04276	27.92206	24.66560
115	37.66216	32.22007	28.02467	24.72514
120	37.93369	32.37302	28.11108	24.77409
125	38.17368	32.50496	28.18382	24.81432
130	38.38580	32.61877	28.24508	24.84738
135	38.57328	32.71695	28.29665	24.87456
140	38.73899	32.80163	28.34007	24.89690
145	38.88545	32.87468	28.37663	24.91526
150	39.01490	32.93770	28.40742	24.93035
155	39.12932	32.99205	28.43333	24.94275
160	39.23044	33.03894	28.45516	24.95295

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
5	4.38998	4.32948	4.27028	4.21236
10	7.91272	7.72173	7.53763	7.36009
15	10.73955	10.37966	10.03758	9.71225
20	13.00794	12.46221	11.95038	11.46992
25	14.82821	14.09394	13.41393	12.78336
30	16.28889	15.37245	14.53375	13.76483
35	17.46101	16.37419	15.39055	14.49825
40	18.40158	17.15909	16.04612	15.04630
45	19.15635	17.77407	16.54773	15.45583
50	19.76201	18.25593	16.93152	15.76186
55	20.24802	18.63347	17.22517	15.99054
60	20.63802	18.92929	17.44985	16.16143
65	20.95098	19.16107	17.62177	16.28912
70	21.20211	19.34268	17.75330	16.38454
75	21.40363	19.48497	17.85395	16.45585
80	21.56534	19.59646	17.93095	16.50913
85	21.69511	19.68382	17.98987	16.54895
90	21.79924	19.75226	18.03495	16.57870
95	21.88280	19.80589	18.06945	16.60093
100	21.94985	19.84791	18.09584	16.61755
105	22.00366	19.88083	18.11603	16.62996
110	22.04684	19.90663	18.13148	16.63924
115	22.08148	19.92684	18.14331	16.64617
120	22.10929	19.94268	18.15235	16.65135
125	22.13160	19.95509	18.15927	16.65522
130	22.14950	19.96481	18.16457	16.65811
135	22.16387	19.97243	18.16862	16.66028
140	22.17539	19.97840	18.17172	16.66189
145	22.18464	19.98307	18.17409	16.66310
150	22.19207	19.98674	18.17591	16.66400
155	22.19803	19.98961	18.17729	16.66467
160	22.20281	19.99186	18.17836	16.66518

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TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
5	4.10020	3.99271	3.88965	3.79079
10	7.02358	6.71008	6.41766	6.14457
15	9.10791	8.55948	8.06069	7.60608
20	10.59401	9.81815	9.12855	8.51356
25	11.65358	10.67478	9.82258	9.07704
30	12.40904	11.25778	10.27365	9.42691
35	12.94767	11.65457	10.56682	9.64416
40	13.33171	11.92461	10.75736	9.77905
45	13.60552	12.10840	10.88120	9.86281
50	13.80075	12.23348	10.96168	9.91481
55	13.93994	12.31861	11.01399	9.94711
60	14.03918	12.37655	11.04799	9.96716
65	14.10994	12.41598	11.07009	9.97961
70	14.16039	12.44282	11.08445	9.98734
75	14.19636	12.46108	11.09378	9.99214
80	14.22201	12.47351	11.09985	9.99512
85	14.24029	12.48197	11.10379	9.99697
90	14.25333	12.48773	11.10635	9.99812
95	14.26262	12.49165	11.10802	9.99883
100	14.26925	12.49432	11.10910	9.99927
105	14.27398	12.49613	11.10981	9.99955
110	14.27735	12.49737	11.11026	9.99972
115	14.27975	12.49821	11.11056	9.99983
120	14.28146	12.49878	11.11075	9.99989
125	14.28268	12.49917	11.11088	9.99993
130	14.28355	12.49944	11.11096	9.99996
135	14.28417	12.49962	11.11101	9.99997
140	14.28461	12.49974	11.11105	9.99998
145	14.28493	12.49982	11.11107	9.99999
150	14.28516	12.49988	11.11108	9.99999
155	14.28532	12.49992	11.11109	10.00000
160	14.28543	12.49994	11.11110	10.00000

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TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
EARS	.110	.120	.130	.140
5	3.69590	3.60478	3.51723	3.43308
10	5.88923	5.65022	5.42624	5.21612
15	7.19087	6.81086	6.46238	6.14217
20	7.96333	7.46944	7.02475	6.62313
25	8.42174	7.84314	7.32998	6.87293
30	8.69379	8.05518	7.49565	7.00266
35	8.85524	8.17550	7.58557	7.07005
40	8.95105	8.24378	7.63438	7.10504
45	9.00791	8.28252	7.66086	7.12322
50	9.04165	8.30450	7.67524	7.13266
55	9.06168	8.31697	7.68304	7.13756
60	9.07356	8.32405	7.68728	7.14011
65	9.08061	8.32807	7.68958	7.14143
70	9.08480	8.33034	7.69083	7.14211
75	9.08728	8.33164	7.69150	7.14247
80	9.08876	8.33237	7.69187	7.14266
85	9.08963	8.33279	7.69207	7.14275
90	9.09015	8.33302	7.69218	7.14280
95	9.09046	8.33316	7.69224	7.14283
100	9.09064	8.33323	7.69227	7.14284
105	9.09075	8.33328	7.69229	7.14285
110	9.09082	8.33330	7.69230	7.14285
115	9.09085	8.33332	7.69230	7.14286
120	9.09088	8.33332	7.69230	7.14286
125	9.09089	8.33333	7.69231	7.14286
130	9.09090	8.33333	7.69231	7.14286
135	9.09090	8.33333	7.69231	7.14286
140	9.09090	8.33333	7.69231	7.14286
145	9.09091	8.33333	7.69231	7.14286
150	9.09091	8.33333	7.69231	7.14286
155	9.09091	8.33333	7.69231	7.14286
160	9.09091	8.33333	7.69231	7.14286

TABLE 6. DISCOUNTED ANNUAL PAYMENT MULTIPLIER
THE PRESENT VALUE OF AN ANNUAL PAYMENT OF ONE DOLLAR FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
5	3.35216	2.99061	2.68928	2.43557
10	5.01877	4.19247	3.57050	3.09154
15	5.84737	4.67547	3.85926	3.26821
20	6.25933	4.86958	3.95388	3.31579
25	6.46415	4.94759	3.98489	3.32861
30	6.56598	4.97894	3.99505	3.33206
35	6.61661	4.99154	3.99838	3.33299
40	6.64178	4.99660	3.99947	3.33324
45	6.65429	4.99863	3.99983	3.33331
50	6.66051	4.99945	3.99994	3.33333
55	6.66361	4.99978	3.99998	3.33333
60	6.66515	4.99991	3.99999	3.33333
65	6.66591	4.99996	4.00000	3.33333
70	6.66629	4.99999	4.00000	3.33333
75	6.66648	4.99999	4.00000	3.33333
80	6.66657	5.00000	4.00000	3.33333
85	6.66662	5.00000	4.00000	3.33333
90	6.66664	5.00000	4.00000	3.33333
95	6.66666	5.00000	4.00000	3.33333
100	6.66666	5.00000	4.00000	3.33333
105	6.66666	5.00000	4.00000	3.33333
110	6.66667	5.00000	4.00000	3.33333
115	6.66667	5.00000	4.00000	3.33333
120	6.66667	5.00000	4.00000	3.33333
125	6.66667	5.00000	4.00000	3.33333
130	6.66667	5.00000	4.00000	3.33333
135	6.66667	5.00000	4.00000	3.33333
140	6.66667	5.00000	4.00000	3.33333
145	6.66667	5.00000	4.00000	3.33333
150	6.66667	5.00000	4.00000	3.33333
155	6.66667	5.00000	4.00000	3.33333
160	6.66667	5.00000	4.00000	3.33333

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Table 7.--Investment Increase Multiplier

The increase in value of \$1 invested for n years.

$$(1+i)^n - 1$$

This multiplier is used to find the increase in value (V'_n), such as compounded investment earnings or interest rate charges, over a period of n years of an initial investment (V_o) compounded for n years at interest rate i.

To find the increase in investment value, multiply the initial investment by the multiplier for the appropriate interest rate i and years n:

$$V'_n = V_o \left[(1+i)^n - 1 \right]$$

Example: Determine the accumulated interest cost at the end of a 40-year rotation for an initial land cost of \$10 per acre, compounded at 4-percent interest, when the land value at the end of the rotation remains at \$10 per acre.

Land value:	V_o = \$10 per acre
Interest rate:	i = .04
Years:	n = 40
Multiplier:	= 3.80102 (page 90)
Compounded interest charges:	V'_n = \$10 (3.8010) = \$38.01 per acre.

Notes

TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
1	.00500	.01000	.01500	.02000
2	.01002	.02010	.03022	.04040
3	.01508	.03030	.04568	.06121
4	.02015	.04060	.06136	.08243
5	.02525	.05101	.07728	.10408
6	.03038	.06152	.09344	.12616
7	.03553	.07214	.10984	.14869
8	.04071	.08286	.12649	.17166
9	.04591	.09369	.14339	.19509
10	.05114	.10462	.16054	.21899
11	.05640	.11567	.17795	.24337
12	.06168	.12683	.19562	.26824
13	.06699	.13809	.21355	.29361
14	.07232	.14947	.23176	.31948
15	.07768	.16097	.25023	.34587
16	.08307	.17258	.26899	.37279
17	.08849	.18430	.28802	.40024
18	.09393	.19615	.30734	.42825
19	.09940	.20811	.32695	.45681
20	.10490	.22019	.34686	.48595
21	.11042	.23239	.36706	.51567
22	.11597	.24472	.38756	.54598
23	.12155	.25716	.40838	.57690
24	.12716	.26973	.42950	.60844
25	.13280	.28243	.45095	.64061
26	.13846	.29526	.47271	.67342
27	.14415	.30821	.49480	.70689
28	.14987	.32129	.51722	.74102
29	.15562	.33450	.53998	.77584
30	.16140	.34785	.56308	.81136
31	.16721	.36133	.58653	.84759
32	.17304	.37494	.61032	.88454
33	.17891	.38869	.63448	.92223
34	.18480	.40258	.65900	.96068
35	.19073	.41660	.68388	.99989
36	.19668	.43077	.70914	1.03989
37	.20266	.44508	.73478	1.08069
38	.20868	.45953	.76080	1.12230
39	.21472	.47412	.78721	1.16474
40	.22079	.48886	.81402	1.20804

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TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
1	.02500	.03000	.03500	.04000
2	.05062	.06090	.07122	.08160
3	.07689	.09273	.10872	.12486
4	.10381	.12551	.14752	.16986
5	.13141	.15927	.18769	.21665
6	.15969	.19405	.22926	.26532
7	.18869	.22987	.27228	.31593
8	.21840	.26677	.31681	.36857
9	.24886	.30477	.36290	.42331
10	.28008	.34392	.41060	.48024
11	.31209	.38423	.45997	.53945
12	.34489	.42576	.51107	.60103
13	.37851	.46853	.56396	.66507
14	.41297	.51259	.61869	.73168
15	.44830	.55797	.67535	.80094
16	.48451	.60471	.73399	.87298
17	.52162	.65285	.79468	.94790
18	.55966	.70243	.85749	1.02582
19	.59865	.75351	.92250	1.10685
20	.63862	.80611	.98979	1.19112
21	.67958	.86029	1.05943	1.27877
22	.72157	.91610	1.13151	1.36992
23	.76461	.97359	1.20611	1.46472
24	.80873	1.03279	1.28333	1.56330
25	.85394	1.09378	1.36324	1.66584
26	.90029	1.15659	1.44596	1.77247
27	.94780	1.22129	1.53157	1.88337
28	.99650	1.28793	1.62017	1.99870
29	1.04641	1.35657	1.71188	2.11865
30	1.09757	1.42726	1.80679	2.24340
31	1.15001	1.50008	1.90503	2.37313
32	1.20376	1.57508	2.00671	2.50806
33	1.25885	1.65234	2.11194	2.64838
34	1.31532	1.73191	2.22086	2.79432
35	1.37321	1.81386	2.33359	2.94609
36	1.43254	1.89828	2.45027	3.10393
37	1.49335	1.98523	2.57103	3.26809
38	1.55568	2.07478	2.69601	3.43881
39	1.61957	2.16703	2.82537	3.61637
40	1.68506	2.26204	2.95926	3.80102

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TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
1	.04500	.05000	.05500	.06000
2	.09202	.10250	.11303	.12360
3	.14117	.15762	.17424	.19102
4	.19252	.21551	.23882	.26248
5	.24618	.27628	.30696	.33823
6	.30226	.34010	.37884	.41852
7	.36086	.40710	.45468	.50363
8	.42210	.47746	.53469	.59385
9	.48610	.55133	.61909	.68948
10	.55297	.62889	.70814	.79085
11	.62285	.71034	.80209	.89830
12	.69588	.79586	.90121	1.01220
13	.77220	.88565	1.00577	1.13293
14	.85194	.97993	1.11609	1.26090
15	.93528	1.07893	1.23248	1.39656
16	1.02237	1.18287	1.35526	1.54035
17	1.11338	1.29202	1.48480	1.69277
18	1.20848	1.40662	1.62147	1.85434
19	1.30786	1.52695	1.76565	2.02560
20	1.41171	1.65330	1.91776	2.20714
21	1.52024	1.78596	2.07823	2.39956
22	1.63365	1.92526	2.24754	2.60354
23	1.75217	2.07152	2.42615	2.81975
24	1.87601	2.22510	2.61459	3.04893
25	2.00543	2.38635	2.81339	3.29187
26	2.14068	2.55567	3.02313	3.54938
27	2.28201	2.73346	3.24440	3.82235
28	2.42970	2.92013	3.47784	4.11169
29	2.58404	3.11614	3.72412	4.41839
30	2.74532	3.32194	3.98395	4.74349
31	2.91386	3.53804	4.25807	5.08810
32	3.08998	3.76494	4.54726	5.45339
33	3.27403	4.00319	4.85236	5.84059
34	3.46636	4.25335	5.17424	6.25103
35	3.66735	4.51602	5.51383	6.68609
36	3.87738	4.79182	5.87209	7.14725
37	4.09686	5.08141	6.25005	7.63609
38	4.32622	5.38548	6.64880	8.15425
39	4.56590	5.70475	7.06949	8.70351
40	4.81636	6.03999	7.51331	9.28572

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TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
1	.07000	.08000	.09000	.10000
2	.14490	.16640	.18810	.21000
3	.22504	.25971	.29503	.33100
4	.31080	.36049	.41158	.46410
5	.40255	.46933	.53862	.61051
6	.50073	.58687	.67710	.77156
7	.60578	.71382	.82804	.94872
8	.71819	.85093	.99256	1.14359
9	.83846	.99900	1.17189	1.35795
10	.96715	1.15892	1.36736	1.59374
11	1.10485	1.33164	1.58043	1.85312
12	1.25219	1.51817	1.81266	2.13843
13	1.40985	1.71962	2.06580	2.45227
14	1.57853	1.93719	2.34173	2.79750
15	1.75903	2.17217	2.64248	3.17725
16	1.95216	2.42594	2.97031	3.59497
17	2.15882	2.70002	3.32763	4.05447
18	2.37993	2.99602	3.71712	4.55992
19	2.61653	3.31570	4.14166	5.11591
20	2.86968	3.66096	4.60441	5.72750
21	3.14056	4.03383	5.10881	6.40025
22	3.43040	4.43654	5.65860	7.14027
23	3.74053	4.87146	6.25787	7.95430
24	4.07237	5.34118	6.91108	8.84973
25	4.42743	5.84848	7.62308	9.83471
26	4.80735	6.39635	8.39916	10.91818
27	5.21387	6.98806	9.24508	12.10999
28	5.64884	7.62711	10.16714	13.42099
29	6.11426	8.31727	11.17218	14.86309
30	6.61226	9.06266	12.26768	16.44940
31	7.14511	9.86767	13.46177	18.19434
32	7.71527	10.73708	14.76333	20.11378
33	8.32534	11.67605	16.18203	22.22515
34	8.97811	12.69013	17.72841	24.54767
35	9.67658	13.78534	19.41397	27.10244
36	10.42394	14.96817	21.25123	29.91268
37	11.22362	16.24563	23.25384	33.00395
38	12.07927	17.62528	25.43668	36.40434
39	12.99482	19.11530	27.81598	40.14478
40	13.97446	20.72452	30.40942	44.25926

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TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
1	.11000	.12000	.13000	.14000
2	.23210	.25440	.27690	.29960
3	.36763	.40493	.44290	.48154
4	.51807	.57352	.63047	.68896
5	.68506	.76234	.84244	.92541
6	.87041	.97382	1.08195	1.19497
7	1.07616	1.21068	1.35261	1.50227
8	1.30454	1.47596	1.65844	1.85259
9	1.55804	1.77308	2.00404	2.25195
10	1.83942	2.10585	2.39457	2.70722
11	2.15176	2.47855	2.83586	3.22623
12	2.49845	2.89598	3.33452	3.81790
13	2.88328	3.36349	3.89801	4.49241
14	3.31044	3.88711	4.53475	5.26135
15	3.78459	4.47357	5.25427	6.13794
16	4.31089	5.13039	6.06733	7.13725
17	4.89509	5.86604	6.98608	8.27646
18	5.54355	6.68997	8.02427	9.57517
19	6.26334	7.61276	9.19742	11.05569
20	7.06231	8.64629	10.52309	12.74349
21	7.94917	9.80385	12.02109	14.66758
22	8.93357	11.10031	13.71383	16.86104
23	10.02627	12.55235	15.62663	19.36158
24	11.23916	14.17863	17.78809	22.21221
25	12.58546	16.00006	20.23054	25.46192
26	14.07986	18.04007	22.99051	29.16658
27	15.73865	20.32488	26.10928	33.38991
28	17.57990	22.88387	29.63349	38.20449
29	19.62369	25.74993	33.61584	43.69312
30	21.89230	28.95992	38.11590	49.95016
31	24.41045	32.55511	43.20096	57.08318
32	27.20560	36.58173	48.94709	65.21483
33	30.30821	41.09153	55.44021	74.48490
34	33.75212	46.14252	62.77744	85.05279
35	37.57485	51.79962	71.06851	97.10018
36	41.81808	58.13557	80.43741	110.83420
37	46.52807	65.23184	91.02428	126.49099
38	51.75616	73.17966	102.98743	144.33973
39	57.55934	82.08122	116.50580	164.68729
40	64.00087	92.05097	131.78155	187.88351

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TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
1	.15000	.20000	.25000	.30000
2	.32250	.44000	.56250	.69000
3	.52087	.72800	.95313	1.19700
4	.74901	1.07360	1.44141	1.85610
5	1.01136	1.48832	2.05176	2.71293
6	1.31306	1.98598	2.81470	3.82681
7	1.66002	2.58318	3.76837	5.27485
8	2.05902	3.29982	4.96046	7.15731
9	2.51788	4.15978	6.45058	9.60450
10	3.04556	5.19174	8.31323	12.78585
11	3.65239	6.43008	10.64153	16.92160
12	4.35025	7.91610	13.55192	22.29809
13	5.15279	9.69932	17.18989	29.28751
14	6.07571	11.83918	21.73737	38.37376
15	7.13706	14.40702	27.42171	50.18589
16	8.35762	17.48843	34.52714	65.54166
17	9.76126	21.18611	43.40892	85.50416
18	11.37545	25.62333	54.51115	111.45541
19	13.23177	30.94800	68.38894	145.19203
20	15.36654	37.33760	85.73617	189.04964
21	17.82152	45.00512	107.42022	246.06453
22	20.64475	54.20614	134.52527	320.18389
23	23.89146	65.24737	168.40659	416.53905
24	27.62518	78.49685	210.75824	541.80077
25	31.91895	94.39622	263.69780	704.64100
26	36.85680	113.47546	329.87225	916.33330
27	42.53531	136.37055	412.59031	1191.53329
28	49.06561	163.84466	515.98788	1549.29328
29	56.57545	196.81359	645.23485	2014.38126
30	65.21177	236.37631	806.79357	2618.99564
31	75.14354	283.85158	1008.74196	3404.99434
32	86.56507	340.82189	1261.17745	4426.79264
33	99.69983	409.18627	1576.72181	5755.13043
34	114.80480	491.22352	1971.15226	7481.96956
35	132.17552	589.66823	2464.19033	9726.86043
36	152.15185	707.80187	3080.48791	12645.21855
37	175.12463	849.56225	3850.85989	16439.08412
38	201.54332	1019.67470	4813.82486	21371.10935
39	231.92482	1223.80964	6017.53108	27782.74216
40	266.86355	1468.77157	7522.16385	36117.86481

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TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
5	.02525	.05101	.07728	.10408
10	.05114	.10462	.16054	.21899
15	.07768	.16097	.25023	.34587
20	.10490	.22019	.34686	.48595
25	.13280	.28243	.45095	.64061
30	.16140	.34785	.56308	.81136
35	.19073	.41660	.68388	.99989
40	.22079	.48886	.81402	1.20804
45	.25162	.56481	.95421	1.43785
50	.28323	.64463	1.10524	1.69159
55	.31563	.72852	1.26794	1.97173
60	.34885	.81670	1.44322	2.28103
65	.38291	.90937	1.63204	2.62252
70	.41783	1.00676	1.83546	2.99956
75	.45363	1.10913	2.05459	3.41584
80	.49034	1.21672	2.29066	3.87544
85	.52797	1.32979	2.54498	4.38288
90	.56655	1.44863	2.81895	4.94313
95	.60611	1.57354	3.11409	5.56170
100	.64667	1.70481	3.43205	6.24465
105	.68825	1.84279	3.77457	6.99867
110	.73088	1.98780	4.14357	7.83118
115	.77459	2.14020	4.54109	8.75034
120	.81940	2.30039	4.96932	9.76516
125	.86534	2.46874	5.43066	10.88561
130	.91244	2.64568	5.92764	12.12267
135	.96073	2.83165	6.46304	13.48849
140	1.01024	3.02710	7.03981	14.99647
145	1.06100	3.23252	7.66116	16.66139
150	1.11305	3.44842	8.33053	18.49960
155	1.16640	3.67534	9.05163	20.52914
160	1.22111	3.91383	9.82846	22.76991

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
5	.13141	.15927	.18769	.21665
10	.28008	.34392	.41060	.48024
15	.44830	.55797	.67535	.80094
20	.63862	.80611	.98979	1.19112
25	.85394	1.09378	1.36324	1.66584
30	1.09757	1.42726	1.80679	2.24340
35	1.37321	1.81386	2.33359	2.94609
40	1.68506	2.26204	2.95926	3.80102
45	2.03790	2.78160	3.70236	4.84118
50	2.43711	3.38391	4.58493	6.10668
55	2.88877	4.08215	5.63314	7.64637
60	3.39979	4.89160	6.87809	9.51963
65	3.97796	5.82998	8.35670	11.79874
70	4.63210	6.91782	10.11283	14.57162
75	5.37221	8.17893	12.19855	17.94525
80	6.20957	9.64089	14.67574	22.04980
85	7.15696	11.33571	17.61786	27.04360
90	8.22886	13.30047	21.11218	33.11933
95	9.44160	15.57816	25.26233	40.51139
100	10.81372	18.21863	30.19141	49.50495
105	12.36614	21.27966	36.04561	60.44699
110	14.12256	24.82823	42.99856	73.75966
115	16.10978	28.94200	51.25649	89.95656
120	18.35815	33.71099	61.06432	109.66256
125	20.90197	39.23955	72.71294	133.63793
130	23.78007	45.64866	86.54785	162.80762
135	27.03637	53.07859	102.97938	198.29702
140	30.72058	61.69190	122.49489	241.47530
145	34.88893	71.67710	145.67318	294.00828
150	39.60503	83.25268	173.20173	357.92267
155	44.94086	96.67194	205.89701	435.68431
160	50.97787	112.22855	244.72875	530.29324

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR
N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
5	.24618	.27628	.30696	.33823
10	.55297	.62889	.70814	.79085
15	.93528	1.07893	1.23248	1.39656
20	1.41171	1.65330	1.91776	2.20714
25	2.00543	2.38635	2.81339	3.29187
30	2.74532	3.32194	3.98395	4.74349
35	3.66735	4.51602	5.51383	6.68609
40	4.81636	6.03999	7.51331	9.28572
45	6.24825	7.98501	10.12655	12.76461
50	8.03264	10.46740	13.54196	17.42015
55	10.25631	13.63563	18.00576	23.65032
60	13.02741	17.67919	23.83977	31.98769
65	16.48070	22.83990	31.46459	43.14497
70	20.78414	29.42643	41.42992	58.07593
75	26.14700	37.83269	54.45420	78.05692
80	32.83010	48.56144	71.47643	104.79599
85	41.15846	62.25435	93.72379	140.57890
90	51.53711	79.73037	122.80021	188.46451
95	64.47079	102.03468	160.80192	252.54625
100	80.58852	130.50126	210.46864	338.30208
105	100.67414	166.83263	275.38105	453.06273
110	125.70447	213.20169	360.21898	606.63835
115	156.89683	272.38167	471.09876	812.15719
120	195.76817	347.91199	616.01420	1087.18775
125	244.20894	444.30993	805.41288	1455.24068
130	304.57496	567.34086	1052.94938	1947.77852
135	379.80199	724.36296	1376.46969	2606.90526
140	473.54856	924.76737	1799.29779	3488.96553
145	590.37385	1180.53983	2351.91721	4669.36113
150	735.95941	1506.97750	3074.16870	6248.99672
155	917.38550	1923.60387	4018.12250	8362.90548
160	1143.47542	2455.33644	5251.83237	11191.79224

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR
N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
5	.40255	.46933	.53862	.61051
10	.96715	1.15892	1.36736	1.59374
15	1.75903	2.17217	2.64248	3.17725
20	2.86968	3.66096	4.60441	5.72750
25	4.42743	5.84848	7.62308	9.83471
30	6.61226	9.06266	12.26768	16.44940
35	9.67658	13.78534	19.41397	27.10244
40	13.97446	20.72452	30.40942	44.25926
45	20.00245	30.92045	47.32729	71.89048
50	28.45703	45.90161	73.35752	116.39085
55	40.31500	67.91386	113.40826	188.05914
60	56.94643	100.25706	175.03129	303.48164
65	80.27286	147.77985	269.84596	489.37073
70	112.98939	217.60641	415.73009	788.74696
75	158.87602	320.20453	640.19089	1270.89537
80	223.23439	470.95483	985.55167	2047.40021
85	313.50033	692.45649	1516.93203	3297.96903
90	440.10298	1017.91509	2334.52658	5312.02261
95	617.66975	1496.12055	3592.49715	8555.67605
100	866.71633	2198.76126	5528.04079	13779.61234
105	1216.01703	3231.17098	8506.11461	22192.81398
110	1705.92935	4748.11956	13088.25033	35742.35935
115	2393.05671	6977.01472	20138.43410	57564.03767
120	3356.78838	10251.99294	30986.01575	92708.06882
125	4708.47191	15064.01040	47676.36472	149307.88242
130	6604.27797	22134.44276	73356.53547	240462.44823
135	9263.24406	32523.22754	112868.66134	387267.78801
140	12992.58153	47787.76070	173662.96473	623699.25577
145	18223.17027	70216.36785	267202.53623	999999.99999
150	25559.34155	103171.35007	411124.76168	999999.99999
155	35848.70128	151593.03070	632566.94539	999999.99999
160	50280.06057	222740.36559	973283.19389	999999.99999

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TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
5	.68506	.76234	.84244	.92541
10	1.83942	2.10585	2.39457	2.70722
15	3.78459	4.47357	5.25427	6.13794
20	7.06231	8.64629	10.52309	12.74349
25	12.58546	16.00006	20.23054	25.46192
30	21.89230	28.95992	38.11590	49.95016
35	37.57485	51.79962	71.06851	97.10018
40	64.00087	92.05097	131.78155	187.88351
45	108.53024	162.98760	243.64140	362.67907
50	183.56483	288.00219	449.73593	699.23299
55	310.00247	508.32061	829.45173	1347.23881
60	523.05724	896.59693	1529.05347	2594.91866
65	882.06693	1580.87249	2818.02434	4997.21964
70	1487.01913	2786.79983	5192.86962	9622.64498
75	2506.39877	4912.05584	9568.36811	18528.50639
80	4224.11275	8657.48310	17629.94045	35675.98181
85	7118.56070	15258.20568	32482.86494	68691.98103
90	11995.87381	26890.93422	59848.41552	132261.46738
95	20214.43005	47391.77662	110267.66861	254659.08340
100	34063.17527	83521.26573	203161.87423	490325.23813
105	57399.11633	147193.77037	374313.42661	944080.28902
110	96721.53413	259406.47936	689649.06770	999999.99999
115	162982.09492	457163.61382	999999.99999	999999.99999
120	274634.99325	805679.25501	999999.99999	999999.99999
125	462776.62010	999999.99999	999999.99999	999999.99999
130	779806.20275	999999.99999	999999.99999	999999.99999
135	999999.99999	999999.99999	999999.99999	999999.99999
140	999999.99999	999999.99999	999999.99999	999999.99999
145	999999.99999	999999.99999	999999.99999	999999.99999
150	999999.99999	999999.99999	999999.99999	999999.99999
155	999999.99999	999999.99999	999999.99999	999999.99999
160	999999.99999	999999.99999	999999.99999	999999.99999

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TABLE 7. INVESTMENT INCREASE MULTIPLIER
THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR
N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
5	1.01136	1.48832	2.05176	2.71293
10	3.04556	5.19174	8.31323	12.78585
15	7.13706	14.40702	27.42171	50.18589
20	15.36654	37.33760	85.73617	189.04964
25	31.91895	94.39622	263.69780	704.64100
30	65.21177	236.37631	806.79357	2618.99564
35	132.17552	589.66823	2464.19033	9726.86043
40	266.86355	1468.77157	7522.16385	36117.86481
45	537.76927	3656.26199	22957.87404	134105.81671
50	1082.65744	9099.43815	70063.92322	497928.22298
55	2178.62218	22643.80226	213820.17681	999999.99999
60	4382.99875	56346.51435	652529.44680	999999.99999
65	8816.78739	140209.64692	999999.99999	999999.99999
70	17734.72004	348887.95693	999999.99999	999999.99999
75	35671.86798	868146.36931	999999.99999	999999.99999
80	71749.87940	999999.99999	999999.99999	999999.99999
85	144315.64699	999999.99999	999999.99999	999999.99999
90	290271.32521	999999.99999	999999.99999	999999.99999
95	583840.32764	999999.99999	999999.99999	999999.99999
100	999999.99999	999999.99999	999999.99999	999999.99999
105	999999.99999	999999.99999	999999.99999	999999.99999
110	999999.99999	999999.99999	999999.99999	999999.99999
115	999999.99999	999999.99999	999999.99999	999999.99999
120	999999.99999	999999.99999	999999.99999	999999.99999
125	999999.99999	999999.99999	999999.99999	999999.99999
130	999999.99999	999999.99999	999999.99999	999999.99999
135	999999.99999	999999.99999	999999.99999	999999.99999
140	999999.99999	999999.99999	999999.99999	999999.99999
145	999999.99999	999999.99999	999999.99999	999999.99999
150	999999.99999	999999.99999	999999.99999	999999.99999
155	999999.99999	999999.99999	999999.99999	999999.99999
160	999999.99999	999999.99999	999999.99999	999999.99999

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Table 8.--Discounted Investment Increase Multiplier

The present value of the increase in value
of \$1 invested for n years.

$$\frac{(1+i)^n - 1}{(1+i)^n}$$

This multiplier is used to find the present value of a future increase in value (V'_o), such as compounded investment earnings or interest rate charges, of an initial investment (V_o) compounded for n years at interest rate i.

To find the present value of this future increase in value, multiply the initial investment by the multiplier for the desired interest rate i and years n:

$$V'_o = V_o \left[\frac{(1+i)^n - 1}{(1+i)^n} \right]$$

Example: Find the present value of the cost of using land for one 40-year rotation, where at the end of the rotation the land is valued at its original purchase price, \$10. The interest rate is 4 percent.

Land value: V_o = \$10 per acre

Interest rate: $i = .04$

Years: $n = 40$

Multiplier: = 0.79171 (page 104)

Present value of land costs for one rotation: V'_o = \$10 (0.7917) = \$7.92 per acre

Notes

TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER

THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
1	.00498	.00990	.01478	.01961
2	.00993	.01970	.02934	.03883
3	.01485	.02941	.04368	.05768
4	.01975	.03902	.05782	.07615
5	.02463	.04853	.07174	.09427
6	.02948	.05795	.08546	.11203
7	.03431	.06728	.09897	.12944
8	.03911	.07652	.11229	.14651
9	.04390	.08566	.12541	.16324
10	.04865	.09471	.13833	.17965
11	.05339	.10368	.15107	.19574
12	.05809	.11255	.16361	.21151
13	.06278	.12134	.17597	.22697
14	.06744	.13004	.18815	.24212
15	.07208	.13865	.20015	.25699
16	.07670	.14718	.21197	.27155
17	.08129	.15562	.22361	.28584
18	.08586	.16398	.23509	.29984
19	.09041	.17226	.24639	.31357
20	.09494	.18046	.25753	.32703
21	.09944	.18857	.26850	.34022
22	.10392	.19660	.27931	.35316
23	.10838	.20456	.28996	.36584
24	.11281	.21243	.30046	.37828
25	.11723	.22023	.31079	.39047
26	.12162	.22795	.32098	.40242
27	.12599	.23560	.33101	.41414
28	.13034	.24316	.34090	.42563
29	.13467	.25066	.35064	.43689
30	.13897	.25808	.36024	.44793
31	.14325	.26542	.36969	.45875
32	.14752	.27270	.37901	.46937
33	.15176	.27990	.38818	.47977
34	.15598	.28703	.39723	.48997
35	.16018	.29409	.40613	.49997
36	.16436	.30108	.41491	.50978
37	.16851	.30800	.42356	.51939
38	.17265	.31485	.43208	.52881
39	.17677	.32163	.44047	.53805
40	.18086	.32835	.44874	.54711

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TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
1	.02439	.02913	.03382	.03846
2	.04819	.05740	.06649	.07544
3	.07140	.08486	.09806	.11100
4	.09405	.11151	.12856	.14520
5	.11615	.13739	.15803	.17807
6	.13770	.16252	.18650	.20969
7	.15873	.18691	.21401	.24008
8	.17925	.21059	.24059	.26931
9	.19927	.23358	.26627	.29741
10	.21880	.25591	.29108	.32444
11	.23786	.27758	.31505	.35042
12	.25644	.29862	.33822	.37540
13	.27458	.31905	.36060	.39943
14	.29227	.33888	.38222	.42252
15	.30953	.35814	.40311	.44474
16	.32638	.37683	.42329	.46609
17	.34280	.39498	.44280	.48663
18	.35883	.41261	.46164	.50637
19	.37447	.42971	.47984	.52536
20	.38973	.44632	.49743	.54361
21	.40461	.46245	.51443	.56117
22	.41914	.47811	.53085	.57804
23	.43330	.49331	.54671	.59427
24	.44712	.50807	.56204	.60988
25	.46061	.52239	.57685	.62488
26	.47377	.53631	.59116	.63931
27	.48660	.54981	.60499	.65318
28	.49912	.56292	.61835	.66652
29	.51134	.57565	.63125	.67935
30	.52326	.58801	.64372	.69168
31	.53489	.60001	.65577	.70354
32	.54623	.61166	.66741	.71494
33	.55730	.62297	.67866	.72591
34	.56809	.63396	.68952	.73645
35	.57863	.64462	.70002	.74658
36	.58891	.65497	.71017	.75633
37	.59893	.66502	.71997	.76570
38	.60872	.67477	.72944	.77471
39	.61826	.68425	.73859	.78338
40	.62757	.69344	.74743	.79171

TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER
THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
1	.04306	.04762	.05213	.05660
2	.08427	.09297	.10155	.11000
3	.12370	.13616	.14839	.16038
4	.16144	.17730	.19278	.20791
5	.19755	.21647	.23487	.25274
6	.23210	.25378	.27475	.29504
7	.26517	.28932	.31256	.33494
8	.29681	.32316	.34840	.37259
9	.32710	.35539	.38237	.40810
10	.35607	.38609	.41457	.44161
11	.38380	.41532	.44509	.47321
12	.41034	.44316	.47402	.50303
13	.43573	.46968	.50144	.53116
14	.46003	.49493	.52743	.55770
15	.48328	.51898	.55207	.58273
16	.50553	.54189	.57542	.60635
17	.52682	.56370	.59755	.62864
18	.54720	.58448	.61853	.64966
19	.56670	.60427	.63842	.66949
20	.58536	.62311	.65727	.68820
21	.60321	.64106	.67514	.70584
22	.62030	.65815	.69207	.72249
23	.63665	.67443	.70813	.73820
24	.65230	.68993	.72334	.75302
25	.66727	.70470	.73777	.76700
26	.68160	.71876	.75144	.78019
27	.69531	.73215	.76440	.79263
28	.70843	.74491	.77668	.80437
29	.72098	.75705	.78832	.81544
30	.73300	.76862	.79936	.82589
31	.74450	.77964	.80982	.83575
32	.75550	.79013	.81973	.84504
33	.76603	.80013	.82913	.85381
34	.77610	.80965	.83804	.86209
35	.78575	.81871	.84648	.86989
36	.79497	.82734	.85448	.87726
37	.80380	.83556	.86207	.88421
38	.81225	.84339	.86926	.89076
39	.82033	.85085	.87608	.89694
40	.82807	.85795	.88254	.90278

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TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER
THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
1	.06542	.07407	.08257	.09091
2	.12656	.14266	.15832	.17355
3	.18370	.20617	.22782	.24869
4	.23710	.26497	.29157	.31699
5	.28701	.31942	.35007	.37908
6	.33366	.36983	.40373	.43553
7	.37725	.41651	.45297	.48684
8	.41799	.45973	.49813	.53349
9	.45607	.49975	.53957	.57590
10	.49165	.53681	.57759	.61446
11	.52491	.57112	.61247	.64951
12	.55599	.60289	.64447	.68137
13	.58504	.63230	.67382	.71034
14	.61218	.65954	.70075	.73667
15	.63755	.68476	.72546	.76061
16	.66127	.70811	.74813	.78237
17	.68343	.72973	.76893	.80216
18	.70414	.74975	.78801	.82014
19	.72349	.76829	.80551	.83649
20	.74158	.78545	.82157	.85136
21	.75849	.80134	.83630	.86487
22	.77429	.81606	.84982	.87715
23	.78905	.82968	.86222	.88832
24	.80285	.84230	.87360	.89847
25	.81575	.85398	.88403	.90770
26	.82780	.86480	.89361	.91609
27	.83907	.87481	.90239	.92372
28	.84960	.88409	.91045	.93066
29	.85944	.89267	.91785	.93696
30	.86863	.90062	.92463	.94269
31	.87723	.90798	.93085	.94790
32	.88526	.91480	.93656	.95264
33	.89277	.92111	.94180	.95694
34	.89978	.92695	.94661	.96086
35	.90634	.93237	.95101	.96442
36	.91246	.93738	.95506	.96765
37	.91819	.94201	.95877	.97059
38	.92354	.94631	.96217	.97327
39	.92854	.95029	.96530	.97570
40	.93322	.95397	.96816	.97791

TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
1	.09910	.10714	.11504	.12281
2	.18838	.20281	.21685	.23053
3	.26881	.28822	.30695	.32503
4	.34127	.36448	.38668	.40792
5	.40655	.43257	.45724	.48063
6	.46536	.49337	.51968	.54441
7	.51834	.54765	.57494	.60036
8	.56607	.59612	.62384	.64944
9	.60908	.63939	.66712	.69249
10	.64782	.67803	.70541	.73026
11	.68272	.71252	.73930	.76338
12	.71416	.74332	.76929	.79244
13	.74249	.77083	.79584	.81793
14	.76801	.79538	.81932	.84029
15	.79100	.81730	.84011	.85990
16	.81171	.83688	.85850	.87711
17	.83037	.85436	.87478	.89220
18	.84718	.86996	.88919	.90544
19	.86232	.88389	.90194	.91705
20	.87597	.89633	.91322	.92724
21	.88826	.90744	.92320	.93617
22	.89933	.91736	.93204	.94401
23	.90931	.92621	.93986	.95089
24	.91830	.93412	.94677	.95692
25	.92639	.94118	.95290	.96221
26	.93369	.94748	.95832	.96685
27	.94026	.95311	.96311	.97092
28	.94618	.95813	.96736	.97449
29	.95151	.96262	.97111	.97763
30	.95632	.96662	.97443	.98037
31	.96065	.97020	.97738	.98278
32	.96455	.97339	.97998	.98490
33	.96806	.97624	.98228	.98675
34	.97122	.97879	.98432	.98838
35	.97408	.98106	.98612	.98981
36	.97665	.98309	.98772	.99106
37	.97896	.98490	.98913	.99216
38	.98104	.98652	.99038	.99312
39	.98292	.98796	.99149	.99396
40	.98462	.98925	.99247	.99471

TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER
THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
1	.13043	.16667	.20000	.23077
2	.24386	.30556	.36000	.40828
3	.34248	.42130	.48800	.54483
4	.42825	.51775	.59040	.64987
5	.50282	.59812	.67232	.73067
6	.56767	.66510	.73786	.79282
7	.62406	.72092	.79028	.84063
8	.67310	.76743	.83223	.87741
9	.71574	.80619	.86578	.90570
10	.75282	.83849	.89263	.92746
11	.78506	.86541	.91410	.94420
12	.81309	.88784	.93128	.95708
13	.83747	.90654	.94502	.96698
14	.85867	.92211	.95602	.97460
15	.87711	.93509	.96482	.98046
16	.89314	.94591	.97185	.98497
17	.90707	.95493	.97748	.98844
18	.91919	.96244	.98199	.99111
19	.92973	.96870	.98559	.99316
20	.93890	.97392	.98847	.99474
21	.94687	.97826	.99078	.99595
22	.95380	.98189	.99262	.99689
23	.95983	.98491	.99410	.99761
24	.96507	.98742	.99528	.99816
25	.96962	.98952	.99622	.99858
26	.97358	.99126	.99698	.99891
27	.97703	.99272	.99758	.99916
28	.98003	.99393	.99807	.99935
29	.98263	.99494	.99845	.99950
30	.98490	.99579	.99876	.99962
31	.98687	.99649	.99901	.99971
32	.98858	.99707	.99921	.99977
33	.99007	.99756	.99937	.99983
34	.99136	.99797	.99949	.99987
35	.99249	.99831	.99959	.99990
36	.99347	.99859	.99968	.99992
37	.99432	.99882	.99974	.99994
38	.99506	.99902	.99979	.99995
39	.99571	.99918	.99983	.99996
40	.99627	.99932	.99987	.99997

TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER
THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
5	.02463	.04853	.07174	.09427
10	.04865	.09471	.13833	.17965
15	.07208	.13865	.20015	.25699
20	.09494	.18046	.25753	.32703
25	.11723	.22023	.31079	.39047
30	.13897	.25808	.36024	.44793
35	.16018	.29409	.40613	.49997
40	.18086	.32835	.44874	.54711
45	.20104	.36095	.48829	.58980
50	.22071	.39196	.52500	.62847
55	.23991	.42147	.55907	.66350
60	.25863	.44955	.59070	.69522
65	.27689	.47627	.62007	.72395
70	.29470	.50169	.64732	.74997
75	.31207	.52587	.67262	.77354
80	.32901	.54888	.69611	.79489
85	.34554	.57078	.71791	.81423
90	.36166	.59161	.73815	.83174
95	.37738	.61143	.75693	.84760
100	.39271	.63029	.77437	.86197
105	.40767	.64823	.79056	.87498
110	.42226	.66531	.80558	.88676
115	.43649	.68155	.81953	.89744
120	.45037	.69701	.83248	.90711
125	.46390	.71171	.84449	.91586
130	.47711	.72570	.85565	.92380
135	.48999	.73902	.86601	.93098
140	.50255	.75168	.87562	.93749
145	.51480	.76373	.88454	.94338
150	.52675	.77520	.89282	.94872
155	.53841	.78611	.90051	.95355
160	.54977	.79649	.90765	.95793

TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER
THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
5	.11615	.13739	.15803	.17807
10	.21880	.25591	.29108	.32444
15	.30953	.35814	.40311	.44474
20	.38973	.44632	.49743	.54361
25	.46061	.52239	.57685	.62488
30	.52326	.58801	.64372	.69168
35	.57863	.64462	.70002	.74658
40	.62757	.69344	.74743	.79171
45	.67083	.73556	.78734	.82880
50	.70906	.77189	.82095	.85929
55	.74285	.80323	.84924	.88434
60	.77272	.83027	.87307	.90494
65	.79911	.85359	.89312	.92187
70	.82245	.87370	.91001	.93578
75	.84307	.89105	.92423	.94722
80	.86130	.90602	.93621	.95662
85	.87741	.91893	.94629	.96434
90	.89164	.93007	.95478	.97069
95	.90423	.93968	.96192	.97591
100	.91535	.94797	.96794	.98020
105	.92518	.95512	.97301	.98373
110	.93387	.96128	.97727	.98662
115	.94155	.96660	.98086	.98901
120	.94834	.97119	.98389	.99096
125	.95434	.97515	.98643	.99257
130	.95964	.97856	.98858	.99390
135	.96433	.98151	.99038	.99498
140	.96847	.98405	.99190	.99588
145	.97214	.98624	.99318	.99661
150	.97537	.98813	.99426	.99721
155	.97823	.98976	.99517	.99771
160	.98076	.99117	.99593	.99812

TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER
THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
5	.19755	.21647	.23487	.25274
10	.35607	.38609	.41457	.44161
15	.48328	.51898	.55207	.58273
20	.58536	.62311	.65727	.68820
25	.66727	.70470	.73777	.76700
30	.73300	.76862	.79936	.82589
35	.78575	.81871	.84648	.86989
40	.82807	.85795	.88254	.90278
45	.86204	.88870	.91012	.92735
50	.88929	.91280	.93123	.94571
55	.91116	.93167	.94738	.95943
60	.92871	.94646	.95974	.96969
65	.94279	.95805	.96920	.97735
70	.95410	.96713	.97643	.98307
75	.96316	.97425	.98197	.98735
80	.97044	.97982	.98620	.99055
85	.97628	.98419	.98944	.99294
90	.98097	.98761	.99192	.99472
95	.98473	.99029	.99382	.99606
100	.98774	.99240	.99527	.99705
105	.99016	.99404	.99638	.99780
110	.99211	.99533	.99723	.99835
115	.99367	.99634	.99788	.99877
120	.99492	.99713	.99838	.99908
125	.99592	.99775	.99876	.99931
130	.99673	.99824	.99905	.99949
135	.99737	.99862	.99927	.99962
140	.99789	.99892	.99944	.99971
145	.99831	.99915	.99957	.99979
150	.99864	.99934	.99967	.99984
155	.99891	.99948	.99975	.99988
160	.99913	.99959	.99981	.99991

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TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER
THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
5	.28701	.31942	.35007	.37908
10	.49165	.53681	.57759	.61446
15	.63755	.68476	.72546	.76061
20	.74158	.78545	.82157	.85136
25	.81575	.85398	.88403	.90770
30	.86863	.90062	.92463	.94269
35	.90634	.93237	.95101	.96442
40	.93322	.95397	.96816	.97791
45	.95239	.96867	.97931	.98628
50	.96605	.97868	.98655	.99148
55	.97580	.98549	.99126	.99471
60	.98274	.99012	.99432	.99672
65	.98770	.99328	.99631	.99796
70	.99123	.99543	.99760	.99873
75	.99375	.99689	.99844	.99921
80	.99554	.99788	.99899	.99951
85	.99682	.99856	.99934	.99970
90	.99773	.99902	.99957	.99981
95	.99838	.99933	.99972	.99988
100	.99885	.99955	.99982	.99993
105	.99918	.99969	.99988	.99995
110	.99941	.99979	.99992	.99997
115	.99958	.99986	.99995	.99998
120	.99970	.99990	.99997	.99999
125	.99979	.99993	.99998	.99999
130	.99985	.99995	.99999	1.00000
135	.99989	.99997	.99999	1.00000
140	.99992	.99998	.99999	1.00000
145	.99995	.99999	1.00000	1.00000
150	.99996	.99999	1.00000	1.00000
155	.99997	.99999	1.00000	1.00000
160	.99998	1.00000	1.00000	1.00000

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER
THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
5	.40655	.43257	.45724	.48063
10	.64782	.67803	.70541	.73026
15	.79100	.81730	.84011	.85990
20	.87597	.89633	.91322	.92724
25	.92639	.94118	.95290	.96221
30	.95632	.96662	.97443	.98037
35	.97408	.98106	.98612	.98981
40	.98462	.98925	.99247	.99471
45	.99087	.99390	.99591	.99725
50	.99458	.99654	.99778	.99857
55	.99678	.99804	.99880	.99926
60	.99809	.99889	.99935	.99961
65	.99887	.99937	.99965	.99980
70	.99933	.99964	.99981	.99990
75	.99960	.99980	.99990	.99995
80	.99976	.99988	.99994	.99997
85	.99986	.99993	.99997	.99999
90	.99992	.99996	.99998	.99999
95	.99995	.99998	.99999	1.00000
100	.99997	.99999	1.00000	1.00000
105	.99998	.99999	1.00000	1.00000
110	.99999	1.00000	1.00000	1.00000
115	.99999	1.00000	1.00000	1.00000
120	1.00000	1.00000	1.00000	1.00000
125	1.00000	1.00000	1.00000	1.00000
130	1.00000	1.00000	1.00000	1.00000
135	1.00000	1.00000	1.00000	1.00000
140	1.00000	1.00000	1.00000	1.00000
145	1.00000	1.00000	1.00000	1.00000
150	1.00000	1.00000	1.00000	1.00000
155	1.00000	1.00000	1.00000	1.00000
160	1.00000	1.00000	1.00000	1.00000

U.S. DEPT. AGR., FOREST SERVICE. 1970.

TABLE 8. DISCOUNTED INVESTMENT INCREASE MULTIPLIER THE PRESENT VALUE OF THE INCREASE IN VALUE OF ONE DOLLAR INVESTED FOR N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
5	.50282	.59812	.67232	.73067
10	.75282	.83849	.89263	.92746
15	.87711	.93509	.96482	.98046
20	.93890	.97392	.98847	.99474
25	.96962	.98952	.99622	.99858
30	.98490	.99579	.99876	.99962
35	.99249	.99831	.99959	.99990
40	.99627	.99932	.99987	.99997
45	.99814	.99973	.99996	.99999
50	.99908	.99989	.99999	1.00000
55	.99954	.99996	1.00000	1.00000
60	.99977	.99998	1.00000	1.00000
65	.99989	.99999	1.00000	1.00000
70	.99994	1.00000	1.00000	1.00000
75	.99997	1.00000	1.00000	1.00000
80	.99999	1.00000	1.00000	1.00000
85	.99999	1.00000	1.00000	1.00000
90	1.00000	1.00000	1.00000	1.00000
95	1.00000	1.00000	1.00000	1.00000
100	1.00000	1.00000	1.00000	1.00000
105	1.00000	1.00000	1.00000	1.00000
110	1.00000	1.00000	1.00000	1.00000
115	1.00000	1.00000	1.00000	1.00000
120	1.00000	1.00000	1.00000	1.00000
125	1.00000	1.00000	1.00000	1.00000
130	1.00000	1.00000	1.00000	1.00000
135	1.00000	1.00000	1.00000	1.00000
140	1.00000	1.00000	1.00000	1.00000
145	1.00000	1.00000	1.00000	1.00000
150	1.00000	1.00000	1.00000	1.00000
155	1.00000	1.00000	1.00000	1.00000
160	1.00000	1.00000	1.00000	1.00000

The annual payment which will recover an original investment of \$1 plus interest in n years.

$$\frac{i(1+i)^n}{(1+i)^n - 1}$$

This multiplier is used to determine the annual payment (a) which in n years will amount to the original investment (V_o) plus interest at rate i.

To find the annual payment, multiply the original investment by the multiplier:

$$a = V_o \left[\frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

Example: Determine how much should be charged annually for the use of equipment so as to recover the original purchase price of \$1,000 plus interest at 10 percent by the end of a 10-year period.

Original investment: $V_o = \$1,000$

Interest rate: $i = .10$

Years: $n = 10$

Multiplier: $= 0.16275$ (page 120)

Annual payment: $a = \$1,000\ (0.16275) = \$162.75$ per year

Notes

TABLE 9. CAPITAL RECOVERY MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YRS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
1	1.00500	1.01000	1.01500	1.02000
2	.50375	.50751	.51128	.51505
3	.33667	.34002	.34338	.34675
4	.25313	.25628	.25944	.26262
5	.20301	.20604	.20909	.21216
6	.16960	.17255	.17553	.17853
7	.14573	.14863	.15156	.15451
8	.12783	.13069	.13358	.13651
9	.11391	.11674	.11961	.12252
10	.10277	.10558	.10843	.11133
11	.09366	.09645	.09929	.10218
12	.08607	.08885	.09168	.09456
13	.07964	.08241	.08524	.08812
14	.07414	.07690	.07972	.08260
15	.06936	.07212	.07494	.07783
16	.06519	.06794	.07077	.07365
17	.06151	.06426	.06708	.06997
18	.05823	.06098	.06381	.06670
19	.05530	.05805	.06088	.06378
20	.05267	.05542	.05825	.06116
21	.05028	.05303	.05587	.05878
22	.04811	.05086	.05370	.05663
23	.04613	.04889	.05173	.05467
24	.04432	.04707	.04992	.05287
25	.04265	.04541	.04826	.05122
26	.04111	.04387	.04673	.04970
27	.03969	.04245	.04532	.04829
28	.03836	.04112	.04400	.04699
29	.03713	.03990	.04278	.04578
30	.03598	.03875	.04164	.04465
31	.03490	.03768	.04057	.04360
32	.03389	.03667	.03958	.04261
33	.03295	.03573	.03864	.04169
34	.03206	.03484	.03776	.04082
35	.03122	.03400	.03693	.04000
36	.03042	.03321	.03615	.03923
37	.02967	.03247	.03541	.03851
38	.02896	.03176	.03472	.03782
39	.02829	.03109	.03405	.03717
40	.02765	.03046	.03343	.03656

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TABLE 9. CAPITAL RECOVERY MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL
INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YR

		RATE OF INTEREST		
YEARS	.025	.030	.035	.040
1	1.02500	1.03000	1.03500	1.04000
2	.51883	.52261	.52640	.53020
3	.35014	.35353	.35693	.36035
4	.26582	.26903	.27225	.27549
5	.21525	.21835	.22148	.22463
6	.18155	.18460	.18767	.19076
7	.15750	.16051	.16354	.16661
8	.13947	.14246	.14548	.14853
9	.12546	.12843	.13145	.13449
10	.11426	.11723	.12024	.12329
11	.10511	.10808	.11109	.11415
12	.09749	.10046	.10348	.10655
13	.09105	.09403	.09706	.10014
14	.08554	.08853	.09157	.09467
15	.08077	.08377	.08683	.08994
16	.07660	.07961	.08268	.08582
17	.07293	.07595	.07904	.08220
18	.06967	.07271	.07582	.07899
19	.06676	.06981	.07294	.07614
20	.06415	.06722	.07036	.07358
21	.06179	.06487	.06804	.07128
22	.05965	.06275	.06593	.06920
23	.05770	.06081	.06402	.06731
24	.05591	.05905	.06227	.06559
25	.05428	.05743	.06067	.06401
26	.05277	.05594	.05921	.06257
27	.05138	.05456	.05785	.06124
28	.05009	.05329	.05660	.06001
29	.04889	.05211	.05545	.05888
30	.04778	.05102	.05437	.05783
31	.04674	.05000	.05337	.05686
32	.04577	.04905	.05244	.05595
33	.04486	.04816	.05157	.05510
34	.04401	.04732	.05076	.05431
35	.04321	.04654	.05000	.05358
36	.04245	.04580	.04928	.05289
37	.04174	.04511	.04861	.05224
38	.04107	.04446	.04798	.05163
39	.04044	.04384	.04739	.05106
40	.03984	.04326	.04683	.05052

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TABLE 9. CAPITAL RECOVERY MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL
INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YRS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
1	1.04500	1.05000	1.05500	1.06000
2	.53400	.53780	.54162	.54544
3	.36377	.36721	.37065	.37411
4	.27874	.28201	.28529	.28859
5	.22779	.23097	.23418	.23740
6	.19388	.19702	.20018	.20336
7	.16970	.17282	.17596	.17914
8	.15161	.15472	.15786	.16104
9	.13757	.14069	.14384	.14702
10	.12638	.12950	.13267	.13587
11	.11725	.12039	.12357	.12679
12	.10967	.11283	.11603	.11928
13	.10328	.10646	.10968	.11296
14	.09782	.10102	.10428	.10758
15	.09311	.09634	.09963	.10296
16	.08902	.09227	.09558	.09895
17	.08542	.08870	.09204	.09544
18	.08224	.08555	.08892	.09236
19	.07941	.08275	.08615	.08962
20	.07688	.08024	.08368	.08718
21	.07460	.07800	.08146	.08500
22	.07255	.07597	.07947	.08305
23	.07068	.07414	.07767	.08128
24	.06899	.07247	.07604	.07968
25	.06744	.07095	.07455	.07823
26	.06602	.06956	.07319	.07690
27	.06472	.06829	.07195	.07570
28	.06352	.06712	.07081	.07459
29	.06241	.06605	.06977	.07358
30	.06139	.06505	.06881	.07265
31	.06044	.06413	.06792	.07179
32	.05956	.06328	.06710	.07100
33	.05874	.06249	.06633	.07027
34	.05798	.06176	.06563	.06960
35	.05727	.06107	.06497	.06897
36	.05661	.06043	.06437	.06839
37	.05598	.05984	.06380	.06786
38	.05540	.05928	.06327	.06736
39	.05486	.05876	.06278	.06689
40	.05434	.05828	.06232	.06646

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TABLE 9. CAPITAL RECOVERY MULTIPLIER

THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL
INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YR

RATE OF INTEREST

YEARS	.070	.080	.090	.100
1	1.07000	1.08000	1.09000	1.10000
2	.55309	.56077	.56847	.57619
3	.38105	.38803	.39505	.40211
4	.29523	.30192	.30867	.31547
5	.24389	.25046	.25709	.26380
6	.20980	.21632	.22292	.22961
7	.18555	.19207	.19869	.20541
8	.16747	.17401	.18067	.18744
9	.15349	.16008	.16680	.17364
10	.14238	.14903	.15582	.16275
11	.13336	.14008	.14695	.15396
12	.12590	.13270	.13965	.14676
13	.11965	.12652	.13357	.14078
14	.11434	.12130	.12843	.13575
15	.10979	.11683	.12406	.13147
16	.10586	.11298	.12030	.12782
17	.10243	.10963	.11705	.12466
18	.09941	.10670	.11421	.12193
19	.09675	.10413	.11173	.11955
20	.09439	.10185	.10955	.11746
21	.09229	.09983	.10762	.11562
22	.09041	.09803	.10590	.11401
23	.08871	.09642	.10438	.11257
24	.08719	.09498	.10302	.11130
25	.08581	.09368	.10181	.11017
26	.08456	.09251	.10072	.10916
27	.08343	.09145	.09973	.10826
28	.08239	.09049	.09885	.10745
29	.08145	.08962	.09806	.10673
30	.08059	.08883	.09734	.10608
31	.07980	.08811	.09669	.10550
32	.07907	.08745	.09610	.10497
33	.07841	.08685	.09556	.10450
34	.07780	.08630	.09508	.10407
35	.07723	.08580	.09464	.10369
36	.07672	.08534	.09424	.10334
37	.07624	.08492	.09387	.10303
38	.07580	.08454	.09354	.10275
39	.07539	.08419	.09324	.10249
40	.07501	.08386	.09296	.10226

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TABLE 9. CAPITAL RECOVERY MULTIPLIER

THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL
INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YRS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
1	1.11000	1.12000	1.13000	1.14000
2	.58393	.59170	.59948	.60729
3	.40921	.41635	.42352	.43073
4	.32233	.32923	.33619	.34320
5	.27057	.27741	.28431	.29128
6	.23638	.24323	.25015	.25716
7	.21222	.21912	.22611	.23319
8	.19432	.20130	.20839	.21557
9	.18060	.18768	.19487	.20217
10	.16980	.17698	.18429	.19171
11	.16112	.16842	.17584	.18339
12	.15403	.16144	.16899	.17667
13	.14815	.15568	.16335	.17116
14	.14323	.15087	.15867	.16661
15	.13907	.14682	.15474	.16281
16	.13552	.14339	.15143	.15962
17	.13247	.14046	.14861	.15692
18	.12984	.13794	.14620	.15462
19	.12756	.13576	.14413	.15266
20	.12558	.13388	.14235	.15099
21	.12384	.13224	.14081	.14954
22	.12231	.13081	.13948	.14830
23	.12097	.12956	.13832	.14723
24	.11979	.12846	.13731	.14630
25	.11874	.12750	.13643	.14550
26	.11781	.12665	.13565	.14480
27	.11699	.12590	.13498	.14419
28	.11626	.12524	.13439	.14366
29	.11561	.12466	.13387	.14320
30	.11502	.12414	.13341	.14280
31	.11451	.12369	.13301	.14245
32	.11404	.12328	.13266	.14215
33	.11363	.12292	.13234	.14188
34	.11326	.12260	.13207	.14165
35	.11293	.12232	.13183	.14144
36	.11263	.12206	.13162	.14126
37	.11236	.12184	.13143	.14111
38	.11213	.12164	.13126	.14097
39	.11191	.12146	.13112	.14085
40	.11172	.12130	.13099	.14075

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TABLE 9. CAPITAL RECOVERY MULTIPLIER

THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL
INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YRS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
1	1.15000	1.20000	1.25000	1.30000
2	.61512	.65455	.69444	.73478
3	.43798	.47473	.51230	.55063
4	.35027	.38629	.42344	.46163
5	.29832	.33438	.37185	.41058
6	.26424	.30071	.33882	.37839
7	.24036	.27742	.31634	.35687
8	.22285	.26061	.30040	.34192
9	.20957	.24808	.28876	.33124
10	.19925	.23852	.28007	.32346
11	.19107	.23110	.27349	.31773
12	.18448	.22526	.26845	.31345
13	.17911	.22062	.26454	.31024
14	.17469	.21689	.26150	.30782
15	.17102	.21388	.25912	.30598
16	.16795	.21144	.25724	.30458
17	.16537	.20944	.25576	.30351
18	.16319	.20781	.25459	.30269
19	.16134	.20646	.25366	.30207
20	.15976	.20536	.25292	.30159
21	.15842	.20444	.25233	.30122
22	.15727	.20369	.25186	.30094
23	.15628	.20307	.25148	.30072
24	.15543	.20255	.25119	.30055
25	.15470	.20212	.25095	.30043
26	.15407	.20176	.25076	.30033
27	.15353	.20147	.25061	.30025
28	.15306	.20122	.25048	.30019
29	.15265	.20102	.25039	.30015
30	.15230	.20085	.25031	.30011
31	.15200	.20070	.25025	.30009
32	.15173	.20059	.25020	.30007
33	.15150	.20049	.25016	.30005
34	.15131	.20041	.25013	.30004
35	.15113	.20034	.25010	.30003
36	.15099	.20028	.25008	.30002
37	.15086	.20024	.25006	.30002
38	.15074	.20020	.25005	.30001
39	.15065	.20016	.25004	.30001
40	.15056	.20014	.25003	.30001

TABLE 9. CAPITAL RECOVERY MULTIPLIER

THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL
INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YRS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
5	.20301	.20604	.20909	.21216
10	.10277	.10558	.10843	.11133
15	.06936	.07212	.07494	.07783
20	.05267	.05542	.05825	.06116
25	.04265	.04541	.04826	.05122
30	.03598	.03875	.04164	.04465
35	.03122	.03400	.03693	.04000
40	.02765	.03046	.03343	.03656
45	.02487	.02771	.03072	.03391
50	.02265	.02551	.02857	.03182
55	.02084	.02373	.02683	.03014
60	.01933	.02224	.02539	.02877
65	.01806	.02100	.02419	.02763
70	.01697	.01993	.02317	.02667
75	.01602	.01902	.02230	.02586
80	.01520	.01822	.02155	.02516
85	.01447	.01752	.02089	.02456
90	.01383	.01690	.02032	.02405
95	.01325	.01636	.01982	.02360
100	.01273	.01587	.01937	.02320
105	.01226	.01543	.01897	.02286
110	.01184	.01503	.01862	.02255
115	.01146	.01467	.01830	.02229
120	.01110	.01435	.01802	.02205
125	.01078	.01405	.01776	.02184
130	.01048	.01378	.01753	.02165
135	.01020	.01353	.01732	.02148
140	.00995	.01330	.01713	.02133
145	.00971	.01309	.01696	.02120
150	.00949	.01290	.01680	.02108
155	.00929	.01272	.01666	.02097
160	.00909	.01256	.01653	.02088

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TABLE 9. CAPITAL RECOVERY MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YRS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
5	.21525	.21835	.22148	.22463
10	.11426	.11723	.12024	.12329
15	.08077	.08377	.08683	.08994
20	.06415	.06722	.07036	.07358
25	.05428	.05743	.06067	.06401
30	.04778	.05102	.05437	.05783
35	.04321	.04654	.05000	.05358
40	.03984	.04326	.04683	.05052
45	.03727	.04079	.04445	.04826
50	.03526	.03887	.04263	.04655
55	.03365	.03735	.04121	.04523
60	.03235	.03613	.04009	.04420
65	.03128	.03515	.03919	.04339
70	.03040	.03434	.03846	.04275
75	.02965	.03367	.03787	.04223
80	.02903	.03311	.03738	.04181
85	.02849	.03265	.03699	.04148
90	.02804	.03226	.03666	.04121
95	.02765	.03193	.03639	.04099
100	.02731	.03165	.03616	.04081
105	.02702	.03141	.03597	.04066
110	.02677	.03121	.03581	.04054
115	.02655	.03104	.03568	.04044
120	.02636	.03089	.03557	.04036
125	.02620	.03076	.03548	.04030
130	.02605	.03066	.03540	.04025
135	.02592	.03057	.03534	.04020
140	.02581	.03049	.03529	.04017
145	.02572	.03042	.03524	.04014
150	.02563	.03036	.03520	.04011
155	.02556	.03031	.03517	.04009
160	.02549	.03027	.03514	.04008

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TABLE 9. CAPITAL RECOVERY MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL
INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YRS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
5	.22779	.23097	.23418	.23740
10	.12638	.12950	.13267	.13587
15	.09311	.09634	.09963	.10296
20	.07688	.08024	.08368	.08718
25	.06744	.07095	.07455	.07823
30	.06139	.06505	.06881	.07265
35	.05727	.06107	.06497	.06897
40	.05434	.05828	.06232	.06646
45	.05220	.05626	.06043	.06470
50	.05060	.05478	.05906	.06344
55	.04939	.05367	.05805	.06254
60	.04845	.05283	.05731	.06188
65	.04773	.05219	.05675	.06139
70	.04717	.05170	.05633	.06103
75	.04672	.05132	.05601	.06077
80	.04637	.05103	.05577	.06057
85	.04609	.05080	.05559	.06043
90	.04587	.05063	.05545	.06032
95	.04570	.05049	.05534	.06024
100	.04556	.05038	.05526	.06018
105	.04545	.05030	.05520	.06013
110	.04536	.05023	.05515	.06010
115	.04529	.05018	.05512	.06007
120	.04523	.05014	.05509	.06006
125	.04518	.05011	.05507	.06004
130	.04515	.05009	.05505	.06003
135	.04512	.05007	.05504	.06002
140	.04510	.05005	.05503	.06002
145	.04508	.05004	.05502	.06001
150	.04506	.05003	.05502	.06001
155	.04505	.05003	.05501	.06001
160	.04504	.05002	.05501	.06001

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TABLE 9. CAPITAL RECOVERY MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YRS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
5	.24389	.25046	.25709	.26380
10	.14238	.14903	.15582	.16275
15	.10979	.11683	.12406	.13147
20	.09439	.10185	.10955	.11746
25	.08581	.09368	.10181	.11017
30	.08059	.08883	.09734	.10608
35	.07723	.08580	.09464	.10369
40	.07501	.08386	.09296	.10226
45	.07350	.08259	.09190	.10139
50	.07246	.08174	.09123	.10086
55	.07174	.08118	.09079	.10053
60	.07123	.08080	.09051	.10033
65	.07087	.08054	.09033	.10020
70	.07062	.08037	.09022	.10013
75	.07044	.08025	.09014	.10008
80	.07031	.08017	.09009	.10005
85	.07022	.08012	.09006	.10003
90	.07016	.08008	.09004	.10002
95	.07011	.08005	.09003	.10001
100	.07008	.08004	.09002	.10001
105	.07006	.08002	.09001	.10000
110	.07004	.08002	.09001	.10000
115	.07003	.08001	.09000	.10000
120	.07002	.08001	.09000	.10000
125	.07001	.08001	.09000	.10000
130	.07001	.08000	.09000	.10000
135	.07001	.08000	.09000	.10000
140	.07001	.08000	.09000	.10000
145	.07000	.08000	.09000	.10000
150	.07000	.08000	.09000	.10000
155	.07000	.08000	.09000	.10000
160	.07000	.08000	.09000	.10000

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TABLE 9. CAPITAL RECOVERY MULTIPLIER

THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL
INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YRS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
5	.27057	.27741	.28431	.29128
10	.16980	.17698	.18429	.19171
15	.13907	.14682	.15474	.16281
20	.12558	.13388	.14235	.15099
25	.11874	.12750	.13643	.14550
30	.11502	.12414	.13341	.14280
35	.11293	.12232	.13183	.14144
40	.11172	.12130	.13099	.14075
45	.11101	.12074	.13053	.14039
50	.11060	.12042	.13029	.14020
55	.11035	.12024	.13016	.14010
60	.11021	.12013	.13009	.14005
65	.11012	.12008	.13005	.14003
70	.11007	.12004	.13003	.14001
75	.11004	.12002	.13001	.14001
80	.11003	.12001	.13001	.14000
85	.11002	.12001	.13000	.14000
90	.11001	.12000	.13000	.14000
95	.11001	.12000	.13000	.14000
100	.11000	.12000	.13000	.14000
105	.11000	.12000	.13000	.14000
110	.11000	.12000	.13000	.14000
115	.11000	.12000	.13000	.14000
120	.11000	.12000	.13000	.14000
125	.11000	.12000	.13000	.14000
130	.11000	.12000	.13000	.14000
135	.11000	.12000	.13000	.14000
140	.11000	.12000	.13000	.14000
145	.11000	.12000	.13000	.14000
150	.11000	.12000	.13000	.14000
155	.11000	.12000	.13000	.14000
160	.11000	.12000	.13000	.14000

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TABLE 9. CAPITAL RECOVERY MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL RECOVER ORIGINAL INVESTMENT OF ONE DOLLAR PLUS INTEREST IN N YRS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
5	.29832	.33438	.37185	.41058
10	.19925	.23852	.28007	.32346
15	.17102	.21388	.25912	.30598
20	.15976	.20536	.25292	.30159
25	.15470	.20212	.25095	.30043
30	.15230	.20085	.25031	.30011
35	.15113	.20034	.25010	.30003
40	.15056	.20014	.25003	.30001
45	.15028	.20005	.25001	.30000
50	.15014	.20002	.25000	.30000
55	.15007	.20001	.25000	.30000
60	.15003	.20000	.25000	.30000
65	.15002	.20000	.25000	.30000
70	.15001	.20000	.25000	.30000
75	.15000	.20000	.25000	.30000
80	.15000	.20000	.25000	.30000
85	.15000	.20000	.25000	.30000
90	.15000	.20000	.25000	.30000
95	.15000	.20000	.25000	.30000
100	.15000	.20000	.25000	.30000
105	.15000	.20000	.25000	.30000
110	.15000	.20000	.25000	.30000
115	.15000	.20000	.25000	.30000
120	.15000	.20000	.25000	.30000
125	.15000	.20000	.25000	.30000
130	.15000	.20000	.25000	.30000
135	.15000	.20000	.25000	.30000
140	.15000	.20000	.25000	.30000
145	.15000	.20000	.25000	.30000
150	.15000	.20000	.25000	.30000
155	.15000	.20000	.25000	.30000
160	.15000	.20000	.25000	.30000

Table 10.--Sinking Fund Multiplier

The annual payment which will accumulate with
interest to \$1 in n years.

$$\frac{i}{(1+i)^n - 1}$$

This multiplier is used to determine the annual payment (a) which will accumulate with interest to the original investment (V_o) in n years.

To find the annual payment, multiply the original investment by the multiplier:

$$a = V_o \left[\frac{i}{(1+i)^n - 1} \right]$$

Example: Determine how much should be set aside annually to provide for the replacement of a \$10,000 truck at the end of 5 years, the annual payments earning 7-percent compound interest.

Initial investment: V_o = \$10,000

Interest rate: $i = .07$

Years: $n = 5$

Multiplier: = 0.17389 (page 134)

Annual payment: a = \$10,000 (0.17389) = \$1,738.90 per year

Notes

TABLE 10. SINKING FUND MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH
INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
1	1.00000	1.00000	1.00000	1.00000
2	.49875	.49751	.49628	.49505
3	.33167	.33002	.32838	.32675
4	.24813	.24628	.24444	.24262
5	.19801	.19604	.19409	.19216
6	.16460	.16255	.16053	.15853
7	.14073	.13863	.13656	.13451
8	.12283	.12069	.11858	.11651
9	.10891	.10674	.10461	.10252
10	.09777	.09558	.09343	.09133
11	.08866	.08645	.08429	.08218
12	.08107	.07885	.07668	.07456
13	.07464	.07241	.07024	.06812
14	.06914	.06690	.06472	.06260
15	.06436	.06212	.05994	.05783
16	.06019	.05794	.05577	.05365
17	.05651	.05426	.05208	.04997
18	.05323	.05098	.04881	.04670
19	.05030	.04805	.04588	.04378
20	.04767	.04542	.04325	.04116
21	.04528	.04303	.04087	.03878
22	.04311	.04086	.03870	.03663
23	.04113	.03889	.03673	.03467
24	.03932	.03707	.03492	.03287
25	.03765	.03541	.03326	.03122
26	.03611	.03387	.03173	.02970
27	.03469	.03245	.03032	.02829
28	.03336	.03112	.02900	.02699
29	.03213	.02990	.02778	.02578
30	.03098	.02875	.02664	.02465
31	.02990	.02768	.02557	.02360
32	.02889	.02667	.02458	.02261
33	.02795	.02573	.02364	.02169
34	.02706	.02484	.02276	.02082
35	.02622	.02400	.02193	.02000
36	.02542	.02321	.02115	.01923
37	.02467	.02247	.02041	.01851
38	.02396	.02176	.01972	.01782
39	.02329	.02109	.01905	.01717
40	.02265	.02046	.01843	.01656

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TABLE 10. SINKING FUND MULTIPLIER

THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
1	1.00000	1.00000	1.00000	1.00000
2	.49383	.49261	.49140	.49020
3	.32514	.32353	.32193	.32035
4	.24082	.23903	.23725	.23549
5	.19025	.18835	.18648	.18463
6	.15655	.15460	.15267	.15076
7	.13250	.13051	.12854	.12661
8	.11447	.11246	.11048	.10853
9	.10046	.09843	.09645	.09449
10	.08926	.08723	.08524	.08329
11	.08011	.07808	.07609	.07415
12	.07249	.07046	.06848	.06655
13	.06605	.06403	.06206	.06014
14	.06054	.05853	.05657	.05467
15	.05577	.05377	.05183	.04994
16	.05160	.04961	.04768	.04582
17	.04793	.04595	.04404	.04220
18	.04467	.04271	.04082	.03899
19	.04176	.03981	.03794	.03614
20	.03915	.03722	.03536	.03358
21	.03679	.03487	.03304	.03128
22	.03465	.03275	.03093	.02920
23	.03270	.03081	.02902	.02731
24	.03091	.02905	.02727	.02559
25	.02928	.02743	.02567	.02401
26	.02777	.02594	.02421	.02257
27	.02638	.02456	.02285	.02124
28	.02509	.02329	.02160	.02001
29	.02389	.02211	.02045	.01888
30	.02278	.02102	.01937	.01783
31	.02174	.02000	.01837	.01686
32	.02077	.01905	.01744	.01595
33	.01986	.01816	.01657	.01510
34	.01901	.01732	.01576	.01431
35	.01821	.01654	.01500	.01358
36	.01745	.01580	.01428	.01289
37	.01674	.01511	.01361	.01224
38	.01607	.01446	.01298	.01163
39	.01544	.01384	.01239	.01106
40	.01484	.01326	.01183	.01052

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TABLE 10. SINKING FUND MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
1	1.00000	1.00000	1.00000	1.00000
2	.48900	.48780	.48662	.48544
3	.31877	.31721	.31565	.31411
4	.23374	.23201	.23029	.22859
5	.18279	.18097	.17918	.17740
6	.14888	.14702	.14518	.14336
7	.12470	.12282	.12096	.11914
8	.10661	.10472	.10286	.10104
9	.09257	.09069	.08884	.08702
10	.08138	.07950	.07767	.07587
11	.07225	.07039	.06857	.06679
12	.06467	.06283	.06103	.05928
13	.05828	.05646	.05468	.05296
14	.05282	.05102	.04928	.04758
15	.04811	.04634	.04463	.04296
16	.04402	.04227	.04058	.03895
17	.04042	.03870	.03704	.03544
18	.03724	.03555	.03392	.03236
19	.03441	.03275	.03115	.02962
20	.03188	.03024	.02868	.02718
21	.02960	.02800	.02646	.02500
22	.02755	.02597	.02447	.02305
23	.02568	.02414	.02267	.02128
24	.02399	.02247	.02104	.01968
25	.02244	.02095	.01955	.01823
26	.02102	.01956	.01819	.01690
27	.01972	.01829	.01695	.01570
28	.01852	.01712	.01581	.01459
29	.01741	.01605	.01477	.01358
30	.01639	.01505	.01381	.01265
31	.01544	.01413	.01292	.01179
32	.01456	.01328	.01210	.01100
33	.01374	.01249	.01133	.01027
34	.01298	.01176	.01063	.00960
35	.01227	.01107	.00997	.00897
36	.01161	.01043	.00937	.00839
37	.01098	.00984	.00880	.00786
38	.01040	.00928	.00827	.00736
39	.00986	.00876	.00778	.00689
40	.00934	.00828	.00732	.00646

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TABLE 10. SINKING FUND MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
1	1.00000	1.00000	1.00000	1.00000
2	.48309	.48077	.47847	.47619
3	.31105	.30803	.30505	.30211
4	.22523	.22192	.21867	.21547
5	.17389	.17046	.16709	.16380
6	.13980	.13632	.13292	.12961
7	.11555	.11207	.10869	.10541
8	.09747	.09401	.09067	.08744
9	.08349	.08008	.07680	.07364
10	.07238	.06903	.06582	.06275
11	.06336	.06008	.05695	.05396
12	.05590	.05270	.04965	.04676
13	.04965	.04652	.04357	.04078
14	.04434	.04130	.03843	.03575
15	.03979	.03683	.03406	.03147
16	.03586	.03298	.03030	.02782
17	.03243	.02963	.02705	.02466
18	.02941	.02670	.02421	.02193
19	.02675	.02413	.02173	.01955
20	.02439	.02185	.01955	.01746
21	.02229	.01983	.01762	.01562
22	.02041	.01803	.01590	.01401
23	.01871	.01642	.01438	.01257
24	.01719	.01498	.01302	.01130
25	.01581	.01368	.01181	.01017
26	.01456	.01251	.01072	.00916
27	.01343	.01145	.00973	.00826
28	.01239	.01049	.00885	.00745
29	.01145	.00962	.00806	.00673
30	.01059	.00883	.00734	.00608
31	.00980	.00811	.00669	.00550
32	.00907	.00745	.00610	.00497
33	.00841	.00685	.00556	.00450
34	.00780	.00630	.00508	.00407
35	.00723	.00580	.00464	.00369
36	.00672	.00534	.00424	.00334
37	.00624	.00492	.00387	.00303
38	.00580	.00454	.00354	.00275
39	.00539	.00419	.00324	.00249
40	.00501	.00386	.00296	.00226

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TABLE 10. SINKING FUND MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
1	1.00000	1.00000	1.00000	1.00000
2	.47393	.47170	.46948	.46729
3	.29921	.29635	.29352	.29073
4	.21233	.20923	.20619	.20320
5	.16057	.15741	.15431	.15128
6	.12638	.12323	.12015	.11716
7	.10222	.09912	.09611	.09319
8	.08432	.08130	.07839	.07557
9	.07060	.06768	.06487	.06217
10	.05980	.05698	.05429	.05171
11	.05112	.04842	.04584	.04339
12	.04403	.04144	.03899	.03667
13	.03815	.03568	.03335	.03116
14	.03323	.03087	.02867	.02661
15	.02907	.02682	.02474	.02281
16	.02552	.02339	.02143	.01962
17	.02247	.02046	.01861	.01692
18	.01984	.01794	.01620	.01462
19	.01756	.01576	.01413	.01266
20	.01558	.01388	.01235	.01099
21	.01384	.01224	.01081	.00954
22	.01231	.01081	.00948	.00830
23	.01097	.00956	.00832	.00723
24	.00979	.00846	.00731	.00630
25	.00874	.00750	.00643	.00550
26	.00781	.00665	.00565	.00480
27	.00699	.00590	.00498	.00419
28	.00626	.00524	.00439	.00366
29	.00561	.00466	.00387	.00320
30	.00502	.00414	.00341	.00280
31	.00451	.00369	.00301	.00245
32	.00404	.00328	.00266	.00215
33	.00363	.00292	.00234	.00188
34	.00326	.00260	.00207	.00165
35	.00293	.00232	.00183	.00144
36	.00263	.00206	.00162	.00126
37	.00236	.00184	.00143	.00111
38	.00213	.00164	.00126	.00097
39	.00191	.00146	.00112	.00085
40	.00172	.00130	.00099	.00075

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TABLE 10. SINKING FUND MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.150	.200	.250	.300
1	1.00000	1.00000	1.00000	1.00000
2	.46512	.45455	.44444	.43478
3	.28798	.27473	.26230	.25063
4	.20027	.18629	.17344	.16163
5	.14832	.13438	.12185	.11058
6	.11424	.10071	.08882	.07839
7	.09036	.07742	.06634	.05687
8	.07285	.06061	.05040	.04192
9	.05957	.04808	.03876	.03124
10	.04925	.03852	.03007	.02346
11	.04107	.03110	.02349	.01773
12	.03448	.02526	.01845	.01345
13	.02911	.02062	.01454	.01024
14	.02469	.01689	.01150	.00782
15	.02102	.01388	.00912	.00598
16	.01795	.01144	.00724	.00458
17	.01537	.00944	.00576	.00351
18	.01319	.00781	.00459	.00269
19	.01134	.00646	.00366	.00207
20	.00976	.00536	.00292	.00159
21	.00842	.00444	.00233	.00122
22	.00727	.00369	.00186	.00094
23	.00628	.00307	.00148	.00072
24	.00543	.00255	.00119	.00055
25	.00470	.00212	.00095	.00043
26	.00407	.00176	.00076	.00033
27	.00353	.00147	.00061	.00025
28	.00306	.00122	.00048	.00019
29	.00265	.00102	.00039	.00015
30	.00230	.00085	.00031	.00011
31	.00200	.00070	.00025	.00009
32	.00173	.00059	.00020	.00007
33	.00150	.00049	.00016	.00005
34	.00131	.00041	.00013	.00004
35	.00113	.00034	.00010	.00003
36	.00099	.00028	.00008	.00002
37	.00086	.00024	.00006	.00002
38	.00074	.00020	.00005	.00001
39	.00065	.00016	.00004	.00001
40	.00056	.00014	.00003	.00001

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TABLE 10. SINKING FUND MULTIPLIER

THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.005	.010	.015	.020
5	.19801	.19604	.19409	.19216
10	.09777	.09558	.09343	.09133
15	.06436	.06212	.05994	.05783
20	.04767	.04542	.04325	.04116
25	.03765	.03541	.03326	.03122
30	.03098	.02875	.02664	.02465
35	.02622	.02400	.02193	.02000
40	.02265	.02046	.01843	.01656
45	.01987	.01771	.01572	.01391
50	.01765	.01551	.01357	.01182
55	.01584	.01373	.01183	.01014
60	.01433	.01224	.01039	.00877
65	.01306	.01100	.00919	.00763
70	.01197	.00993	.00817	.00667
75	.01102	.00902	.00730	.00586
80	.01020	.00822	.00655	.00516
85	.00947	.00752	.00589	.00456
90	.00883	.00690	.00532	.00405
95	.00825	.00636	.00482	.00360
100	.00773	.00587	.00437	.00320
105	.00726	.00543	.00397	.00286
110	.00684	.00503	.00362	.00255
115	.00646	.00467	.00330	.00229
120	.00610	.00435	.00302	.00205
125	.00578	.00405	.00276	.00184
130	.00548	.00378	.00253	.00165
135	.00520	.00353	.00232	.00148
140	.00495	.00330	.00213	.00133
145	.00471	.00309	.00196	.00120
150	.00449	.00290	.00180	.00108
155	.00429	.00272	.00166	.00097
160	.00409	.00256	.00153	.00088

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TABLE 10. SINKING FUND MULTIPLIER

THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH
INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.025	.030	.035	.040
5	.19025	.18835	.18648	.18463
10	.08926	.08723	.08524	.08329
15	.05577	.05377	.05183	.04994
20	.03915	.03722	.03536	.03358
25	.02928	.02743	.02567	.02401
30	.02278	.02102	.01937	.01783
35	.01821	.01654	.01500	.01358
40	.01484	.01326	.01183	.01052
45	.01227	.01079	.00945	.00826
50	.01026	.00887	.00763	.00655
55	.00865	.00735	.00621	.00523
60	.00735	.00613	.00509	.00420
65	.00628	.00515	.00419	.00339
70	.00540	.00434	.00346	.00275
75	.00465	.00367	.00287	.00223
80	.00403	.00311	.00238	.00181
85	.00349	.00265	.00199	.00148
90	.00304	.00226	.00166	.00121
95	.00265	.00193	.00139	.00099
100	.00231	.00165	.00116	.00081
105	.00202	.00141	.00097	.00066
110	.00177	.00121	.00081	.00054
115	.00155	.00104	.00068	.00044
120	.00136	.00089	.00057	.00036
125	.00120	.00076	.00048	.00030
130	.00105	.00066	.00040	.00025
135	.00092	.00057	.00034	.00020
140	.00081	.00049	.00029	.00017
145	.00072	.00042	.00024	.00014
150	.00063	.00036	.00020	.00011
155	.00056	.00031	.00017	.00009
160	.00049	.00027	.00014	.00008

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TABLE 10. SINKING FUND MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.045	.050	.055	.060
5	.18279	.18097	.17918	.17740
10	.08138	.07950	.07767	.07587
15	.04811	.04634	.04463	.04296
20	.03188	.03024	.02868	.02718
25	.02244	.02095	.01955	.01823
30	.01639	.01505	.01381	.01265
35	.01227	.01107	.00997	.00897
40	.00934	.00828	.00732	.00646
45	.00720	.00626	.00543	.00470
50	.00560	.00478	.00406	.00344
55	.00439	.00367	.00305	.00254
60	.00345	.00283	.00231	.00188
65	.00273	.00219	.00175	.00139
70	.00217	.00170	.00133	.00103
75	.00172	.00132	.00101	.00077
80	.00137	.00103	.00077	.00057
85	.00109	.00080	.00059	.00043
90	.00087	.00063	.00045	.00032
95	.00070	.00049	.00034	.00024
100	.00056	.00038	.00026	.00018
105	.00045	.00030	.00020	.00013
110	.00036	.00023	.00015	.00010
115	.00029	.00018	.00012	.00007
120	.00023	.00014	.00009	.00006
125	.00018	.00011	.00007	.00004
130	.00015	.00009	.00005	.00003
135	.00012	.00007	.00004	.00002
140	.00010	.00005	.00003	.00002
145	.00008	.00004	.00002	.00001
150	.00006	.00003	.00002	.00001
155	.00005	.00003	.00001	.00001
160	.00004	.00002	.00001	.00001

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TABLE 10. SINKING FUND MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.070	.080	.090	.100
5	.17389	.17046	.16709	.16380
10	.07238	.06903	.06582	.06275
15	.03979	.03683	.03406	.03147
20	.02439	.02185	.01955	.01746
25	.01581	.01368	.01181	.01017
30	.01059	.00883	.00734	.00608
35	.00723	.00580	.00464	.00369
40	.00501	.00386	.00296	.00226
45	.00350	.00259	.00190	.00139
50	.00246	.00174	.00123	.00086
55	.00174	.00118	.00079	.00053
60	.00123	.00080	.00051	.00033
65	.00087	.00054	.00033	.00020
70	.00062	.00037	.00022	.00013
75	.00044	.00025	.00014	.00008
80	.00031	.00017	.00009	.00005
85	.00022	.00012	.00006	.00003
90	.00016	.00008	.00004	.00002
95	.00011	.00005	.00003	.00001
100	.00008	.00004	.00002	.00001
105	.00006	.00002	.00001	.00000
110	.00004	.00002	.00001	.00000
115	.00003	.00001	.00000	.00000
120	.00002	.00001	.00000	.00000
125	.00001	.00001	.00000	.00000
130	.00001	.00000	.00000	.00000
135	.00001	.00000	.00000	.00000
140	.00001	.00000	.00000	.00000
145	.00000	.00000	.00000	.00000
150	.00000	.00000	.00000	.00000
155	.00000	.00000	.00000	.00000
160	.00000	.00000	.00000	.00000

TABLE 10. SINKING FUND MULTIPLIER
THE ANNUAL PAYMENT WHICH WILL ACCUMULATE WITH INTEREST TO ONE DOLLAR IN N YEARS

	RATE OF INTEREST			
YEARS	.110	.120	.130	.140
5	.16057	.15741	.15431	.15128
10	.05980	.05698	.05429	.05171
15	.02907	.02682	.02474	.02281
20	.01558	.01388	.01235	.01099
25	.00874	.00750	.00643	.00550
30	.00502	.00414	.00341	.00280
35	.00293	.00232	.00183	.00144
40	.00172	.00130	.00099	.00075
45	.00101	.00074	.00053	.00039
50	.00060	.00042	.00029	.00020
55	.00035	.00024	.00016	.00010
60	.00021	.00013	.00009	.00005
65	.00012	.00008	.00005	.00003
70	.00007	.00004	.00003	.00001
75	.00004	.00002	.00001	.00001
80	.00003	.00001	.00001	.00000
85	.00002	.00001	.00000	.00000
90	.00001	.00000	.00000	.00000
95	.00001	.00000	.00000	.00000
100	.00000	.00000	.00000	.00000
105	.00000	.00000	.00000	.00000
110	.00000	.00000	.00000	.00000
115	.00000	.00000	.00000	.00000
120	.00000	.00000	.00000	.00000
125	.00000	.00000	.00000	.00000
130	.00000	.00000	.00000	.00000
135	.00000	.00000	.00000	.00000
140	.00000	.00000	.00000	.00000
145	.00000	.00000	.00000	.00000
150	.00000	.00000	.00000	.00000
155	.00000	.00000	.00000	.00000
160	.00000	.00000	.00000	.00000

ABOUT THE FOREST SERVICE .

As our Nation grows, people expect and need more from their forests— more wood, more water, fish, and wildlife; more recreation and natural beauty, more special forest products and forage. The Forest Service of the U.S. Department of Agriculture helps to fulfill these expectations and needs through three major activities:

- Conducting forest and range research at over 75 locations ranging from Puerto Rico to Alaska to Hawaii.
- Participating with all State forestry agencies in cooperative programs to protect, improve, and wisely use our Country's 395 million acres of State, local, and private forest lands.
- Managing and protecting the 187-million acre National Forest System.

The Forest Service does this by encouraging use of the new knowledge that research scientists develop; by setting an example in managing, under sustained yield, the National Forests and Grasslands for multiple use purposes, and by cooperating with all States and with private citizens in their efforts to achieve better management, protection, and use of forest resources.

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USDA FOREST SERVICE
RESEARCH PAPER NC-52
1971

Ecological Studies of the TIMBER WOLF in Northeastern Minnesota

NORTH CENTRAL FOREST EXPERIMENT STATION
FOREST SERVICE
U. S. DEPARTMENT OF AGRICULTURE

FOREWORD

The largest population of timber wolves remaining in the United States (excluding Alaska) lives in northern Minnesota. Many of these wolves inhabit the Superior National Forest, so protecting the habitat of this endangered species is largely a Forest Service responsibility.

As the "Age of Ecology" broadens into the 1970's, wolves and wolf habitat will become a subject of concerted research. Forest land managers will have to know more about how the timber wolf fits into a forest system. Building on nearly 50 years of research in northern forests, we at the North Central Station intend to expand our studies of wildlife habitat. We are happy to publish the enclosed papers as one step in this direction.

D. B. King, Director

North Central Forest Experiment Station
D. B. King, Director
Forest Service — U.S. Department of Agriculture
Folwell Avenue
St. Paul, Minnesota 55101

ECOLOGICAL STUDIES OF THE TIMBER WOLF IN NORTHEASTERN MINNESOTA

L. David Mech and L. D. Frenzel, Jr. (Editors)

CONTENTS

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MOVEMENTS, BEHAVIOR, AND ECOLOGY OF TIMBER WOLVES IN NORTHEASTERN MINNESOTA

L. David Mech, L. D. Frenzel, Jr.,
Robert R. Ream, and John W. Winship

The largest population of wolves (*Canis* *pus*) remaining today in the continental United tates outside of Alaska is in northern Minne- ta. As of mid-1970 this population was not gally protected, and the species, which once nged over almost all of North America, is now nsidered by the U.S. Department of the In- rior to be in danger of extinction in the contig- ous 48 States. Until the present research, the ly field studies of Minnesota wolves were those Olson (1938 a, b) and Stenlund (1955). Those vestigations provided much useful general in- rmation about Minnesota wolves and gave the esent authors an excellent background with hich to begin more detailed investigations.

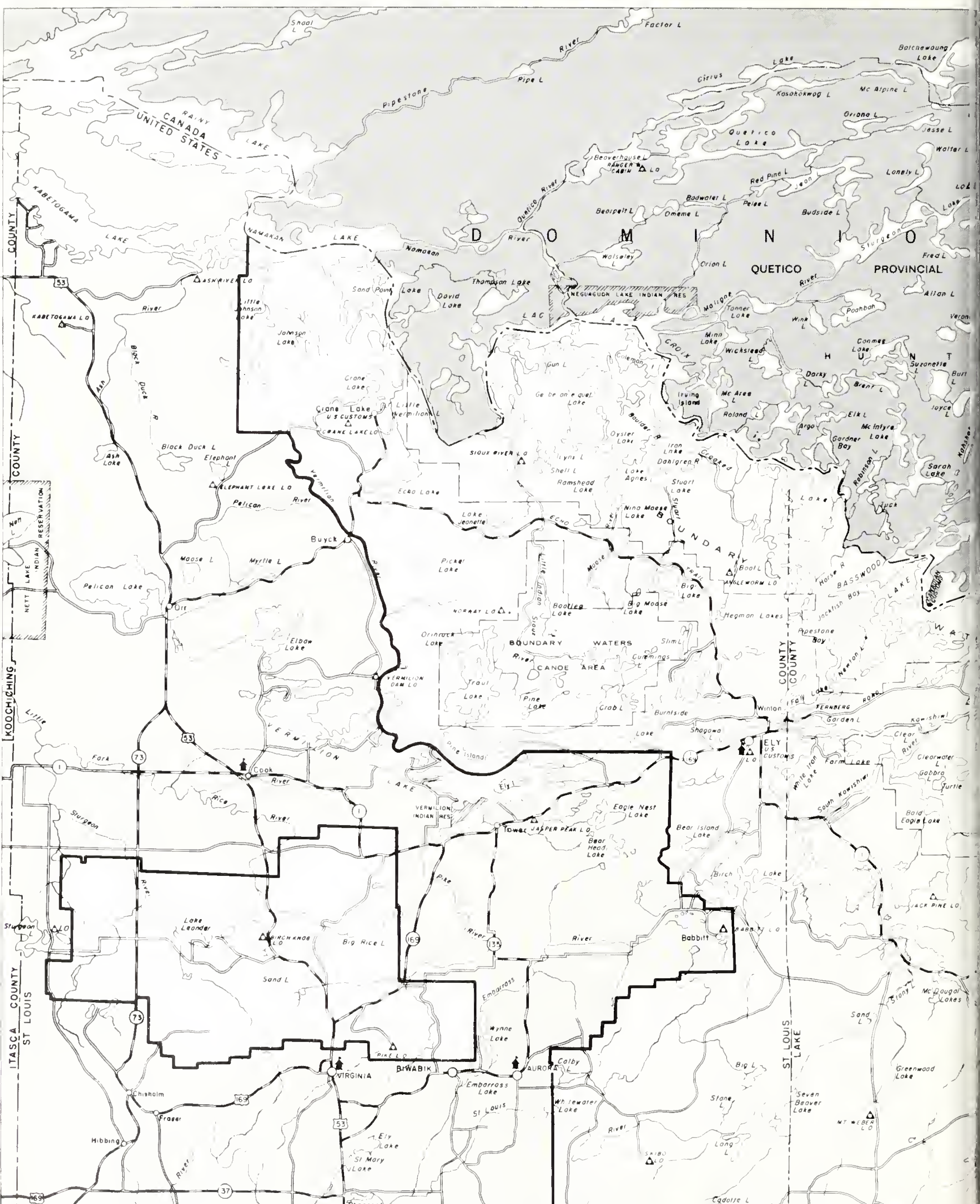
This paper reports on the basic aspects of a ries of studies that began in 1964, and concen- ates primarily on wolf movements and activity, cial behavior, hunting behavior, and population ganization. Most of the data were collected uring January, February, and March 1967; Feb- ary, November, and December 1968; and Jan- ary through August 1969. A total of 192 days as spent in the field.

According to a distribution map of wolf sub- pecies (Goldman 1944), the race of wolves in ur study area is *Canis lupus lycaon*. However, vidence presented by Mech and Frenzel (see age 60) suggests that there may be strong nfluence by *C. 1. nubilus*, a more western race f wolf formerly thought to be extinct (Goldman 944).

Between 1965 and the present, wolves in the udy area were neither protected nor bountied, nd the influence of trapping and hunting is hought to have been negligible.

THE STUDY AREA

This study was conducted in the Superior National Forest (fig. 1) in northern St. Louis, Lake, and Cook Counties of northeastern Minnesota (92° west longitude, 48° north latitude), an area well described by Stenlund (1955). Most of the data were collected from within and immediately south of the Boundary Waters Canoe Area, a special wilderness region in which travel by motorized vehicles is restricted. The total study area encompasses approximately 1.5 million acres, and numerous lakes and rivers comprise about 15 percent of this area (fig. 2). The topography varies from large stretches of swamps to rocky ridges, with altitudes ranging from 1,000 to 2,300 feet above sea level (fig. 3). Winter temperatures lower than -30° F. are not unusual, and snow depths generally range from 20 to 30 inches on the level. However, an important exception occurred in early 1969 when depths of 45 inches and more accumulated in much of the area. Further details on snow conditions in the study area during the period of this investigation are given by Mech *et al.* (see page 51). Conifers predominate in the forest overstory, with the following species present: jack pine (*Pinus banksiana* Lamb.), white pine (*P. strobus* L.), red pine (*P. resinosa* Ait.), black spruce (*Picea mariana* (Mill.) B.S.P.), white spruce (*P. glauca* (Moench) Voss), balsam fir (*Abies balsamea* (L.) Mill.), white cedar (*Thuja occidentalis* L.), and tamarack (*Larix laricina* (DuRoi) K. Koch). However, as a result of extensive cutting and fires much of the conifer cover is interspersed with large stands of white birch (*Betula papyrifera* Marsh.) and aspen (*Populus tremuloides* Michx.). Detailed descriptions of the forest vegetation were presented by Ohmann and Ream (1969).



Shoal L
Factor L
River
Pipe L
Pipestone
Batchewaung Lake
Cirrus Lake
Kasakokwog L
Mc Alpine L
Oriana L
Jesse L
Walter L
Quetico Lake
RAINY
CANADA
UNITED STATES
LAKE
Beaverhouse L
RANGER CABIN
LO
Red Pine L
Jean
Lonely L
KABETOGAMA
LAKE
Quetico River
Badwater L
Pelee L
Bearpelt L
Omeme L
Budside L
NAMAKAN
LAKE
D O M I N I O
River
Namakan
Walsaley L
Orion L
Sturgeon
QUETICO
PROVINCIAL
Fred L
COUNTY
53
ASH RIVER LO
Thompson Lake
River
Allan L
NEGUAGUON LAKE INDIAN RES
KABETOGAMA LO
Sand Point Lake
David Lake
Maligne
Tanner Lake
Wink L
Poohbah
LAC
LA
CROIX
Veron
River
Little Johnson Lake
Johnson Lake
Minn Lake
Wicksteed L
Conmee Lake
Black Duck R
Gun L
Coleman
H
U
N
T
Suzanette L
Burt L
Ash
Crane Lake
Crane Lake
US CUSTOMS
CRANE LAKE LO
Little Vermilion L
Ge be on e quet Lake
Darky L
Brent L
Irving Island
Mc Aree L
Roland L
Elk L
Mc Intyre Lake
Joyce L
Argo
Gardner Bay
Oyster Lake
Boulder R
Iron Lake
Dahlgren R
Crooked
Robinson L
Kahshahpiwi
Black Duck L
Elephant L
Ash Lake
SIOUX RIVER LO
Lynx L
Shell L
Ramshead Lake
Lake Agnes
Stuart Lake
Sarah Lake
COUNTY
ELEPHANT LAKE LO
Vermilion
Echo Lake
Lake
Tuck
Pelican
River
Nina Moose Lake
BOUNDARY
Lake Jeanette
ECHO
Stuart
Buyck
Picker Lake
Moose
Little Indian Sioux
TRAIL
Boot L
Horse R
BASSWOOD
Moose L
Myrtle L
ANGLEWORM LO
CANADIAN CUSTOMS
Pelican Lake
Big Lake
LAKE
Bootleg Lake
Big Moose Lake
Hegmon Lakes
Jackfish Bay
Orr
NORWAY LO
Pipestone Bay
Sioux
Newton L
WAT
NETT LAKE INDIAN RESERVATION
Elbow Lake
Orinrock Lake
BOUNDARY
WATERS
Slim L
River
CANOE
AREA
Cummings L
COUNTY
COUNTY
Fall Lake
Little
KOOCHICHING
VERMILION DAM LO
Trout Lake
Pine Lake
Crab L
Winton
FERNBERG
ROAD
Garden L
Kawishiwi
VERMILION
Burntside
Lake
Shagawa L
ELY
US CUSTOMS
LO
Clear L
River
Clearwater L
Gabbro L
Turtle
1
Fork
73
53
Pine Island
Cook
169
Rice
River
LAKE
Ely
White Iron Lake
Farm Lake
South Kawishiwi
River
Sturgeon
River
Eagle Nest Lake
VERMILION INDIAN RES
1
Tower
JASPER PEAK LO
Bear Head Lake
Bear Island Lake
Bald Eagle Lake
Pike
Birch
Lake
1
River
JACK PINE LO
Sturgeon L
LO
Lake Leander
BIRCH KNOB LO
Big Rice L
169
River
135
River
Babbitt
BABBITT LO
Sand L
Embarrass
Stony L
Mc Dougal Lakes
Wynne Lake
Sand L
ITASCA COUNTY
ST LOUIS
73
PINE LO
Colby L
VIRGINIA
BIWABIK
AURORA
Big L
Greenwood Lake
Chisholm
169
Embarrass Lake
Whitewater Lake
Stone L
ST LOUIS
LAKE
Seven Beaver Lake
Fraser
St Louis
53
River
MT WEBER LO
Hibbing
Ely Lake
St Mary Lake
SKIBO LO
Long L
River
169
37
Pheasant L
Cadotte L
Bassett Lake
River
Anchor L
Murphy L
Long L
Linwood L
Swan
73
St Louis
Stone L
WHITEFACE RIVER RESERVOIR
DULUTH

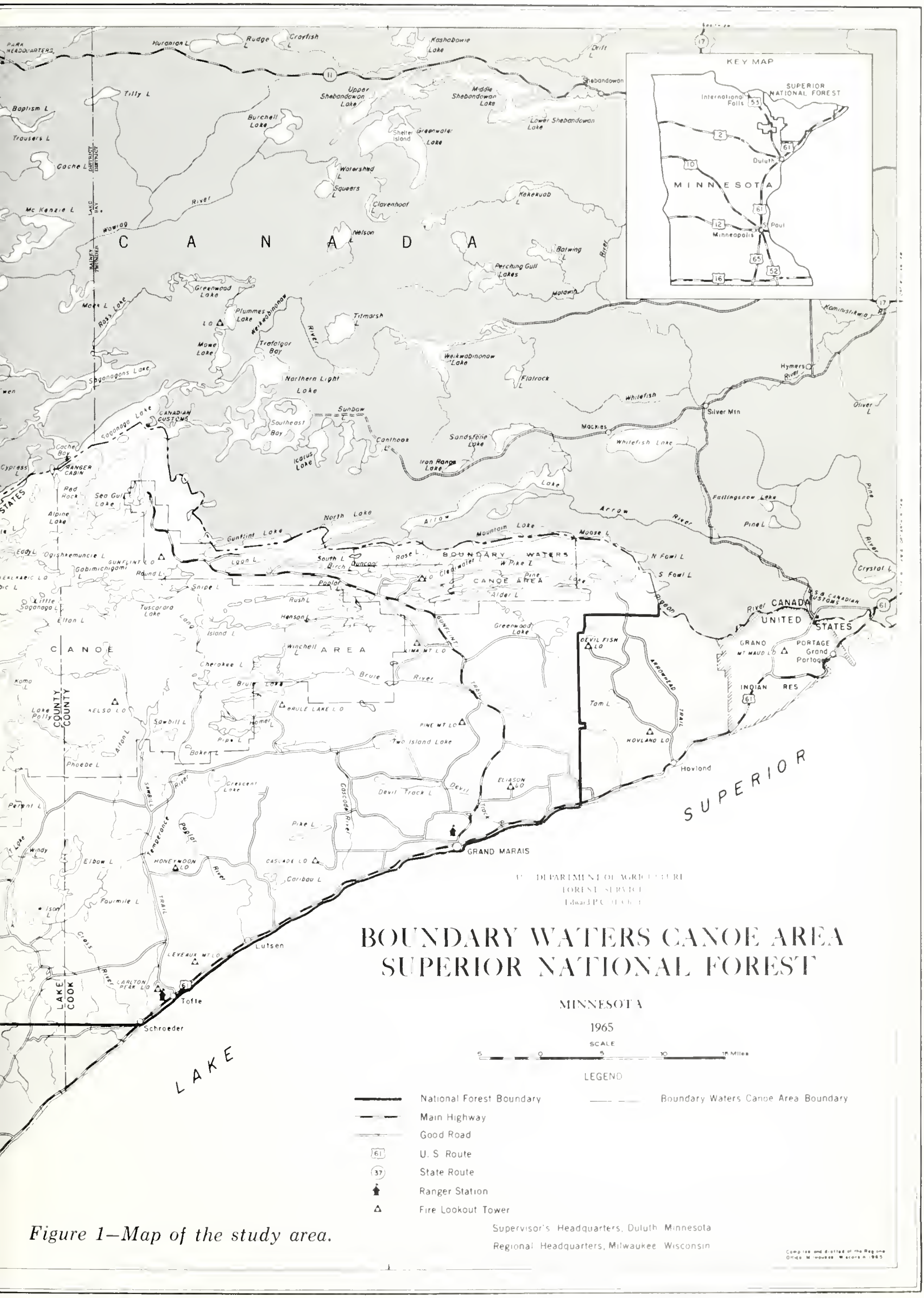


Figure 1—Map of the study area.

Figure 2.—Lakes are common throughout most of the study area. (Photo courtesy of L. D. Mech.)

Figure 3.—Ridges, islands, swamps, and bays are part of the variable topography in the Superior National Forest. (Photo courtesy of L. D. Mech.)

METHODS

The observations discussed in this paper were l made from aircraft, the method of flying being at reported by Burkholder (1959) and Mech 1966a). The following aircraft were used (in der of size): Aeronca Champ,[1] Supercub, Cess- a 172, Cessna 180, and Cessna 206. The smaller rcraft were excellent for holding in tight circles uring observations but had the disadvantage being slow and cold; the larger planes could ver the study area much more quickly and ere more comfortable, but were not as maneu- erable during observations. For radiotracking, be discussed below, the best compromise emed to be a Cessna 172.

To make observations of wolves, we flew over ozen waterways until tracks were found, and en followed the tracks until we lost them or w the wolves (fig. 4). Several times we located olves directly just by scanning the lakes. How- ver, because there seemed to be a number of acks in the area, and because most wolves were e same color (with the exception of a few black or white individuals (see Mech and Frenzel, page 60), it usually was not possible to follow packs from one day to the next and be certain of identification. Moreover, it was impossible to locate any pack at will because most wolves also spent much time inland.

Therefore, to facilitate our observations and to obtain data on wolf movements and extent of range, we began a radiotracking program in 1968-69. A professional trapper, Robert Himes, was employed to capture the wolves. Using Newhouse No. 4 and 14 steel traps at scent-post sets, he caught two wolves, and captured another with a live-snare similar to that used by Nellis (1968); the senior author trapped two additional wolves (fig. 5).

The four wolves held in steel traps were restrained by a choker (fig. 6), and then anesthetized by intramuscular injections (fig. 7A, B) of a combination of 30 mg. of phencyclidine hydrochloride (Sernylan, Parke-Davis Co.) and 25 mg. promazine hydrochloride (Sparine, Wyeth Laboratories) as prescribed by Seal and Erickson (1969); these drugs proved most satisfactory.

The fifth wolf (a female), which was captured around the chest by the live-snare, was handled without drugs. A forked stick was used to hold down her head (Kolenosky and Johnston

Figure 4.—An important technique used in the study involved aerial tracking and observing of wolf packs. (Photo courtesy of L. D. Frenzel.)

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[1] *Mention of trade names does not constitute ndorsement by the USDA Forest Service.*

Figure 5.—A wolf caught in a trap. (Photo courtesy of D. L. Breneman.)

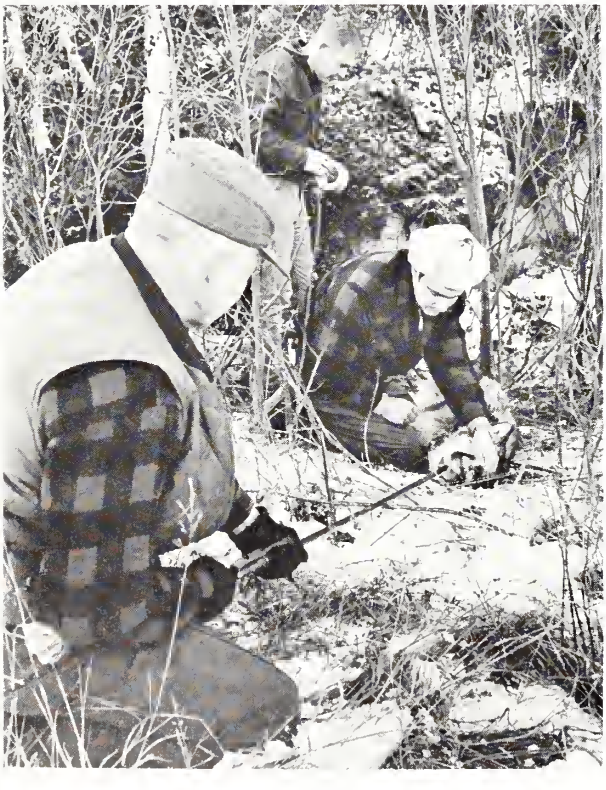

Figure 6.—A choker was used to restrain wolves caught in traps. (Photo courtesy of D. L. Breneman.)

Figure 7.—A. A small hypodermic syringe is loaded with drugs. B. The loaded syringe is used on the end of a pole. (Photos courtesy of D. L. Breneman.)

1967), and she offered no resistance (fig. 8). Evidently she went into shock or some other psychophysiological state of unconsciousness, for after her release she remained on her side and did not move for 1.5 hours, despite our prodding during the first few minutes (fig. 9). Then suddenly she leaped up and ran off.

Each wolf was examined, outfitted with a radio transmitter collar 15 inches inside circumference (fig. 10) and tagged with identification numbers in both ears (fig. 11). Each transmitter was of a different frequency in the 150 MH_Z range, emitted a pulsed signal ranging from 75 to 350 pulses per minute, and had a calculated

gure 8.–Once pinned by the forked stick, the wolf ceased struggling. (Photo courtesy of Richard Bend.)

Figure 10.–A radio transmitter collar was placed around the neck of each trapped wolf. (Photo courtesy of D. L. Breneman.)

gure 9–After release, the wolf lay still for 1½ hours before jumping up and running off. (Photo courtesy of L. D. Mech.)

Figure 11.–Each ear of the wolf was tagged with identifying numbers. (Photo courtesy of Richard Bend.)

life of at least 300 days (fig. 12). Two types of 12-inch whip antennas were used on the transmitters: one type extended up the side of the collar and then stuck out above for 6 inches; the other was fully attached inside the collar and extended up one side, around the top, and partly down the other side. The transmitter, batteries, and antenna were molded into a collar of acrylic weighing 11 ounces (Mech *et al.* 1965).[2] All radio equipment functioned flawlessly for at least 5 months, and one transmitter continued operating for at least 9 months.

Figure 12.—Each radio collar had a different frequency tuned to special receivers, which allowed each wolf to be identified. (Photo courtesy of D. L. Breneman.)

For tracking radio-equipped wolves, a directional yagi antenna (fig. 13) was attached to each of the wing struts of an aircraft and connected inside to a portable receiver. The usual tracking technique was to fly at 1,500 to 3,000 feet elevation to the last known location of the wolf being sought (fig. 14). If a signal was not obtained at that point, the aircraft spiraled upward until the signal was found or until 10,000 feet altitude had been reached. If the signal still was not heard, a search pattern was flown at 10,000 feet. The range of the signal from this altitude was 15 to 35 miles; at 3,000 feet it was 10 to 15 miles. Collars with antennas molded fully inside gave only about two-thirds the range of those protruding partly, but could be expected to last longer because the antennas could not break off. It is unknown whether any protruding antennas did break during the study, but on January 5, 1970, one wolf was recaptured, and its antenna had broken.

Figure 13.—Directional yagi antennas fastened to the wing struts of the aircraft were necessary to "home in" on the wolves. (Photo courtesy of U.S. Bureau of Sport Fisheries and Wildlife.)

Figure 14.—The tracking aircraft was usually flown at altitudes of 1,500 to 3,000 feet. (Photo courtesy of Dick Shank.)

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[2] *The acrylic collar was fashioned by the Davidson Co., Minneapolis, Minnesota, which also produced some of the transmitters. Other transmitters and two radio receivers were manufactured by the AVM Instrument Co., Cham-[illegible] Illinois.*

When a signal was received, the aircraft was eaded in the approximate direction of the source ntil the signal strength reached a peak; a 90° urn was then made in the direction the signal eemed the strongest. A series of these maneuers soon narrowed the area to the point where isual search was possible. After practice and xperience with this technique, we could locate he approximate source of the signal within 10 30 minutes after first receiving it.

Even though the radiotagged wolves spent ost of their time inland, often in stands of onifers, they were frequently observed from the ircraft. The technique was to circle at 300 to 00 feet altitude around a radius of a quarter ile from the point where the strongest signal nanated. From December through April, 65 perent of the wolves located by radio were sighted; he rate was much higher for more experienced ersonnel. A pack of five wolves that was racked was seen 31 times out of 33 attempts uring February and March.

Whenever wolves were located, radiotagged not, observations were made from an altitude hat did not disturb them. Packs varied in the oncern shown the aircraft, but only one or two an from it. The radiotagged wolves, and a pack of 10 to 13 animals, were habituated to the aircraft and usually could be observed from altitudes of 500 feet and less without disturbance (fig. 15).

Almost all the radiotracking was done from aircraft, but when inclement weather prevented flying, some attempts from the ground succeeded when wolves were close enough to roads. The usual range on the ground was 0.75 to 1.50 miles. One wolf was approached to within 35 feet through radiotracking.

RESULTS AND OBSERVATIONS

Aerial observations made during this study involved 490 hours distributed as follows: January, February, March 1967 – 124 hours; February 1968 – 10 hours; December 1968 through August 1969 – 356 hours. Seventy-seven observations involving a total of 323 wolves were made (table 1), excluding animals located through radiotracking.

One male and four female wolves were radiotagged, and they and their associates were followed intermittently for periods of 5 to 8 months (table 2). All except one initially suffered some injury to a foot. Three of these animals were

Figure 15.–The wolves studied soon became accustomed to the aircraft and could then be observed during their natural activity. (Photo courtesy of L. D. Mech.)

Table 1.—Sizes of wolf population units observed in northeastern Minnesota

Population unit[1] (number of wolves)	Wolf observations							
	Winter 1966-67		Winter 1968-69		Total		Winters[2] 1948-53	
	Number	Percent	Number	Percent	Number	Percent	Number	Percent
1	8	31	17	33	25	32	48	43
2	3	12	6	12	9	12	24	22
3	3	12	2	4	5	6	7	6
4	--	--	7	14	7	9	7	6
5	2	7	4	8	6	8	8	7
6	3	12	4	8	7	9	7	6
7	2	7	1	2	3	4	4	4
8	3	12	3	6	6	8	3	3
9	--	--	2	4	2	3	3	3
10	2	7	2	4	4	5	--	--
11	--	--	--	--	--	--	--	--
12	--	--	1	2	1	1	1	1
13	--	--	2	4	2	3	--	--
Total number of wolves	109	--	214	--	323	--	318	--
Total number of observations	26	--	51	--	77	--	112	--
Mean population unit size	4.2	--	4.2	--	4.2	--	2.8	--

[1] Because wolf packs sometimes split temporarily, these figures may not strictly represent actual pack sizes; nevertheless they should provide reasonably accurate approximations.

[2] From Stenlund (1955).

Table 2.—Background information on five radiotagged wolves studied in northeastern Minnesota

Wolf		Estimated weight[1] (pounds)	Usual associations	Location captured	Date captured	Last date located	Days located	General condition
Number	Sex							
							Number	
1051	M	75	None[2]	T62N-R7W-S18	Nov. 27/68	Apr. 24/69	84	Good, but two toes frozen in trap; animal limped lightly for 5-6 wks.
1053	F	60	None	T62N-R8W-S13	Dec. 10/68	Aug. 29/69	72	Thin; top of foot cut in trap but no broken bones or frozen toes; limped for at least 10 wks.
1055	F	60	Another wolf intermittently	T61N-R10W-S26	Jan. 5/69	May 30/69	65	Thin; two toes lightly frozen; no limp ever noticed.
1057	F	60	Pack of 13[3]	T66N-R5W-S33	Jan. 8/69	Apr. 24/69	47	Thin; front foot frozen in trap; lost use of foot and could not stay with pack.
1059	F	65	Pack of 5	T62N-R11W-S26	Jan. 22/69	Aug. 29/69	51	Good but thin; captured in snare; no apparent injury.

[1] Wolf 1059, when killed by a trapper on January 10, 1970, appeared to be of the same size and condition as when radiotagged; she only weighed 53 pounds, however, indicating that probably all the weights are overestimated.

[2] Tracks of a pack of at least two other wolves came by trap where 1051 was caught; however, there was never any other indication that 1051 may have been a member of a pack.

[3] A frozen foot prevented 1057 from staying with her pack; but she did associate with other wolves intermittently and with the whole pack when it came by her restricted area.

seen limping, but only in one case was the limp judged extreme enough to have significantly affected the movements or behavior of the animal. In that one case, the wolf (No. 1057) was caught in a steel trap on an extremely cold night, and her foot froze. After that she was often seen hopping on three legs. She was not able to keep up with her pack, which consisted of 10 to 13 members, and her movements were much restricted compared with those of other wolves. However, she was frequently observed feeding on fresh kills, and may even have made them herself.

The precise ages of the radiotagged wolves were unknown. All individuals, however, had sharp unworn teeth, indicating that they were ll relatively young. No. 1051, the only male tudied, had testes 2.0 cm. long and 1.5 cm. wide; heir volume therefore would be less than 4.5 cc. The small size of these testes, compared with the to 28 cc. reported by Fuller and Novakowski 1955) as the volume of the testes from wolves aken during fall, would indicate that 1051 had not yet matured. Since the animal's testes and canine lengths were considerably greater than hose of pups caught in a later study, we presume 1051 was 18 or 30 months old.

Two of the females, No. 1055 and No. 1059, both captured in January, had vulvas that seemed to be beginning to swell. No. 1059 was killed by a trapper about a year later, on January 10, 1970, and an examination revealed that she had bred in 1969 and carried five fetuses. Sectioning her incisors and reading the apparent annulations indicated that she probably was 3+ or 4+ years old.[3]

Three of the wolves were basically lone individuals. One of these, No. 1051, was captured on a night when tracks of at least two other wolves came by the trap, and this could mean that he had been part of a pack. However, it is also possible that these were merely tracks of nonassociated wolves that were also traveling through the area. In any case, 1051 was not seen associating with any other wolf until 4 months after he was caught, and even then the association seemed to be temporary and casual. It could be argued that capture, handling by humans, or wearing a collar prevented him from regaining old associations or making new ones. However, the wolves radiotagged by Kolenosky and Johnston (1967) were quickly accepted back into their packs, and so were two of ours. Thus we conclude that 1051 probably was a lone wolf when captured.

When 1053 was trapped, her tracks were the only ones in the area, and she was never seen closely associating with another wolf. No. 1055 probably was with another wolf when captured, as evidenced by tracks. About a month after she was radiotagged she associated with another wolf intermittently for about 2 weeks, after which she was only seen alone.

No. 1057 and No. 1059 were both members of packs. No. 1057 was captured during the night after a pack of 13 wolves was seen heading toward the area; 5 days later she was seen with 10 other wolves, which no doubt represented this same pack. This wolf's association with the pack was interrupted, however, because of the foot injury sustained during capture. When 1059 was caught, tracks of two other wolves were seen in the immediate vicinity, and one of the animals was seen within a quarter mile of the trapped wolf. Three days after 1059's release, and perhaps sooner, she was back with her pack, with which she remained at least through March.

The detailed histories of the associations of the radiotagged wolves will be discussed in a later section.

Radiotagged wolves were tracked every day that weather permitted during December, January, and February; every week during March, April, and May; and once a month during June, July, and August (fig. 16). Information was obtained for a total of 570 "wolf-days" – a wolf-day being a day in which one radiotagged wolf was located; a pack of five being located for 1 day would constitute 5 wolf-days.

The last day that animals 1051 and 1057 were heard from was April 24, 1969. Both had traveled long distances during the previous week and may have moved out of range. Signals from wolf 1055 were last heard on May 30; this animal had also been ranging widely. Circles with radii of at least 50 miles around the last known locations of each wolf were searched unsuccessfully for the signals. During all subsequent tracking flights for the remaining wolves, the missing animals were also sought, but to no avail. Before the last dates that signals from these animals were heard, attempts to locate marked animals from the air had failed in only three instances.

Daytime Activity Patterns

When radiotagged wolves were located, notes were kept on the type of activity they were engaged in; the results are summarized in figure 17. In a total of 171 observations made between 9:00 a.m. and 6:00 p.m., the wolves were resting 62 percent of the time, traveling 28 percent and feeding 10 percent. They tended to travel more

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[3] *David W. Kuehn. Personal correspondence to L. D. Mech, 1970.*

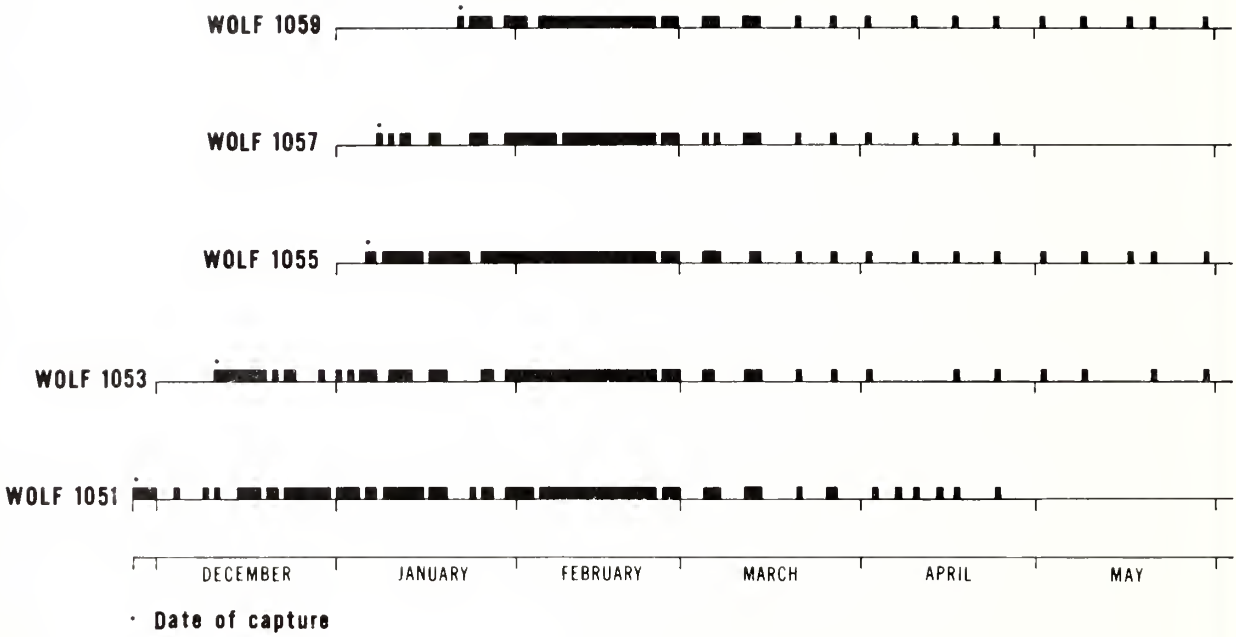

Figure 16.—Distribution of the days on which data were obtained for each of the radiotagged wolves. Because tracking success was 99 percent, this also represents the distribution of effort. During June, July, and August, wolves 1053 and 1059 were located 1 day each month.

before 11:00 a.m. and after 3:00 p.m., although resting still composed at least 45 percent of the activity during every hour (fig. 18).

These results generally agree with the statement by Mech (1966a) that wolves on nearby Isle Royale tend to rest about 11:00 a.m. and begin traveling again about 4:00 p.m. However, it does appear that the Minnesota wolves spend much more of the day resting than do the Isle Royale animals. The difference may be caused by the difference in pack sizes studied. The Isle Royale pack of 15 to 16 may have had to travel more to find enough food to feed all its members than did the lone wolves and pack of five in the present study.

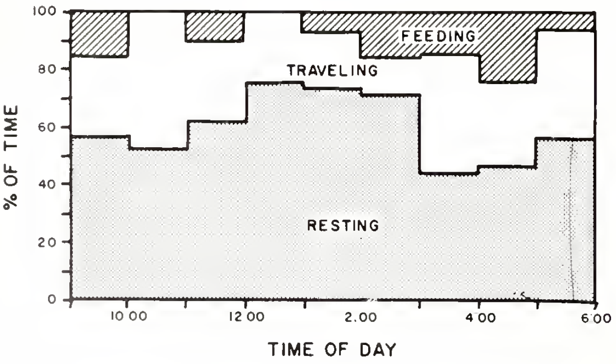

Figure 17.—Percentage of time spent by radiotagged wolves in various types of activity throughout the day, from December through April.

Figure 18.—Generally the wolves rested during most of the day. (Photo courtesy of L. D. Mech.)

Movements and Range

Wolf movement is greatly hindered by deep, soft snow, so during winter travel, wolves frequently use areas where they sink into the snow the least. In our study area, frozen waterways are used extensively where possible, just as reported by Stenlund (1955). Where few lakes or rivers exist, wolves follow railroad beds and logging roads, often soon after a plow or other vehicle has driven on them. In cutting cross country through deep snow, wolves travel single file and tend to stick to windblown ridges and to trails of deer and moose. Wolves that have ranges small enough to cover in a few days form a network of their own trails, which they can maintain merely by traveling regularly over them. Packs on Isle Royale depended a great deal on such a system of trails (Mech 1966a), and so did Pack No. 1059 in our study area.

Wolf packs can travel up to 45 miles in a day but it is usually larger packs that do so (Stenlund 1955, Burkholder 1959, Mech 1966a, Pimlott *et al.* 1969). In our study area we sometimes saw evidence of long moves by large packs along strings of lakes and waterways. However, most of our movement data pertain to lone wolves and a pack of five. The daily travel of these animals was usually much less than that reported for large packs.

Our radiotracking data provide an index to the extent of travel for each wolf rather than the actual amount of travel, for it is based on straight line distances between consecutive points at which an animal was found. This measure will be referred to as the "net daily distance."

Much variation was found in the net daily distances of wolves, with the longest ranging from 4.5 miles for 1057 to 12.8 for 1055 (table 3). The mean net daily distance for each animal, excluding days with no net movement, varied from 1.5 to 3.6 miles. The movements of these wolves may have been affected by the snow depth and penetrability, for mean and maximum net daily distances suddenly increased for all animals between February 23 and 28, when snow penetrability had decreased to a point where walking wolves would be expected to sink in only about 6 inches (table 4). Other possible explanations for the wolves' sudden increase in movements will be discussed below.

The straight line distances traveled between consecutive weekly locations (called the "net weekly distances") showed a similar variation (table 3). The maximum net weekly distance for each wolf varied from 4.6 miles for 1059 to 49.0 for 1055, with means ranging from 2.9 to 15.6 miles for the same wolves. No doubt 1059's net weekly distances were relatively short because her total range and that of her pack were much smaller than those of the other wolves.

It is difficult to obtain comparable measures of the extent of the ranges covered by each of the radiotagged wolves because their patterns of travel varied so much. Thus the figures given in table 5 should be regarded only as gross indicators of the minimum range of each animal. The area figures are especially deceiving in the case of 1055, for she had a horseshoe-shaped range, much of which apparently was not used.

Nevertheless, one major piece of information is obvious from the figures: 1059's pack of five

Table 3.—Straight line distances (miles) between consecutive locations of radiotagged wolves

Wolf number	Net daily distances								Net weekly distances		
	Days data obtained	Days no net movement		Days movement		Mean net distance per day	Mean net distance per day excluding days of no movement	Range	Weeks data	Mean net distance per week	Range
	Number	Number	Percent	Number	Percent	Miles	Miles	Miles	Number	Miles	Miles
1051	54	13	24	41	76	2.0	2.6	0.0-12.0	22	12.7	1.0-46.0
1053	37	20	54	17	46	1.0	2.1	0.0- 5.0	23	6.3	0.0-23.6
1055	46	7	15	39	85	2.9	3.6	0.0-12.8	21	15.6	1.7-49.0
1057	29	11	38	18	62	1.0	1.5	0.0- 4.5	15	4.6	0.0-31.0
1059	26	1	4	25	96	2.5	2.6	0.0- 5.6	18	2.9	0.0- 4.6

Table 4.—Straight line distances (miles) traveled between consecutive days ("net daily distance") by radiotagged wolves in northeastern Minnesota during February 1969

Wolf number	Mean net daily distance		Greatest net daily distance	
	Feb. 1-23	Feb. 23-28	Feb. 1-23	Feb. 23-28
1051	1.1	3.5	2.3	4.8
1053	0.7	2.5	3.0	5.0
1055	2.7	6.2	8.0	12.8
1057	1.0	1.5	4.0	4.5
1059	2.2	3.1	4.0	5.6

wolves had a much smaller range than any of the other uninjured animals — approximately 43 square miles when figured by the minimum-area method (Mohr 1947). The next smallest range was that of 1051 (excluding the area of his later dispersal — see below), which was some seven times the size of the pack's range.

There is little published information on the movements and ranges of lone wolves with which to compare our data. Mech (1970) summarized information regarding ranges of packs. Reported ranges varied from 36 square miles for a pack of two wolves in Minnesota (Stenlund 1955) to 5,000 square miles for a pack of 10 in Alaska (Burkholder 1959). Considering only data based on intensive study in the same general region (Minnesota, Isle Royale, and Ontario) as our study area, the largest range reported was 210 square miles for a pack of 15 to 21 wolves on Isle Royale (Mech 1966, Jordan *et al.* 1967). On a per-wolf basis, the ranges in this region varied from 6 to 28 square miles per wolf. Our pack of five with its range of 43 square miles would have about 9 square miles per wolf.

A more accurate assessment of the ranges of the radiotagged wolves requires an individual discussion for each.

No. 1051.—The range of 1051 was composed basically of three distinct areas (fig. 19). Within 10 days after being released, the wolf left the general area of his capture (Area A near Isabella Lake) and traveled to Area B along Highway 1, some 17 miles to the southwest. From December 9 to January 4 wolf 1051 remained in Area B, which covers about 45 square miles. Between January 4 and 6 he returned to Area A and stayed in 13 square miles until February 3. Between February 3 and 5 he shifted to Area C east of Snowbank Lake, 11 miles northwest of Area A. He remained in that 16-square-mile area until February 25, then suddenly left and headed 8 miles to the northeast.

Table 5.—Extent of ranges used by radiotagged wolves

Wolf number	Greatest length	Greatest width	Total area[1]	Area[1] of intense use (before late Feb.)
	Miles	Miles	Sq. miles	Sq. miles
1051[2]	28.5	13.6	318	13 (Location A[3]) 45 (Location B) 16 (Location C)
1053	31.1	22.0	392	31
1055	55.4	24.9	997	40
1057	32.3	3.8	77	14
1059[4]	8.4	8.0	43	39

[1] Minimum area method (Mohr 1947).
[2] Before dispersal.
[3] See text and figure 19.
[4] Pack of five.

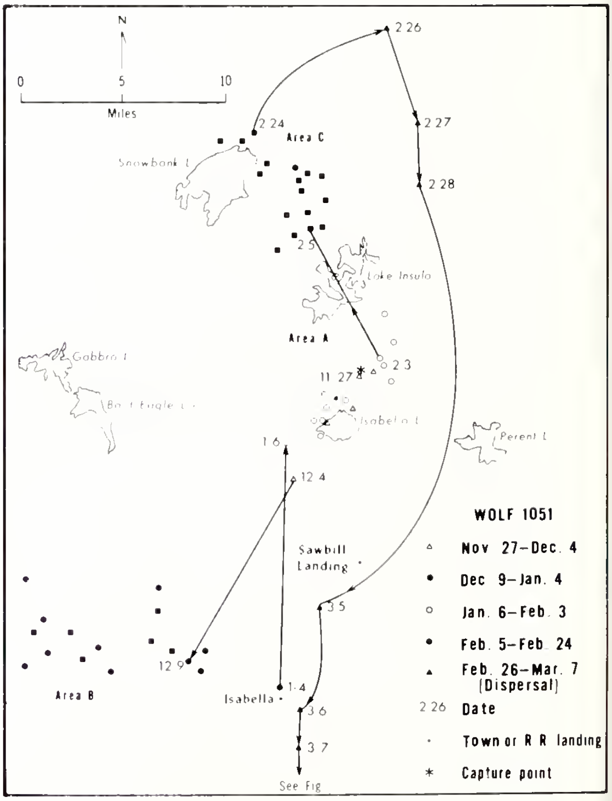

Figure 19.—Locations and range of wolf 1051. Lines are NOT travel routes; rather they merely indicate sequence of locations. Only selected lakes are shown.

From February 26 until April 24 the movenents of 1051 were strongly indicative of dispersal (fig. 20). His average weekly straight line nove during that period was 25 miles (compared vith 6 miles per week before this pericd), and ntil March 14 he maintained an almost straight outh-southwest heading to a location west of he town of Castle Danger. After that the animal raveled a series of northwest-southwest alternaions that on April 3 took him east of Big Sandy ake to a point 129 miles southwest of where e had begun the dispersal. There he remained or about 2 weeks, but between April 17 and 24 e traveled 26 miles northwest. We last saw him t 3:30 p.m. on April 24 heading northwest hrough a swamp 15 miles southeast of Grand Rapids, approximately 122 miles from where he ad started. The total of straight line distances etween 16 consecutive pairs of locations taken t intervals of from 1 to 8 days was 226 miles, vhich is the minimum distance the wolf traveled uring his dispersal.

We observed 1051 for distances of up to 5 niles during these travels; he maintained a teady trot that seemed faster than usual, and e appeared intent on heading in a straight line. He did chase deer during his travels, and twice was seen feeding on carcasses. In the area where he remained for about 2 weeks, he was twice seen closely associated with another wolf. This relationship will be discussed later.

An extensive search was made for 1051's signals on May 2 in an area of at least 50 miles radius from his last known location, but it was unsuccessful. On each subsequent tracking flight, the wolf's frequency was also monitored with no success. Possible explanations for the loss of the signal from this wolf include the following: (1) premature expiration of the transmitter, (2) capture of the wolf and breakage of the transmitter, (3) loss of the exposed antenna and consequent reduction of range, and (4) travel of the wolf out of range of the tracking aircraft.

During 1051's travels a number of interesting events took place:

Nov. 27, 1968 – Captured and radiotagged
Dec. 4, 1968 – Crossed road in front of tracking truck
Dec. 8, 1968 – Moved to Area B
Dec. 9, 1968 – Surprised on the ground at distance of 35 feet
Dec. 18, 1968 – Chased by loggers with axes

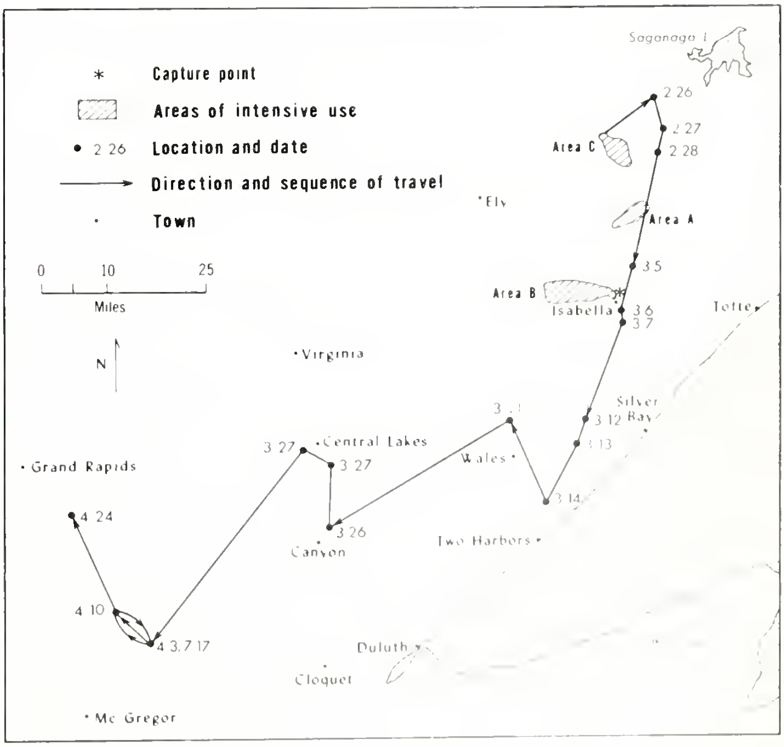

Figure 20. – Dispersal of wolf 1051. Lines merely indicate sequence of locations. Only selected lakes are shown.

Dec. 25, 1968 – Almost shot by trapper who saw collar and withheld fire
Jan. 5, 1969 – Returned to Area A
Jan. 13, 1969 – "Bumped" twice on logging road by loggers in auto but no apparent injury
Feb. 4, 1969 – Moved to Area C
Feb. 26, 1969 – Began long-range southwest movement considered to be dispersal
Mar. 14, 1969 – Seen feeding on old carcass within 200 yards of houses, dogs, and a man walking
Mar. 27, 1969—Chased two deer across 4-lane State highway 53
Apr. 3, 1969 – Found with another wolf at point farthest south in his range
Apr. 24, 1969 – Last contact with this animal; was seen traveling NW

Wolf 1053.—This wolf was basically a scavenger who subsisted for long periods on the remains of old carcasses. She was known to have visited the remains of at least four deer and three moose, and she stayed near one moose carcass from February 8 to 20, at least during the day. Between her date of capture, December 10, and February 28, 1053 traveled about in an area of 31 square miles in the Arrow Lake-Maniwaki Lake region (fig. 21).

Between February 28 and March 6 she suddenly moved 13 miles to the east-southeast near the Sawbill Trail, and during the next week she traveled a straight line distance of 24 miles southwest to a point southeast of the town of Isabella. Her subsequent travels eventually took her over a much larger area. Before February 28, 1053's average weekly straight line distance was 2 miles, but after that date it increased to 11 miles.

Wolf 1055.—The range of this animal from January 5, when she was captured, to February 23 covered about 40 square miles near Stony Lake, Slate Lake, and the Jack Pine Lookout Tower (fig. 22), and her mean weekly distance was 4 miles. Between February 23 and 24, however, she traveled 13 miles northeastward, the beginning of a series of long moves. By March 5, 1055 had reached Crescent Lake, a point 39 miles east-northeast of her previous area of intensive use. She then gradually headed back toward the west and south during the next 10 days and within the next month repeated this pattern. When her signal was heard last on May 30, 1055 was near Martin Landing in the center of her range. Her mean net weekly distance after February 23 had increased to 22 miles.

Wolf 1057.—The movements of 1057 cannot

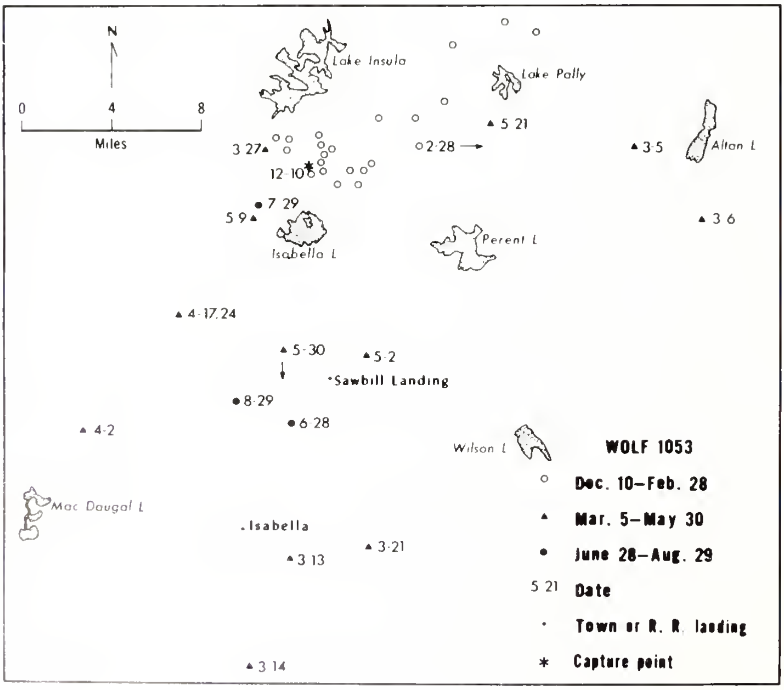

Figure 21. – Locations and range of wolf 1053. Only selected lakes are shown.

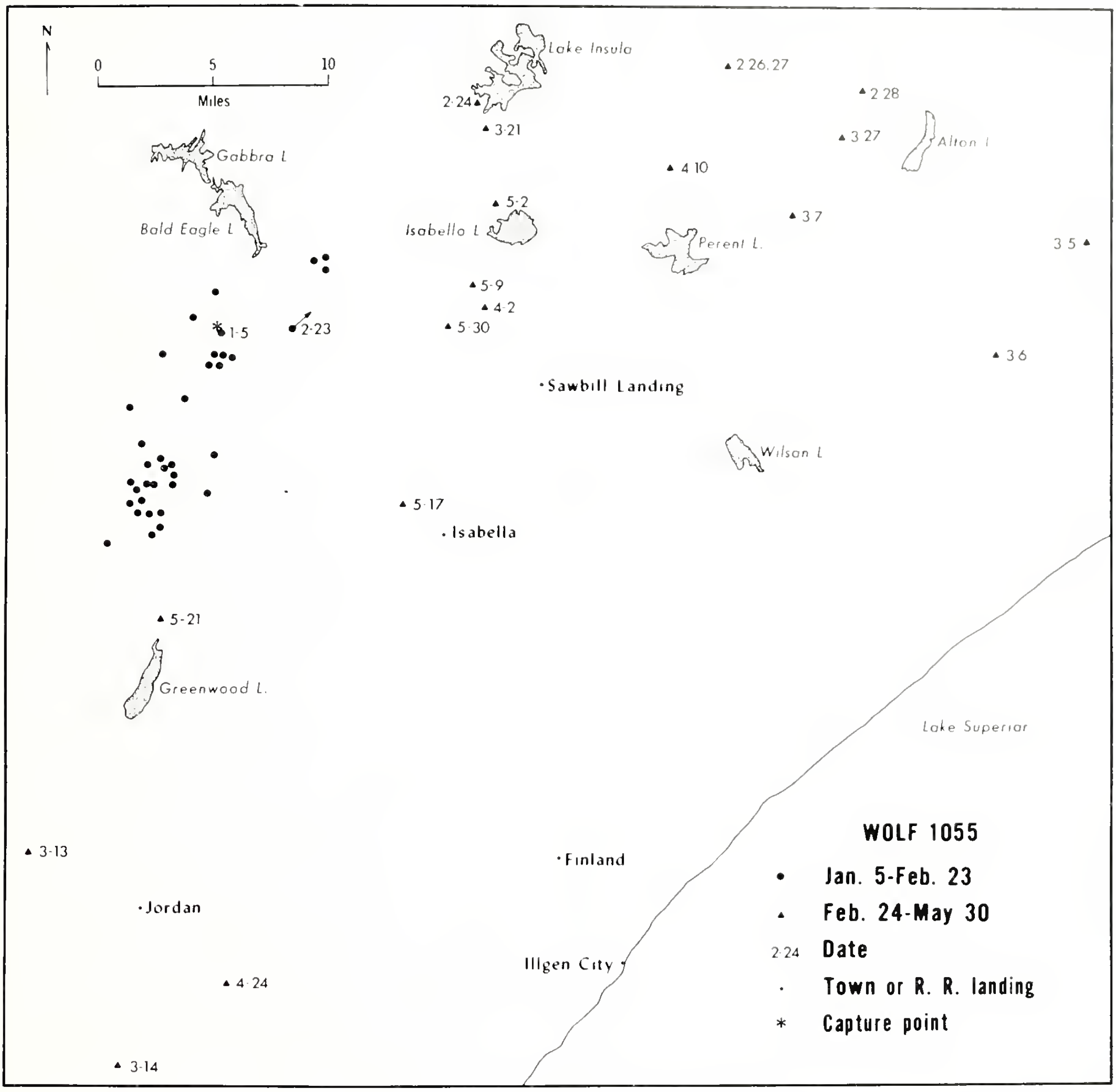

Figure 22.—Locations and range of wolf 1055. Only selected lakes are shown.

e considered normal because freezing of a front oot prevented her accompanying the pack of hich she was a member. Nevertheless, even data rom an abnormal animal can provide some infor- ation. On January 13, 5 days after capture and elease on Red Rock Lake, 1057 was located 4 iles from the capture point with a pack of 10 ther wolves. She was limping and fell behind hen they moved. Five days later she was again een with the pack 12 miles away between Knife ake and Kekekabic Lake. She then remained about 14 square miles of that general area through April 17 (fig. 23).

Suddenly on April 24, 1057 was found in Ontario some 31 miles northeast of her location of the previous week. That was the last time we heard her signal even though on May 2 we scanned an area with a radius of 35 miles from her last known location and listened for her signal during every subsequent flight.

Wolf 1059.—This animal was a member of a pack of three to five wolves (see next section). The movements of the group varied little and were concentrated in the August Lake, Omaday

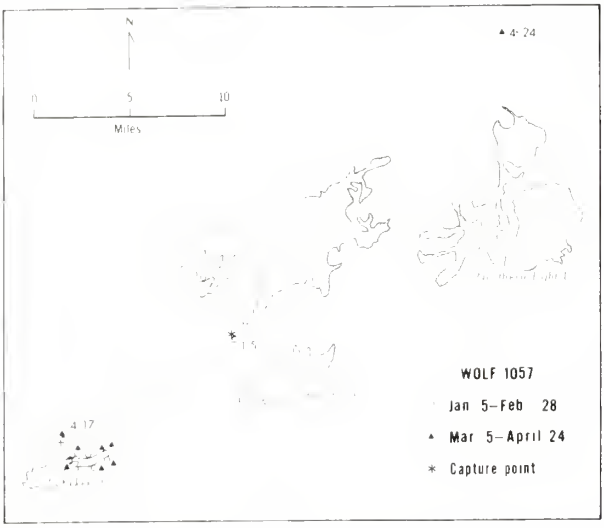

Figure 23.—Locations and range of wolf 1057. Only selected lakes are shown.

Lake, and Keeley Creek area in about 43 square miles (fig. 24). Contrary to animals 1051, 1053, and 1055, this pack did not suddenly begin a series of longer weekly movements in late February. Both before and after February 28, the average weekly straight line movement of the pack was just less than 3 miles.

Probably these animals did begin traveling more in late February, for their net daily distances did increase at that time along with those of the other wolves (table 4). However, the increased travel took place within the restricted area of the pack's usual range rather than in new areas as occurred with the other wolves.

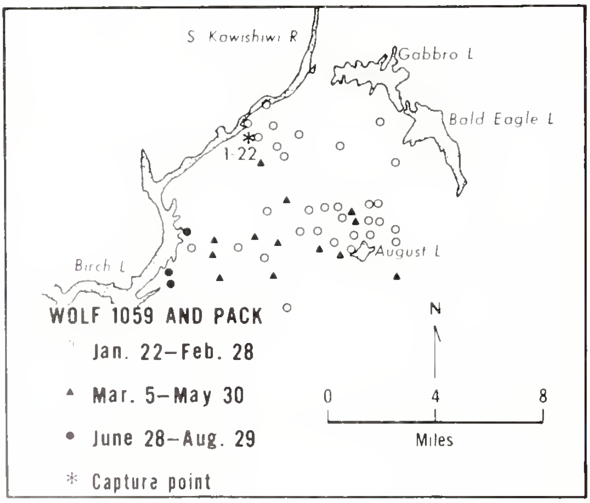

Figure 24.—Location and range of wolf 1059 and pack. Only selected lakes are shown.

Because 1059 was later found to have bred and carried five fetuses, her movements during whelping season (late April and early May) are of interest. Her locations on both April 24 and May 2 were within 250 yards of each other, which might indicate that she was denning. On May 9, however, she was 2.5 miles east of these locations, on the 17th and 21st was 2 miles west of them, and on the 30th was 3 miles north of them.

In early January 1970, Wolf 1059 was killed by a trapper in the southeast corner of her pack's 1969 range.

Summer locations.—Signals from only 1053 and 1059 were heard during summer, and then tracking attempts were made only on June 28, July 29, and August 29. Locations for 1053 on those occasions were near Kelly Landing and Isabella Lake, within her previous range. Wolf 1059 was found each time within 2 miles outside of the southwest corner of the pack's winter and spring range.

Wolf Associations, Social Behavior, and Reproduction

In our study area, population units of wolves exist as both single animals (lone wolves) and packs. In a total of 77 observations, lone wolves constituted 32 percent of the sightings (fig. 25), with packs of from 2 to 13 members making up the remainder (Table 1). On the basis of the

Figure 25.—Only 8 percent of the wolves observed were lone wolves. (Photo courtesy of L. D. Frenzel.)

umber of wolves seen, rather than the number observations, lone wolves accounted for only 5 (8 percent) out of 323.

These figures compare favorably with reports the literature as summarized by Mech (1970). five areas studied, lone wolves made up from 4 to 60 percent of the observations of popula- on units, and from 8 to 28 percent of the wolves en. In our study area during 1948 to 1953, lone olves constituted 43 percent of the observations nd 15 percent of the wolves (Stenlund 1955).

The average size of the population units ob- erved during our study (total number of wolves en divided by the number of observations) was 2, which is significantly larger (95 percent vel) than the average seen in this area (2.8) om 1948 to 1953. This is also larger than that eported from any other area of comparable size able 6).

able 6.—Mean sizes of wolf population units reported from various areas

Area	Observa-tions	Wolves	Mean size of popula-tion unit	Largest pack size	Authority calculated from
	Number	Number			
laska	310	1,041	3.4	12	Kelly 1954
laska	1,268	4,823	3.8	21	R. A. Rausch[1/]
apland	118	311	2.5	12	Pulliainen 1965
. Finland	460	984	2.1	12	Pulliainen 1965
innesota	112	318	2.8	12	Stenlund 1955
innesota	77	323	4.2	13	Present study

[1/] R. A. Rausch. Personal correspondence to L. D. Mech, 1967.

The largest pack seen in our study area in- uded 13 members, and there apparently were t least two such packs. Although larger packs an this have been reported, any group con- aining more than 8 to 10 members is unusually rge (Mech 1970).

Wolf sociology is a complex subject and is till not well understood, so the following detailed bservations of the associations between our adiotagged wolves and others are given. Asso- iations are defined as relationships in which two r more wolves relate in a close, positive manner.

As mentioned earlier, 1051 may or may not ave been associated with other wolves when he as captured. However, although this animal as observed 55 times throughout winter and pring, only twice was he seen associating with nother wolf. Probably the same individual was nvolved each time, because the location was about the same (the vicinity of the juncture of Aitkin, Carlton, and St. Louis Counties).

The first occasion was on April 3. Wolf 1051 in the previous week had moved 46 miles straight line distance from the northeast. He was then observed lying peacefully within 15 feet of another wolf near a freshly killed deer. The very proximity of the two animals implied a positive relationship. On April 7, 10 and 14, 1051 was seen 1 mile, 10 miles, and 8 miles from the kill and was alone each time.

However, on April 17, 1051 was back in the general vicinity of the kill, and he and another wolf were resting on an open hillside about 100 feet from each other. As we descended for a closer look, the smaller animal arose and headed to the larger, presumably 1051 because he had not been disturbed by the aircraft. The larger wolf did not arise for several seconds, but eventually followed the other into the woods. No tail raising or other expressive posturing was seen in either wolf. One week later 1051 was 26 miles northwest of the kill traveling alone.

Wolf 1053 was never seen less than 80 yards from another wolf, and there was no evidence that she ever associated with a conspecific. Even when she was seen 80 yards from the other wolf, both were resting, and when the strange wolf left, 1053 made no attempt to accompany or follow it.

No. 1055 apparently had been traveling with another wolf when caught on January 5, and tracks showed that the individual had remained near her until we arrived to handle her. Tracks found on January 7 and 10 suggested that 1055 was with another animal, but that animal was not seen during any of the six times 1055 was observed through February 1. However, from February 5 to 19, 1055 was with another wolf on eight of the 12 times she was seen. The two animals were observed resting, traveling, hunting, and feeding together. On February 20, and thereafter, 1055 was alone all 14 times she was seen.

It is possible that 1055's associate was killed between February 19 and 20.. About March 6, a 63-pound male wolf pup was found dead (by Mr. Charles Wick, USDA Forest Service) within about 50 feet of a highway and less than a mile from where 1055 and her associate were seen on February 19. Because of the snow conditions, it was judged that the wolf had been killed (probably by an automobile) sometime in February.

Wolf 1057, whose foot froze during capture, was a member of a pack of 10 to 13 wolves, and was seen with the pack on January 13 and 18. After that she was usually found alone, although on at least five occasions she was with one or more wolves:

Period	*No. of observations*	*Associations*
Jan. 13	1	10 other wolves
Jan. 14-17	1	None
Jan. 18	1	10 or 11 other wolves
Jan. 19-29	2	None
Jan. 30	1	1 other wolf
Jan. 31 to Feb. 2	2	None
Feb. 3-4	2	2 other wolves
Feb. 5	1	1 other wolf
Feb. 6-13	6	None
Feb. 14	1	3 other wolves
Feb. 15-22	5	None
Feb. 23	1	10 to 13 other wolves
Feb. 24 to Apr. 24	6	None

February 23 she was with the pack at a kill in her usual area, and although the pack left that night, 1057 remained near the kill the next day. Presumably this animal would have traveled with pack if she could have.

No. 1059 was part of a pack that included three to five members (fig. 26). From January 25, the first time she was observed after release, through April 2, the animal was seen 19 times with two other wolves, eight times with at least three others, and eight times with four others. She was never seen alone until April 17; both times after this when she was seen, May 9 and 21, 1059 was also alone.

Some insight into the fluctuating size of this pack was obtained on February 27 when the five animals were followed for 2 hours. During that time two members (one of which was larger than the other) often lagged behind the other three by as much as a mile. These two romped and played considerably, with one carrying a stick or a bone part of the time. Eventually they caught up again to the other three. The behavior of the two lagging wolves would be consistent with the hypothesis that they were either pups or a courting pair of adults. In either case, they seemed to be an actual part of the pack even though they temporarily traveled separately.

Figure 26.—One of the radiotagged wolves was a member of this pack of five. (Photo courtesy of L. D. Mech.)

The fact that 1059 was observed traveling alone three times from April 17 to May 21 may be further evidence that the pack had a den in the area at that time. The presence of a den allows individual pack members to venture off singly and return each day to a known social center, as Murie (1944) observed, so they do not need to travel with each other to maintain social bonds. Wolves in our area breed during the latter half of February (see below), and the young should be born in the latter half of April Since dens are prepared a few weeks in advance (Young 1944), pack members might be expected to begin traveling singly in mid-April

Some information on social relations within our radiotagged pack of five was also obtained

One of the members could often be distinguished from the others by its reddish cast and this ndividual appeared to be the pack leader or alpha male (Schenkel 1947). In urinating, this animal lifted his leg, a position seen almost exclusively in males. Except for only two temporary occasions, this animal always headed the pack, which usually traveled single file. The second wolf in line generally was noticeably small, possible a female, and the third wolf was twice dentified as 1059 on the basis of sightings of her collar.

The leader often gained a lead on the other wolves, especially during a chase (see below), much as reported for a lead wolf on Isle Royale (Mech 1966a). Upon returning to the lagging members of the pack, this animal usually held his tail vertically, an expression of social dominance (Schenkel 1947). On two occasions he led chases against strange wolves and demonstrated the highest motivation (see below).

The leader was also the most active in his reactions when scent posts were encountered. Because the function of scent-marking behavior is still unknown, it is important that detailed descriptions of the natural behavior of free-ranging wolves around scent posts be made available (fig. 27). Thus the following excerpt from field notes by Mech dated February 27, 1969, is presented:

> "When they [the three wolves] came to a small frozen pond, where the wolf trail [which they had been following] branched and there were some packed down areas, they became quite excited [fig. 28]. This was especially true of the reddish wolf. He nosed several spots, and scratched around them. Usually his tail was vertical. He defecated at one spot, and right afterwards another wolf did. After about 2 minutes that pack went on.
>
> "About 15 minutes later the 2 'satellite' wolves arrived at this spot, hesitated, nosed around but continued on after less than a minute.
>
> "The three wolves meanwhile came to a junction of 2 logging roads. There they nosed around, scratched, and acted much as described above. Again the reddish wolf was most active and had its tail up.
>
> "When the last 2 wolves came to this spot, they nosed around, ran back and forth, and 1 defecated. They then headed on a different branch of the trail than the first 3 had gone on just 10 minutes before.
>
> "The first 3 wolves meanwhile were running along a logging road but eventually they circled and one other than the reddish one headed across a swamp toward the last 2. Then the reddish one and the other followed this one, and they met the last 2 on a ridge. There was the usual tail wagging, then all headed off together in a new direction. They passed the first scent post again and there was some nosing by the reddish wolf but little hesitation.
>
> "When they traveled, one wolf lagged behind by 150 yards. The wolf just ahead of it had its tail vertical part of the time, as did the reddish leader.

Figure 27.—Feces, urine, and scratching in a conspicuous spot indicate a wolf "scent post." (Photo courtesy of L. D. Mech.)

Figure 28.—A pack of wolves investigating a scent post. The raised tails indicate their excitement. (Photo courtesy of L. D. Mech.)

> "Soon the pack came upon another area packed with wolf tracks on a pond. There they followed every little trail, nose to the ground, wagged tails, grouped together often, chased each other, rolled over, etc. for 6 minutes. The reddish animal had tail up most of the time.
>
> "The wolves continued on, and we left them about 1 mile S.W. of the S.W. arm of Bald Eagle Lake [at 6:05 p.m.]."

Unfortunately it was not known whether the trails that the wolves were following were their own or those made by other wolves.

Significant aspects of the above observation are (1) the spirited initiative of the leader, (2) the amount of time spent in scratching, urinating, and defecating, (3) the decision of the last two wolves to take a different route from that of the first three even though their goal seemed to be to catch up to the first three, and (4) the fact that the scent posts were located at trail junctions. In the last regard, we often noted from the ground that wolves urinated at the junction of newly formed human trails heading perpendicularly from roads they were following.

Copulation in wolves was only observed once during our study, on February 19, 1969. Two members of a group of four were seen coupled for 2 minutes on Kekekabic Lake. On Isle Royale, which is at the same latitude, copulations were witnessed on February 21, 24 and 27 (Mech 1966a).

On April 17, a den west of Big Moose Lake known to have been used at least intermittently for 13 years was seen from the air to have fresh activity of some kind in the snow in front of it, and on April 24 we saw a wolf at the mound. A few days later, two local human residents unaware of our interests approached this den and looked in. An adult wolf, presumably the bitch, leaped over their heads and fled the area. The men then dug up the den and removed six pups whose eyes had not yet opened.

Intraspecific Intolerance and Indifference

Instances of chasing or attack by a pack of wolves on conspecifics not a part of their group have been described by Murie (1944) and Mech (1966a). Observations of such behavior are important in trying to determine conclusively whether or not wolves are territorial. Pimlott *et al.* (1969, p. 75) wrote "It still is not clear, however, whether or not their use of range should be defined as territorial." Mech (1970) summarized the available evidence for territoriality in wolves and postulated that it may be spatiotemporal such that packs might avoid each other at any particular point in time but over a long period might cover the same area at different times. A number of our observations are pertinent to this question, for we have evidence of both tolerance and intolerance between population units of wolves.

Two direct cases of intolerance were observed, both involving the radiotagged pack and other wolves within the usual range of the pack. Following is a direct quote from the field notes of Mech:

> "Feb. 7, 1969—about 11:30 a.m.—aerial and visual—1059 and 2 other wolves traveling overland about halfway between Heart L. and August L. (R10W-T61 N.

Sect. 17 center). They were traveling quickly and intently along a fresh wolf trail, with a lighter reddish individual in the lead. The other 2 animals were darker colored, and one of them was smaller than the other. One of them must have been 1059.

"We soon found that about half a mile ahead of the pack was a dark wolf hurrying away from the three. This animal often looked back and ran whenever it encountered good running conditions. It soon became obvious that the pack of 3 was chasing this individual. Because it [the lone wolf] often broke its own trail, the pack gradually gained on this animal. The single wolf flushed a deer which ran when the wolf was about 75 feet away and floundered in the snow, but the wolf continued hurrying on by.

"Although the deer ran only about 50 yards and stopped, the pack of 3 also hurried on by. The single wolf flushed another deer, ignored it, and continued by, as did the pack of 3. The chase continued for 2 miles as we watched, into the N.E. corner of Sect. 18 and then into the N. Central part of Sect. 8, and the pack got to within 150 yards of the single wolf.

"However, at this point, the 2 darker members of the pack had fallen about 100 yards behind the lead one. The lead animal stopped and waited for them, as it had done a few times before. It then turned around and headed back to these animals. When they met, the reddish animal's tail was held vertically and there was much tail wagging by all for about 1 minute. Then all animals lay down for a minute and then went up on a knoll. There was much activity and 'playing' on the knoll. (12:10 p.m.)

"The single wolf continued running and looking back for at least another mile. We left at 12:21 p.m.

"At 4:07 p.m. we saw a single wolf running across a small lake and looking behind it about 8 miles N.W. of these animals. The creature behaved the same as the one being chased today, and we wondered whether it could be the same animal."

On February 18, 1969, Ream made a similar observation, as follows (quoted from his field notes):

> "Got visual sighting on 1059 with 3 other wolves at 11:55 about a mile west of Omaday Lake and they were running along fairly fast on a trail. When we circled a second time we saw 2 wolves curled up sleeping on a knoll ahead (south) of the running pack. We then realized the running wolves were on the trail of the sleeping wolves and when the pack of 4 with "red" in the lead was about 50 yards from the knoll the 2 sleeping wolves jumped up and charged away in the opposite direction full tilt, and split and went in 2 directions. When the pack reached the knoll they started off on the trail of the wolf that headed N. E. and then changed and went after the one that headed S. W. The reddish wolf was in the lead and really picked up the pace. Although the reddish wolf seemed to gain on the chased one 3 or 4 times, the pack as a whole couldn't catch up, even though the single was breaking trail. The reddish wolf, after gaining, always stopped and waited for the others or went back to find them. They chased this wolf for 2½ to 3 miles, all the way down to Highway 1 at a point 3.0 miles from the lab [Kawishiwi Field Station, U.S. Forest Service]. There was a dense patch, 10-15 acres, of woods just before Highway 1 and we lost sight of the chased wolf for a while and also the 4 when they entered it, but shortly we found that the chased one had somehow doubled back and was heading N.E. again. The pack was apparently confused for at one point 3 of them were wandering back and forth on Highway 1, apparently looking for the trail of the chased wolf. Two of these paralleled the Highway for a couple hundred yards and then stopped on top of a hill, apparently resting. During this chase both the single wolf and the pack chased up deer from their route of travel and didn't seem to

> pay much attention to them, even though some were really floundering in the deep snow. We finally stopped watching all of this at 1:30 p.m. and proceeded on our rounds."

On February 21 we also saw a single wolf running and looking behind several times on Ojibway Lake. Even when it saw a fisherman on the lake within ⅛ mile, it continued across to the opposite shore seeming most intent on avoiding whatever was on its trail. Presumably it had also been chased by a pack.

The cases of tolerance or indifference that we witnessed between wolves involved our lone animals. On January 27, 1051 was at a kill he had made the day before, and another wolf was sitting within 100 feet looking toward the carcass. Eventually the unidentified wolf left without approaching any closer. A lone wolf was also seen near 1053 in the general vicinity of a moose carcass, which probably both were feeding on at different times. Three such observations were made, on February 10, 15, and 18; and on February 21 another wolf was also seen near 1053 some 2.5 miles away from the moose carcass. In all cases, the two animals were 80 to 200 yards apart in open country and must have been aware of each other's presence.

Hunting, Killing, and Feeding Behavior

The primary prey of most wolves in our study area is the white-tailed deer (fig. 29), but some moose (fig. 30) are also killed. We have examined the remains of six moose that were eaten by wolves, two of which were killed by them (fig. 31). One was found on February 25, 1967, on Gillis Lake and the other on March 7, 1969, on Twinkle Lake. These locations are within 3 miles of each other, suggesting that a wolf pack in that area may be more accustomed to preying on moose than other packs. The other four moose carcasses were found in other parts of the study area, but circumstances were such that the causes of death of those animals could not be determined. A discussion of the details of wolf-moose relations in our study area must await the collection of additional data.

The remains of 93 wolf-killed deer, and 49 probable wolf-kills, were examined for age, sex, and condition and were compared with a sample

Figure 29.—The main prey of wolves in northerr Minnesota is the white-tailed deer. (Photc courtesy of L. D. Mech.)

Figure 30.—Moose are also killed by wolve. (Photo courtesy of Allan Taylor.)

Figure 31.–Only a few wolf-killed moose were located during the study. (Photo courtesy of Laurence Pringle.)

of 433 hunter-killed deer from the same general area. The wolf-killed deer were generally much older than the hunter-kills and had a significantly higher percentage of jaw and limb abnormalities (see Mech and Frenzel, page 35).

Until recently the only observations of wolves hunting deer were those reported by Stenlund (1955) for northern Minnesota. He described two reports of actual observations and two reports of interpretations of tracks in the snow, all successful hunts. Since that time several descriptions of successful and unsuccessful hunts have also been published (Mech 1966b, Rutter and Pimlott 1968, Pimlott *et al.* 1969, Mech 1970). Nevertheless, many more observations must be made before generalizations can be formed.

During the present study we were able to witness a number of hunts from the air and piece together others based on tracks. The following descriptions are quoted from the field notes of Mech:

> "26 January 1967. About ¾ mile N.E. of Alice Lake.
>
> "Jack Burgess [pilot] and I were following a pack of 8 wolves, when at 4:15 they veered from their former line of travel, about 30°. They were then about 200 yards from 2 deer. They began wagging their tails when about 175 yards from the deer. One deer, on the edge of a steep bank, was lying, but one was standing about 75 yards N. of it in open hardwoods. The wolves continued toward the latter deer.
>
> "This deer remained standing in the same place until the wolves approached to within about 100 feet of it. The lead wolf stopped, when that distance from the deer, and the others caught up but also stopped when within about 25 feet behind the lead wolf. By this time the deer, whose body was facing away from the wolves, had its head turned back over its shoulder toward the wolves. The wolves and the deer remained absolutely still while staring at each other, 100 feet apart, for 1-2 minutes, while we made several circles.
>
> "Suddenly the deer bolted, and instantly the wolves pursued. I am fairly certain that it was the deer that bolted first, but could be mistaken. The action was almost simultaneous. The deer headed toward the other deer near the top of the high bank. This animal had been lying but had arisen when the wolves were about 150 yards away.
>
> "The lead wolf followed in the deer's trail, but the others cut toward the bank. This flushed the second deer (near the edge of the bank), which ran down the bank. Meanwhile when the first deer reached the edge of the bank, it headed due W. along the top of it. Only the lead wolf pursued this animal. The other deer had headed down the bank to the S.E., and at least a few of the wolves followed it.
>
> "We could not watch both deer, so we continued following the first. The deer had no trouble in snowdrifts, but the wolf was hindered by them. The wolf followed the deer for about 200 yards along the top of the bank, and then gave up after losing ground. The wolf had run a total distance of about 250-275 yards. He then lay down and rested.
>
> "We noticed at least 3 wolves stopped part way down the bank in the trail of the second deer. However, we did not

see the remaining wolves or the second deer.

"Eventually (after about 5 minutes), these wolves joined the first, and all rested. At 4:25 p.m., one wolf started toward a third deer, which had been lying under a tree while the former chase took place. The deer was about 150 yards from where the wolves rested, and it had stood before the wolf started toward it. We could not see whether the deer or wolf bolted first, but suddenly both animals were bounding away. The wolf chased the deer about 125 yards and gave up after losing ground. The other wolves followed slowly in its trail, and all assembled and rested. The deer continued running for at least ¼-mile."

"27 February 1969. 2 miles N. of August Lake.

"1059's pack of 5 was heading N.E. at 4:10 p.m. when they got to within 100 yards of 2 standing deer. The deer had been standing alertly in a shallow draw, and when at least 2 wolves got to within 100 yards, they fled. The wolves began running after them.

"The deer were in snow up to their bellies and had to hesitate slightly at each bound. But they ran fast. We could only see one wolf very much [of the time]. It was also having a difficult time in the snow, and after a total run of about 250 yards (100 to the deer's original location and 150 after the deer), the wolf lay on the snow and rested about 10 minutes. The deer ran only about 200 yards more and stood alertly for the next 20 minutes at least. The wolves then went on.

"27 March 1969. About 2 miles S. E. of Central Lakes, Minnesota.

"At 3:00 p.m. while we were following wolf 1051 by aircraft in above location, we saw a deer running very quickly on top of the crusted snow and then stand and watch its backtrail. About 1½ minutes later we saw 1051 running along the same route. We did not see when the deer fled again, but saw it running about 100 yards from the wolf and doubling back paralleling its original route. When the wolf got near the approximate doubling-back point, he lay down and rested for about 5 minutes. The deer continued fleeing for about 350 yards, stopped, and for several minutes faced its backtrail. The wolf finally continued on in his original direction, giving up the chase.

"At 4:30 p.m.—1½ miles S. of Central Lakes, Minnesota—Wolf 1051 had come to within 100 yards of [four-lane] Highway 53 and was hesitant to approach it. Several cars were going by in both directions. Thus the wolf headed S. parallel with the highway about 150 yards E. of it.

"Suddenly two deer, which we had noticed S. of the wolf earlier, fled across the highway. The wolf soon got to the point where they crossed, hesitated about a minute and then ran across. No cars came at that time.

"We could not always see the deer or the wolf when W. of the road because there were several patches of evergreens. The wolf did head straight W. after crossing the road. Then about 250 yards W. of this point we saw a deer come out onto an old woods road which lay in a N.W.-S.E. axis. The deer ran N.W. on the road and then we saw the wolf where the deer had come out onto the road. While the deer ran N.W., the wolf cut into the woods to his right, N.E. We could not see it then but presumed it was running N.W. paralleling the road.

"After the deer had run about 50 yards up the road, it also headed N.E. into the evergreens. Within a few seconds it fled right back out and started S. E. down the road. The wolf was about 50 feet behind it and began gaining.

"When the deer got back to where the wolf had headed into the woods from the road before, it also headed N.E. into the woods. The wolf was then about 20 feet away and the deer was headed N. around in a circle with the wolf closing in on the outside. The wolf did not emerge from the evergreens for at least 15 min-

> utes, nor did we see the deer, so I presume the wolf killed the deer. [But see entry for April 1.]
>
> "1 April 1969. Dan Frenzel and I searched the area described on March 27 for 1 hour and found no sign of a kill. Old wolf tracks were seen, but only a single wandering track. No concentration such as usually seen at kills. Best conclusion is that 1051 did *not* kill the deer where seen from the air March 27."

We also saw 1055 and her associate actually kill a deer, on February 6, 1969, but we did not realize what was going on and it happened so fast that we only saw a wolf rushing and biting at the front end of the downed animal. The chase had to have lasted only a few seconds.

In addition to the above direct observations, we also were able to piece together from tracks in the snow the chase and successful encounter between a single wolf and a deer in two instances. In the first case, on January 25, 1967 (11:50 a.m.), we arrived at the scene (near Grub Lake, just N. of Snowbank Lake) within an hour of the encounter, and the wolf was still feeding on the deer, which had been a 2½-year-old female. Mech examined the area from the ground and made the following observations:

> "The deer had come S.W. down the middle of the lake at a fast walk, turned around, backtracked a few yards and headed to the N.W. shore of the lake. Meanwhile a wolf had come at a trot along the deer's track, but it had cut to the N. W. shore about 50 yards N. E. of where the deer had. When still on the the ice about 15 feet from shore, the wolf began running as evidenced by his long bounds. He continued running inland about 50 feet from shore toward the deer. The deer had walked inland from the shore and may have stood there about 25 feet from shore. Suddenly it had bounded away. The bounding wolf track was in the same trail as the deer's for about 25 yards but then it paralleled the deer's about 5 feet away on the inland side. After about 125 yards from where the deer flushed, the deer was pulled down. It was *not* on its side but rather had sunk into the snow in more-or-less of an upright position.
>
> "Apparently the deer had just about reached the shore when the wolf noticed it, and it detected the wolf. At this time the wolf must have been up the shore about 50 yards where his tracks first showed he began bounding. There was no sign that the wolf had spotted the deer on the lake and had tried to cut it off from shore by running inland along the shore and then waiting for the deer to come inland. Once the wolf had begun bounding, he continued until he pulled the deer down . . . Sign showed that the deer dropped within about 20 feet of where she had begun bleeding."

The second case involved a 5½-year-old buck, No. M-28, which had arthritis of his right hind foot and probably had defective gait (see Mech and Frenzel p. 35). The attack took place on Basswood Lake on February 2, 1967, and excerpts from field notes by Mech follow:

> "A single wolf had killed this deer after chasing, following, or tracking the deer about 3.75 miles. The deer's last 350 yards was a fast walk – the tracks were one in front of the other and about 2 feet apart, and there was no leaping or bounding. Same with the wolf – a fast trot.
>
> "Where the tracks came together, the deer apparently had fallen, but there was no blood. From there, the deer dragged its feet or the wolf for about 25 feet and then went down again. The wolf circled the deer, and for the next 150 feet, the 2 animals had fought or scuffled and then the deer had gone down where we found it.
>
> "The 4-mile persistence of this wolf–whether tracking, following, or chasing the deer–is remarkable [compared with most chases] and makes me believe the wolf had good reason to believe it could kill the deer."

Our observations of wounds on fresh kills confirm the following description by Stenlund (1955, p. 31) of the location and manner of attack of wolves on deer: "No evidence of hamstringing of deer was found on freshly killed carcasses, although the possibility does exist. Usually deer are run down from behind, the wolf or wolves biting at the hind flanks and abdomen, or at the hind flanks and head region simultaneously."

On each kill, all the flesh and much of the skin and bones were eaten, at least during the winters of 1966-67 and 1967-68. This was also true during December 1968 and much of January 1969. However, during February and March 1969 when an unusual accumulation of snow had built up, most of the kills were only partly eaten (see Mech *et al.*, page 51). In previous years deer freshly killed by single wolves were sometimes found with only a few pounds of flesh or viscera missing. However, in each case the carcasses were almost completely cleaned up within a few days, often by packs to which the single wolves may have belonged (Mech 1970).

Usually the first parts of a carcass to be eaten are the hams and part of the viscera from the coelomic cavity. In one case where a wolf was interrupted while feeding it was apparent that the animal had been stripping the omental fat from the carcass. This may be the wolf's favorite part of a deer, for the stomach of one wolf that we examined in January 1967 contained nothing but such fat.

The average consumption and kill rate of deer by wolves has not yet been determined, but we have some information bearing on the subject. Because our data were obtained during a winter of unusually deep snow, and it was obvious that wolves were killing more deer than they could eat at the moment (see Mech et al., page 51), our figures should be considered much higher than average. However, they should be useful in that they probably represent the maximum kill rate not only throughout the year but also throughout a period of many years.

By observing each of our radiotagged wolves whenever possible and noting whether or not it was feeding on a kill, we learned that our wolves generally remained close to their kills for periods of from 1 to 7 days, depending on how recently they had eaten (fig. 32). Thus, when a wolf was found at a new location each day, the assumption could be made that the animal did not currently have a kill.

We assumed that wolves found at fresh kills (fig. 33) had made them unless there was evidence to the contrary as with 1053, the scavenger. When a wolf was found at one location for several consecutive days but could not be observed, we assumed it was feeding on a kill, since whenever wolves were observed remaining in the same location for several days they were seen feeding. Thus a range of possible number of kills per wolf was determined, with the lower limit being the known minimum and the upper limit the possible maximum. When more than one wolf fed on a kill, as with the pack, the figures were calculated on a per-wolf basis.

In this way we obtained data on a total of 468 wolf-days and found a total kill of 35 to 48 deer (table 7). This averages out to a kill rate of one deer per 10 to 14 days per wolf. The figure varied considerably among individuals —

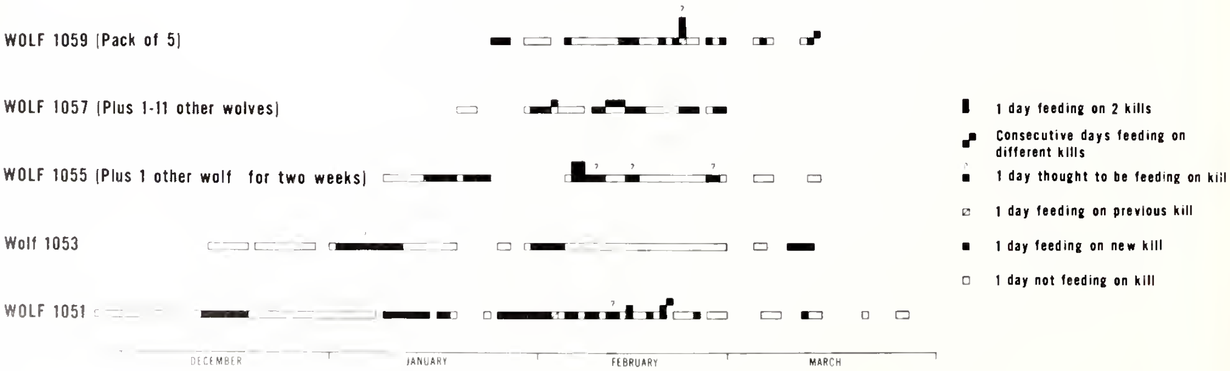

Figure 32.—Periods spent by radiotagged wolves and their associates feeding on kills judged to be their own. This does not include periods when they were known to be feeding on carrion.

Figure 33.—Radiotagged wolf (upper left) found at kill (lower right). (Photo courtesy of L. D. Frenzel.)

1051 had the highest rate of one kill per 6.3 to 7.2 days, and each wolf in 1059's pack had the lowest rate (except for 1053, the scavenger) of one deer per 14.0 to 18.0 days.

It is significant that the pack of five wolves had a lower kill rate per wolf than did single wolves and pairs. This is explainable because the ability of wolves to kill deer during early 1969 was much greater than usual (see Mech *et al.*, p. 51). Thus single wolves probably could kill deer just as easily as could packs, but they did not need to share them. This differs markedly from the situation on Isle Royale, where lone wolves usually feed only on moose remains left by packs (Mech 1966a, Jordan *et al.* 1967).

That lone wolves had more of a food surplus than those in the pack is confirmed by the figures on the average number of days that the various wolves fed on kills (table 7). Wolf 1051 spent an average of only 2.2 to 2.4 days feeding at each of his kills, whereas 1059's pack of five spent an average of 5.8 to 7.5 wolf-days at each kill. Further confirmation is found in the fact that even when most wolves were leaving their kills partly uneaten, a pack of 8 to 10 wolves (probably that to which 1057 belonged) was seen completely devouring a kill.

Table 7.—Kill rate of deer by radiotagged wolves and their associates

Wolf number	Wolves	Dates	Wolf-days of data	Kills	Wolf-days per kill[1]	Wolf-days feeding	Wolf-days feeding per kill
	Number		Number	Number	Mean number	Number	Mean number
1051	1	Nov. 26 to Apr. 3	101	14-16	6.3- 7.2	33- 40	2.2-2.4
1053[2]	1	Dec. 14 to Mar. 27	75	2- 3	25.0-37.5	9- 18	4.5-6.0
1055	1- 2	Jan. 9 to Mar. 14	61	4- 9	6.7-15.0	13- 25	2.8-3.3
1057	1-13	Jan. 24 to Feb. 28	51	5- 7	7.3-10.2	25- 33	4.7-5.0
1059	5	Jan. 25 to Mar. 14	180	10-13	14.0-18.0	75	5.8-7.5
Summary	22	Nov. 27 to Apr. 3	468	35-48	[3] 9.8-13.4	145-181	[4] 3.8-4.1
		Before Feb. 1	142	7- 9	[5] 15.7-20.3	39- 56	5.1-5.6
		After Jan. 31	326	28-39	8.4-11.6	106-125	3.2-3.8

[1] Kill rate per wolf.

[2] Figures for this animal are so low because she was basically a scavenger.

[3] Average kill rate per wolf for all radiotagged wolves and their associates, derived by dividing total number of wolf-days by total number of kills.

[4] Average number of days that each wolf spent at each kill, derived by dividing total number of wolf-days spent feeding by the total number of kills.

[5] This figure probably is the closest to the actual kill rate during most winters.

Therefore it is probable that the kill rate per wolf for members of the pack of five is much closer to the usual average winter kill rate. It can still be considered higher than the usual winter rate, however, because this pack also was leaving some of its kills partly uneaten.

A reasonable approximation of the average kill rate during most winters would be the rate found for our radiotagged wolves before February 1, because the relations among the wolves, the deer, and the snow during that period were not unlike those of most winters. The average kill rate per wolf before February 1 was estimated at one deer per 15.7 to 20.3 days.

After this period, the rate increased to about one deer per 8.4 to 11.6 days, and an estimated 50 percent of the available food was left uneaten (see Mech *et al.*, page 51). This implies that the kill rate during February and March was about twice as high as usual. On this basis, the usual kill rate would be estimated at one deer per 16.8 to 23.2 days, which checks well with the rate found before February (one deer per 15.7 to 20.3 days). Thus we feel that an estimated kill rate of about one deer per 18 days per wolf is a close approximation of the average kill rate for most winters. This is about 50 percent less than the kill rate of one deer per 4 days estimated by Stenlund (1955) for two packs of three wolves (one deer per 12 days per wolf). However, it compares favorably with the actual kill rate of one deer per wolf per 17.6 days found for a pack of eight wolves in Ontario.[4]

Once the average rate of kill is known, the average food consumption per wolf can be calculated. The average deer (considering both fawns and adults) from the Superior National Forest during winter weighs about 113 pounds (calculated from Erickson *et al.* 1961), and an arbitrary 13 pounds can be deducted from this for inedible portions. This leaves 100 pounds of deer per wolf per 18 days, or 5.6 pounds per wolf per day. This figure is much less than the 10 to 14 pounds estimated consumption rate for wolves feeding on moose on Isle Royale (Mech 1966a). However, much variation can be expected in an animal whose physiology must be adapted to a feast-or-famine existence.

Wolves can be maintained in captivity on 2.5 pounds of meat per day, and large active dogs (*Canis familiaris*) require 3.7 pounds per day, so it is likely that the minimum daily requirement for wolves in the wild is about 4.0 pounds per day (Mech 1970). This figure agrees well with the estimated consumption rate for our study area.

Relative Population Density

Censusing wolves in a 1.5-million-acre study area is a difficult task, and we have no direct information on which to base a population estimate. However, some deductions can be made about the relative population densities in our study area between the period 1948 to 1953 and the period of the present study, 1967 to 1969.

R. A. Rausch (1967a) hypothesized that the frequency of large packs is higher when population density is high, and presented evidence supporting this idea. On this assumption, a comparison of pack-size distributions between various periods can indicate relative population densities between periods. The advantage of this method is that it eliminates the usual type of year-to-year biases in wolf censuses such as might result from differences in precise census route, type of aircraft, skill of observers, and other conditions. Only a difference that would cause a bias in the *size* of the packs seen would be of importance.

Therefore, we tested the difference in size distributions of population units between the 1948-53 study period and the present period (table 1), using a Kolmogorov-Smirnov two-sample test (Siegel 1956). The average "pack" size in the earlier years was 2.8, compared with 4.2 at present; thus pack sizes are significantly larger at present (95 percent level). This indicates that the population density from 1967 to 1969 may have been higher than from 1948 to 1953. This apparent change may be attributable to a reduction in snaring, trapping, and aerial hunting that took place between the two periods as a result of changes in State game regulations.

A similar comparison between our observations from 1967 and those from 1968-69 (table 1) shows no significant difference between these

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[4] *Kolenosky, G. B. Wolf movements, activities and predation impact on a wintering deer population in East-Central Ontario. (Manuscript in preparation for publication.)*

years, so it appears that the density of wolves in our area has remained about the same over the period of three winters. This agrees with the esults of several other studies summarized by Mech (1970) in which wolf populations unaffected by man have been found to remain relatively stable from year to year.

DISCUSSION AND CONCLUSIONS

The movements, behavior, and ecology of the wolves in our study area during winter are variable, and are influenced considerably by snow conditions. This may explain the fact that in late February 1969 wolves 1051, 1053, and 1055 suddenly extended their travels and range (fig. F-34 and table 4).

However, increased travel may have resulted from other factors. For one thing, the wolves apparently did not need to spend so much time hunting as before. Because of the deep snow, the ability of wolves to capture deer increased, and the animals had a surplus of food. Perhaps under such conditions wolves may use more of their energy for traveling than for hunting.

In this respect it is interesting that 1051 moved right out of his area and traveled into country that presumably was unknown to him. Wolves 1053 and 1055 each ventured into an area that was almost devoid of deer and that even had few moose in it. Without sufficient fat reserves in all these animals, it would seem disadvantageous for them to have made these travels.

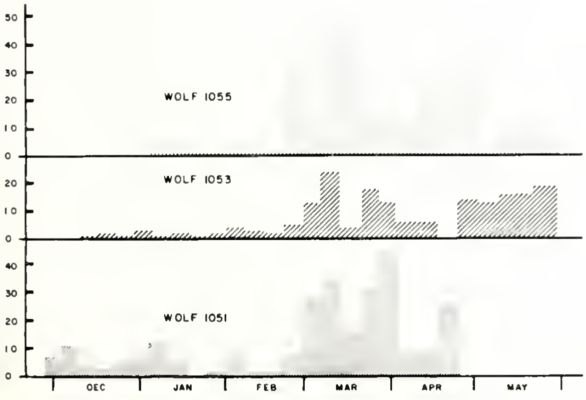

Figure 34.—Net weekly (straight-line) distances traveled by three radiotagged wolves.

Evidently wolves can obtain enough food in much smaller areas than these three animals used after February. Both 1059's pack of five and 1057 lived in relatively small areas throughout the winter and seemed to survive well. Before late February, 1051, 1053, and 1055 did also. Thus some factor other than food must have influenced the movements of these three animals from late February through April.

The fact that the increased movement began during the breeding season makes one suspect a relationship between the two. One possibility is that the factors increasing the hormonal flow associated with breeding in adults stimulate a hormone output in immature or subordinate individuals that causes an increase in their movements. An alternative is that the breeding behavior of resident packs involves the beginning of, or an increase in, aggression toward neighboring nonmembers. This might force the lone animals to shift about over large areas in avoidance of such aggression.

Whatever the cause of the changes in movements of these animals, the fact that the pack used a much smaller area than any of the lone wolves may be of central importance in trying to understand the organization of the wolf population. The following pieces of information are also pertinent to such an understanding: (1) the pack, which can be presumed to include a breeding pair (Mech 1970), chased other wolves in its area; (2) the lone wolves, which apparently did not breed, were tolerant of, or indifferent to, other lone wolves in their areas; (3) the ranges of the lone wolves overlapped considerably (fig. 35); (4) the lone wolves seemed to avoid certain large areas that one might logically think would have been visited by them (fig. 35); and (5) packs of wolves were sometimes observed in these large areas (fig. 35).

From the above information it can be hypothesized that the wolf population consists basically of groups of breeding packs defending territories of limited size, with lone wolves and other nonbreeding population units that are tolerant of each other shifting about in much larger nonexclusive areas among these territories. The information from Isle Royale (Mech 1966a, Jordan

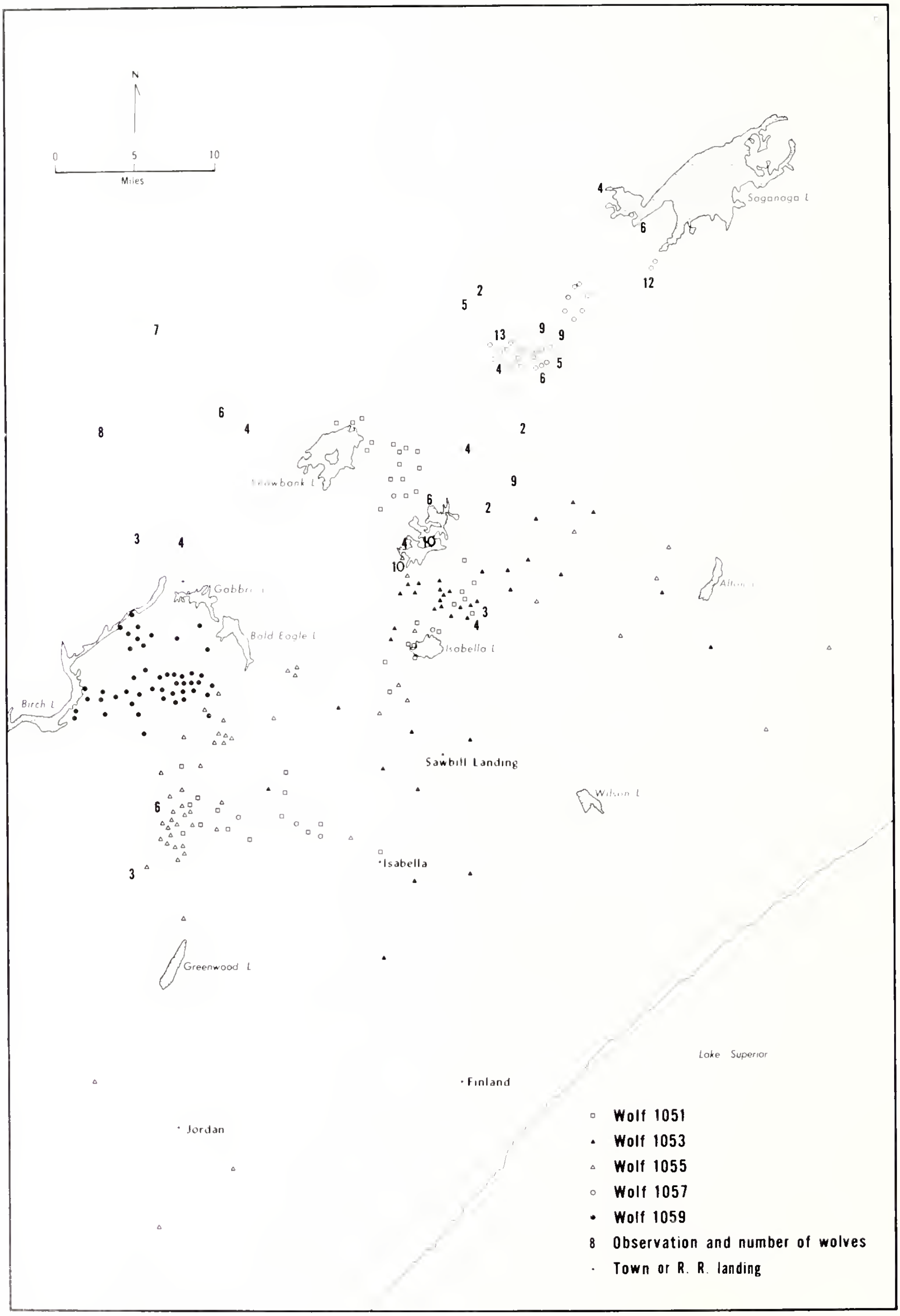

Figure 35.—Locations of all radiotagged wolves and unmarked packs observed during winter 1968-69, *except dispersal of* 1051 *out of the study area. Only selected lakes shown.*

et al. 1967) is consistent with this idea, but the area of that island (210 square miles) is too small to allow untested extrapolations to be made about spacing in much larger wolf populations. Data from Algonquin Park, Ontario (Pimlott *et al.* 1969) also strongly suggest this hypothesis. However, the packs studied there could not be identified with certainty, and little information was obtained about nonbreeding population units.

To test the proposed hypothesis with certainty, a larger number of identifiable breeding and nonbreeding population units from the same general area must be followed during at least one winter. This will be the main objective of our next study.

SUMMARY

During the winters of 1966-67, 1967-68, and 1968-69, aerial observations of timber wolves (*Canis lupus*) were made in the Superior National Forest in northeastern Minnesota, where the primary prey is white-tailed deer (*Odocoileus virginianus*). In 480 hours of flying during the study, 77 sightings involving 323 wolves were made. In addition, during 1968-69, five radiotagged wolves and their associates were tracked via receivers in aircraft for a total of 570 "wolf-days." Visual observations were made during 65 percent of the times the wolves were located from December through April.

The average size of each population unit (including single wolves, pairs, and packs) observed was 4.2, although packs of as many as 13 wolves were sighted. Radiotagged wolves spent most of their daylight hours resting during winter, and when traveling, hunting or feeding during the day, tended to do so before 11:00 a.m. and after 3:00 p.m.

Considerable variation was discovered in the movement patterns of individual wolves, with straight line distances between consecutive daily locations ranging from 0.0 to 12.8 miles, and between weekly locations, 0.0 to 49.0 miles. A pack of five wolves used a range about 43 square miles in extent, whereas lone wolves covered areas many times this size. One animal in an apparent dispersal was tracked a straight line distance of 129 miles between extreme points.

A reddish male wolf was the leader of the pack of five and led two observed chases after alien wolves in the pack's territory. This animal was also most active during scent marking by the pack. Lone wolves were apparently indifferent to other wolves, and thus exclusive areas, or territories, were not observed among lone wolves.

Hunts involving a total of seven deer were observed and described, and two successful attacks on deer were interpreted from tracks in the snow. Wolves generally consumed all the flesh and much of the hair and bones from kills, except during February and March 1969 when extreme snow conditions increased the vulnerability of deer to an unusual degree. At that time kills were found that were partly or totally uneaten. The kill rate by radiotagged wolves and associates during the winter of 1968-69, based on 468 wolf-days of data, varied from one deer per 6.3 days to one per 37.5 days per wolf, with the average being one deer per 10 to 13 days. The rate was much lower per wolf for members of the pack of five than for lone wolves, and much lower before February 1, 1969, than after. The average rate of kill during more usual winters was estimated to be about one deer per 18 days. This is a consumption rate of about 5.6 pounds of deer per wolf per day.

Indirect evidence based on comparisons of pack-size distributions for different periods indicates that the wolf density in the study area may have increased since 1953, but that it has remained the same from 1967 to 1969.

On the basis of data presented in this paper, the following hypothesis about the organization of the wolf population studied is proposed: The wolf population consists basically of groups of breeding packs defending territories of limited size, with lone wolves and other nonbreeding population units, tolerant of each other, shifting about in much larger nonexclusive areas among these territories.

ACKNOWLEDGMENTS

This study was supported by Macalester College, the U.S. Bureau of Sport Fisheries and Wildlife, the USDA Forest Service, the Minnesota Department of Conservation, and the New York Zoological Society. Special thanks are due the following for their help and cooperation with this project: Mr. J. O. Wernham, former Supervisor, Mr. L. T. Magnus, Wildlife Biologist, numerous District Rangers, and other supporting personnel of the Superior National Forest, Mr. J. T. Morgan, North Central Forest Experiment Station; and Mr. S. E. Jorgensen and Mr. C. E. Faulkner, U.S. Bureau of Sport Fisheries and Wildlife.

Drs. C. T. Cushwa, L. F. Ohmann, Catherine Ream, and D. G. Schneider aided in the field work. Mr. W. W. Cochran provided advice and suggestions on the radiotracking technique, Dr. U. S. Seal furnished the drugs and the advice on their use with wolves, and Mr. R. Himes contributed significantly in the wolf trapping. Mr. L. Ringham, Ontario Department of Lands and Forests, granted permission for research personnel to radiotrack wolves crossing into Quetico Park, Canada. Numerous students from Macalester College also contributed to the field effort.

Thanks are also due pilots Robert Hodge, Pat Magie, Ken Bellos, Don Murray, Jack Burgess, and several others, who along with pilot-biologist John Winship, expertly flew the aircraft used in the study.

This report was reviewed by the following biologists: Mr. G. B. Kolenosky, Dr. P. A. Jordan, Mr. M. H. Stenlund, and Dr. D. L. Allen.

Mr. Wallace C. Dayton, Miss Elizabeth Dayton, and the Quetico-Superior Foundation, all of Minneapolis, generously contributed funds to support Mech during the preparation of the paper.

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AN ANALYSIS OF THE AGE, SEX, AND CONDITION OF DEER KILLED BY WOLVES IN NORTHEASTERN MINNESOTA

L. David Mech and L. D. Frenzel, Jr.

The selective effect of predation on prey populations is of significance in studies of evolution and population dynamics. Selective predation can be an important agent in the process of natural selection, and it influences the extent to which predators limit the numbers of their prey.

One of the predators most commonly chosen for investigating the selective effect upon prey is the wolf (*Canis lupus*). Because animals preyed upon by wolves generally are large, their remains can be more easily located and examined. It already has been established that in most areas wolves kill primarily young, old, and other inferior members of such prey populations as Dall sheep (*Ovis dalli*), moose (*Alces alces*), caribou (*Rangifer tarandus*), bison (*Bison bison*), and musk-oxen (*Ovibos moschatus*); evidence for this generalization has been summarized by Mech (1970).

However, only recently has it been shown that this generalization may extend to predation on the smallest hoofed prey of the wolf in North America, the white-tailed deer (*Odocoileus virginianus*). Pimlott *et al.* (1969) demonstrated a difference between the age structure of 331 deer killed by wolves during winter in Algonquin Park, Ontario, and 275 deer assumed to represent the actual population in the same area. Whereas only 13 percent of the deer from the population at large were estimated to be more than 5 years old, 58 percent of the wolf-kills were in this age category.

We employed a similar analysis for deer killed by wolves in northeastern Minnesota, but used a more refined aging technique and included comparisons of the age and sex structures of various subsamples of wolf-kills. Whereas the Ontario research involved a prey population unhunted by man, our work was carried out on both a hunted population and on one relatively unhunted. Further comparisons were made between deer killed during periods of normal snow conditions and those taken during unusually high snow accumulations. The incidence of various abnormalities in wolf-killed deer was also compared with that in hunter-killed animals.

The study was carried out in the Superior National Forest in northern St. Louis, Lake, and Cook Counties of northeastern Minnesota (fig. 1), in conjunction with other aspects of wolf research (see Mech *et al.* p. 1).

METHODS

The investigation began in February 1966 and continued through March 1969; the basic objective was to examine as many wolf-killed deer as possible and compare their ages, sex, and condition with a large sample of deer from the population at large in the same area. Wolf-kills were examined only during December through March when they could be found from the air. Aircraft ranging in size from an Aeronca champ to a Cessna 206 were used to fly over frozen lakes at altitudes up to 2,000 feet to locate wolves (fig. 2), wolf tracks, or kills (fig. 3). We often discovered kills by tracking a wolf pack.

During the winter of 1968-69 this method of finding kills was supplemented by radiotracking five wolves and their associates via aircraft (see Mech *et al.*, p. 1). The latter technique resulted in increased discovery of inland kills.

A deer carcass was judged killed by wolves if the death had been recent, if tracks or other sign indicated that wolves had fed upon it, and if no other possible cause of death was discovered. Carcasses fed on by wolves but not clearly identifiable as kills were labeled "probable" wolf-kills. Although the cause of death of the specimens in this latter category could not be deter-

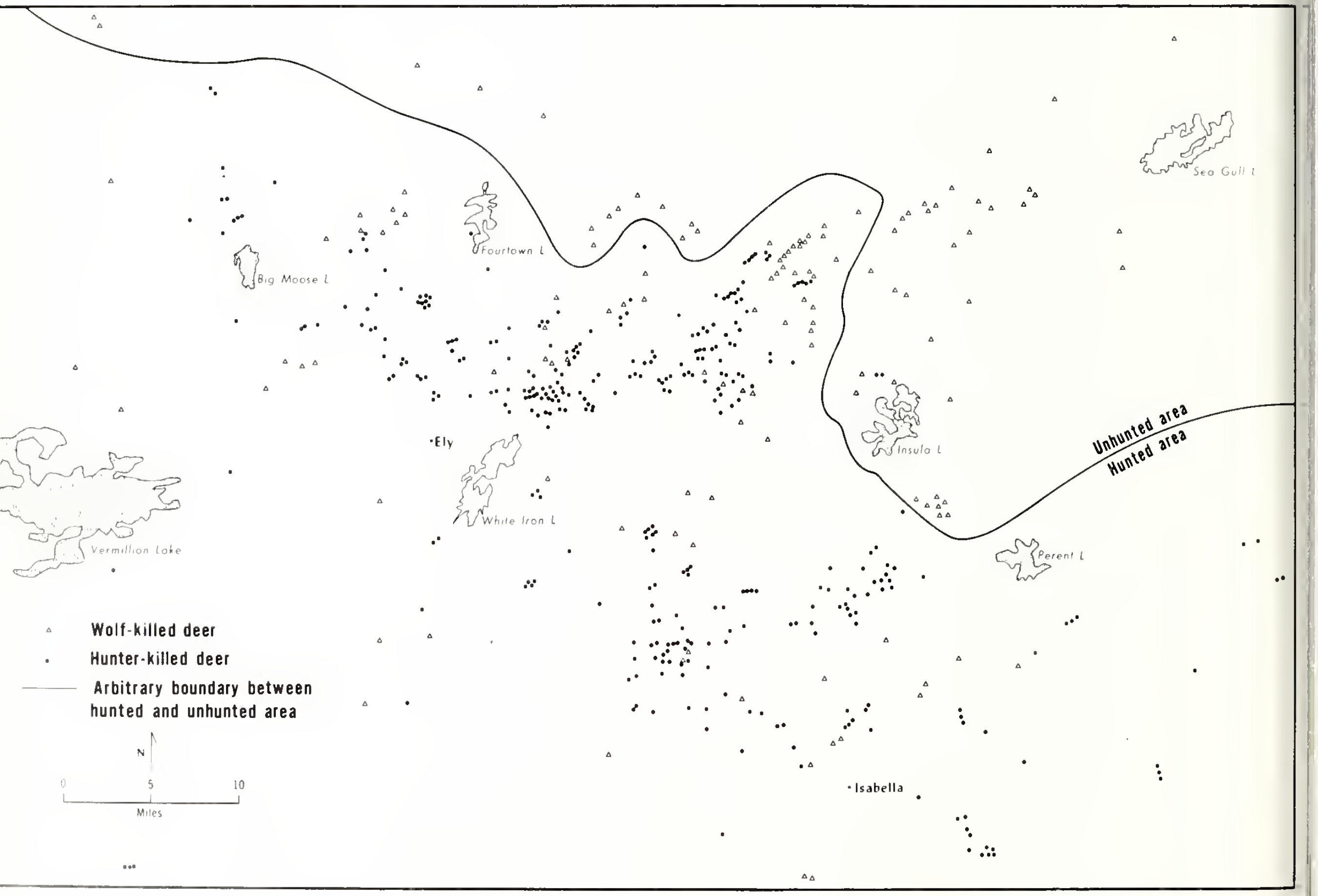

Figure 1.—The study area showing locations where wolf-killed and hunter-killed deer were taken. Line arbitrarily separates the hunted area from the wilderness area.

mined with certainty, there was no reason to believe other agents were involved.

In addition to the wolf-kills examined by project personnel, data and lower jaws from deer judged killed by wolves were contributed by other biologists, game wardens, forest rangers, and others whose competence was known. Nevertheless, if certain identification of carcasses as wolf-kills was not possible, the data were relegated to the "probable" wolf-kill category.

Figure 2.—Wolves were located from the air, usually on frozen lakes. (Photo courtesy of L. D. Mech.)

Whenever possible, kills discovered from the air were examined on the ground (fig. 4). Often only skeletal parts remained, but soft parts were also examined when available. Femur marrow, heart, lungs, liver, kidneys, repreductive tracts, and omenta were usually inspected in the field for fat, parasites, and abnormalities, and the degree of subcutaneous back fat was also noted. Hoofs and lower legs were checked, and those showing pathological conditions or abnormalities were collected and examined by the Veterinary Diagnostic Laboratory of the University of Minnesota. All lower jaws found were collected, aged, and examined for dental abnormalities and pathological conditions.

In November 1967 and 1968 hunter-check stations were operated on the study area (fig. 5), and deer bagged by hunters were field-checked for age (Severinghaus 1949) and hoof abnormalities. As many lower jaws as possible were collected from field-checked deer and other deer killed in the area for age determination and examination for abnormal dentition.

An assumption was made that the age structure and incidence of abnormalities in the sample of hunter-killed deer would be *reasonably representative* of those in the population at large, an assumption also implicit in a similar comparison made by Pimlott *et al.* (1969). In this respect, the following statements by Maguire and Severinghaus (1954, p. 109) about deer in New York State are pertinent: "It may be concluded that, considering the open season as a whole, wariness does not significantly distort the age composition of the [deer] kill in relation to that of the corresponding wild population, except possibly for buck seasons of only 1 or 2 days duration . . . A reliable appraisal of the age composition of the kill by hunting may be obtained through the operation of roadside checking stations." However, in critically reviewing the present paper Severinghaus stated that in States such as Minnesota, with fewer hunters and higher hunter success rates, age compositions of deer from checking stations may not be the same as those of wild populations. Reviewers Peek and Downing also made similar comments.

Nevertheless, for our comparison with wolf-killed deer it is not necessary that the hunter-kill age structure be exactly representative of the age structure of the actual deer population. All that is required is that there be reasonable agreement between the two. The hunting regulations in our study area allow a 9-day period of taking deer of any age or sex, and a single hunter may legally shoot as many deer as he and his party or associates have permits for. Thus there is no reason for selective hunting, and

Figure 3.—Wolf-kills were easily spotted from aircraft. (Photo courtesy of L. D. Mech.)

we feel confident that the age structure of the hunter-kill in our study area does basically represent that of the deer herd at large.

Two laboratory techniques were used for determining the ages of deer from the lower jaws or mandibles — a tooth replacement and wear technique (Severinghaus 1949) and an incisor-sectioning method (Gilbert 1966). The tooth-wear technique requires only the molariform teeth but it is more subjective and inaccurate, particularly in older deer (Ryel *et al.* 1961). Incisor sectioning requires only incisors and appears to be much more accurate.

However, because the incisors had been lost from many of the wolf-kills, and because the tooth-wear technique was used at checking stations, both methods were applied in the laboratory. Mr. David W. Kuehn (1970) sectioned and aged the incisors. Fortunately there was a sufficiently large sample of mandibles with molariform teeth and incisors from both wolf-killed and hunter-killed deer to enable us to devise a table showing the actual ages (based on incisor-sectioning) of each of the jaws assigned to various tooth-wear classes. This table was then used to distribute the ages of specimens that con-

Figure 4.—As many wolf-killed deer as possible were examined from the ground. (Photo courtesy of L. D. Mech.)

tained only molariform teeth. For example, because it was found that 37 percent of the jaws aged 4½ years old by tooth wear were actually 5½ years old, we assigned 37 percent of the incisorless jaws aged 4½ by tooth wear to the 5½-year category. Similarly, another conversion chart comparing field age determinations of hunter-killed deer with ages based on incisor sectioning of the same jaws was employed to distribute the ages of field-aged, hunter-killed deer for which jaws or incisors could not be collected.

RESULTS

We flew a total of 480 hours during this and related research, mainly during January through March 1967 and December 1968 through March 1969; about one-third of this time was devoted primarily to searching for kills. Jaws were examined from 93 wolf-kills and 49 probable wolf-kills.

Figure 5.—Information about hunter-killed deer in the study area was obtained through hunter-check stations. (Photo courtesy of L. D. Frenzel.)

Hunter-check stations yielded information from 335 deer (fig. 6), and data on 98 additional hunter-killed deer were contributed by other hunters. Incisors were collected from 82 of 214 hunter-killed deer checked that were older than yearlings; comparisons were then made between ages of the deer based on incisor sectioning and those based on field checks using the wear method. Similarly, incisors were sectioned from 195 wolf-killed and hunter-killed deer older than yearlings that had been aged by the tooth-wear method in the laboratory, so that these two methods could be compared

Figure 6.—All hunter-killed deer examined were checked for age. (Photo courtesy of L. D. Frenzel.)

(Kuehn 1970). (Note: incisor-sectioning is unnecessary for fawns and yearlings because animals of these ages can be aged objectively by the progress of tooth replacement.)

Because age or sex distributions might differ in the various subsamples of deer examined during this study, these parameters were compared in subsamples of both wolf-kills and hunter-kills (table 1). No significant differences were found in the age or sex structures between the known wolf-kills and "probable" wolf-kills, so these subsamples were pooled and considered wolf-kills for all subsequent comparisons.

Three significant differences in sex ratio were found among the subsamples of wolf-kills: (1) wolves killed more female fawns than male fawns, but more male adults than female adults (table 2); (2) more of the adults killed in the hunted area were females, while in the wildnerness more males were taken (table 3); and (3) after January 1969, when snow was unusually deep, 57 percent of the deer killed were females, compared with only 38 percent before this date.

Table 2.—Sex ratios of hunter-killed deer and wolf-killed deer from northeastern Minnesota

Age	Hunter-killed deer			Wolf-killed deer		
	Number	Percent male	Percent female	Number	Percent male	Percent female
Fawns	108	50	50	22	41	59
Adults	315	68	32	105	54	46

In the comparisons of the subsamples of hunter-kills, the only statistically significant difference found was that the adult subsample had a higher proportion of males than the fawn subsample. No significant difference was found in the age structures of the subsamples, so these were all pooled into a sample of 433 hunter-kills for comparison with the wolf-kills. For the same reason, the entire sample of 142 wolf-killed deer was used for a comparison with the hunter-killed sample.

Wolf-killed deer in our sample, with an average age of 4.7 years, were significantly older (99 percent level) than hunter-killed deer, with an

Table 1.—Results of statistical comparisons between various samples of deer kills from northeastern Minnesota

Sample size	Sample description	VS	Sample size	Sample description	Results of comparisons		Direction of difference
					Age structures[1]	Sex ratios[2]	
	Wolf-kills:[3]			Wolf-kills:[3]			
93	Known		49	Probable	Nonsig.[4]	Nonsig.	--
42	Jan.-Mar. 1967		83	Dec. 1968-Mar. 1969	Nonsig.	Nonsig.	--
66	Male		61	Female	Nonsig.	--	--
50	Wilderness area		92	Hunted area	Nonsig.	Nonsig.	
41	Adult, wilderness		64	Adult, hunted area	--	Sig., 99 percent	More females in hunted area
96	Lakes[5]		32	Inland	Nonsig.[6]	Nonsig.	--
66	Before Feb. 1969		77	After Jan. 1969	Nonsig.[7]	Sig., 95 percent	More females after Jan.
105	Adults		22	Fawns	--	Sig., 95 percent	More female fawns
	Hunter-kills:			Hunter-kills:			
110	Field aged, 1967		225	Field aged, 1968	Nonsig.	Nonsig.	--
335	Field aged		98	Lab. aged	Nonsig.	Nonsig.	--
132	Lab. aged, males		79	Lab. aged, females	Nonsig.	--	--
89	Field aged, fawns		246	Field aged, adults	--	Sig., 95 percent	More male adults
433	Hunter-kills		142	Wolf-kills	Sig., 99 percent	--	Older deer in wolf-kill
321	Hunter-kills excluding fawns		118	Wolf-kills excluding fawns	Sig., 99 percent	--	Older deer in wolf-kill

1/ Kolmogorov-Smirnov two-sample test (Siegel 1956).

2/ ž test (Downie and Heath 1959).

3/ Because test showed no significant differences in age or sex structure between sample of known wolf-kills and probable wolf-kills, these were combined for all subsequent tests and the pooled sample considered "wolf-kills."

4/ At 95 percent level or greater. (NOTE: Lack of a significant difference does not prove that no difference exists. Rather, it means only that the available evidence does not allow the positive conclusion that a difference does exist.)

5/ Wolf-kills found on lakes were compared with those located inland because of the possibility that kills on lakes may not be representative of kills in general.

6/ Sample too small for test, but no apparent difference.

7/ No significant difference in entire age structures. However, when the percentage of yearlings is compared between the two the difference is almost significant at the 95 percent level.

Table 3.—Sex ratios of wolf-killed deer from wilderness areas and from hunted areas

Age	Wilderness area			Hunted area			Total		
	Number	Percent male	Percent female	Number	Percent male	Percent female	Number	Percent male	Percent female
Fawns	4	0	100	18	50	50	22	41	59
Adults	41	71	29	64	44	56	105	56	44

average age of 2.6 years. For example, deer 5 years of age and older made up 48 percent of the wolf-kills but only 10 percent of the hunter-kills (table 4). The oldest hunter-killed deer in our sample was 9½ years old, but the oldest wolf-killed deer was 14½ (fig. 7).

Because of a possible bias against fawns in the method of collecting data from wolf-kills (to be discussed later), the age structure of the sample of wolf-kills excluding fawns was tested against that of the sample of hunter-kills excluding fawns. The result once again was a highly significant difference between these two age structures (table 1).

As an additional test of the degree to which the age structure of the wolf-killed deer might differ from that of the actual population, we compared our wolf-kill age structure with the age structure of a hypothetical deer population. This was considered advisable just in case the hunter-kill data were poorly representative of the age structure of the actual deer herd. Several hypothetical age structures were constructed and compared according to advice from Downing.[1] In all cases, the comparisons produced the same basic results as the tests with the hunter-killed sample. An example of one comparison is given in figure 7.

A further result obtained by aging the wolf-killed deer pertained to the young individuals killed. The deciduous first incisors of fawns and the decidous premolars of yearlings are usually replaced with permanent teeth by December (Severinghaus 1949). Of 24 wolf-killed fawns examined, however, three (13 percent) taken during January, February, and March had not yet replaced their deciduous first incisors. Of the 13 yearlings found during this same period, nine (70 percent) had failed to replace their deciduous premolars, and two (15 percent) had just replaced them (one deer killed in February and one killed in March).

Mandibles from the 142 wolf-killed deer and 259 hunter-killed deer were examined closely for

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[1] *R. L. Downing. Personal correspondence to L. D. Mech, October 2, 1969.*

Table 4.—Age and sex distribution of deer killed by wolves and hunters in northeastern Minnesota

Age (years)	Wolf-killed deer					Hunter-killed deer				
	Number of:				Percent	Number of:				Percent
	Males	Females	Unknown	Total		Males	Females	Unknown	Total	
Fawns	9	13	2	24	17	54	54	4	112	26
1+	5	7	1	13	9	63	26	1	90	21
2+	3	8	5	16	11	42	19	2	63	15
3+	2	4	2	8	6	47	16	1	64	15
4+	6	3	4	13	9	32	22	1	55	13
5+	12	9	--	21	15	15	12	1	28	6
6+	9	2	1	12	8	3	--	--	3	--
7+	12	4	--	16	11	7	4	--	11	3
8+	4	2	--	6	4	5	1	--	6	1
9+	4	2	--	6	4	1	--	--	1	--
10+	--	3	--	3	2	--	--	--	--	--
11+	--	1	--	1	4	--	--	--	--	--
12+	--	--	--	--		--	--	--	--	--
13+	--	1	--	1		--	--	--	--	--
14+	--	2	--	2		--	--	--	--	--
Total	66	61	15	142	100	269	154	10	433	100

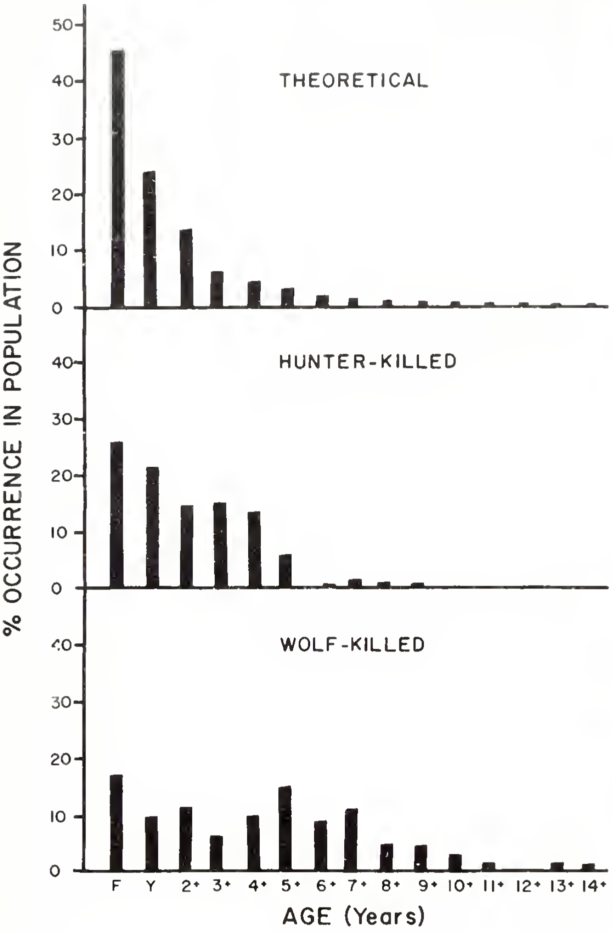

Figure 7.—Comparison between the age structures of deer killed by wolves, deer killed by hunters, and a theoretical population from the same general area of northeastern Minnesota.

abnormal dentition (table 5, figs. 8-10) (Mech *et al.* 1970) and pathological conditions (table 6), and the lower limbs of 75 wolf-kills and 126 hunter-kills were also checked for abnormalities and pathology (table 7, fig. 11). Statistical comparison showed that the incidence of each condition was significantly higher in the sample from wolf-killed deer (table 8).

Jaw necrosis found in our specimens was similar to that described by Murie (1944) for Dall sheep and Mech (1966a) for moose. Generally animals with this condition are old, and ours were no exception.

The following organs were excised from wolf-killed deer end examined grossly in the field for parasites and abnormalities (fig. 12): lungs (six

Figure 8.—Deciduous first premolar (arrow), usually not present in deer, was found in specimen M-31.

Figure 9.—A permanent first premolar (arrow) was discoverd in M-8.

Figure 10.—An extra set of fourth premolars (arrows) occurred in specimens M-96.

Table 5.—Abnormalities in the mandibular dentition of deer from the Superior National Forest, Minnesota

Specimen number	Sex	Age1/	Cause of death	Side of jaw2/	Abnormality
		Years			
M-8	F	3+	Wolves	Right	P_1 present (fig. 9)
				Left	Normal; no P_1 present outside or inside jaw
M-31	F	17 mon.	Wolves	Both	Deciduous P_1 present (fig. 8) and permanent P_1 present inside left ramus; right side not examined internally
M-45	M	4+	Wolves	Right	P_2 rotated 90°
				Left	P_2 absent
M-52	M	4+	Wolves	Right	P_2 absent
				Left	Normal
M-96	F	2+	Hunters	Right	2 permanent P_4s present; both crooked in orientation (fig. 10)
				Left	P_2 diagonal; P_3 normal; P_4 below gumline, pointed posteriorly and wedged against M_1; appears to have pushed out original P_4 (fig. 10)
M-117	M	5+	Hunters	Right	Third column of M_3 reduced
M-191	M	4+	Wolves	Right	Third column of M_3 absent although rudimentary root present
				Left	Third column of M_3 much reduced, peg-like, and almost separate
M-225	--	4+	Wolves	Right	P_2 absent
				Left	P_2 situated diagonally
M-234	F	5+	Wolves	Right	Third column of M_3 reduced
M-254	M	2+	Hunters	Right	P_2 slightly crooked in orientation
				Left	P_2 slanting posteriorly and crowding P_3
M-272	M	5+	Hunters	Right	Third column of M_3 reduced, peg-like, and almost separate
				Left	Third column of M_3 peg-like and separated from second column by 4 mm.
M-296	F	5+	Wolves	Right	Normal
				Left	Extra permanent P_4 crowding original P_4; much like M-96
M-369	M	3+	Hunters	Right	Permanent P_2 still not emerged but appears to be wedged against root of P_3

1/ Based on incisor sectioning method of Gilbert (1966) except that underlined figures are based on tooth replacement or wear (Severinghaus 1949).

2/ Where only one side is listed, the other was not available.

Table 6.—Pathological conditions in the lower jaws of deer killed by wolves or hunters[1]

Specimen number	Sex	Age	Cause of death	Approximate date of death	Condition
		Years			
M-70	M	6½	Wolves	Feb. 1968	Lump in left side of mandible near M_1 and M_2
M-192	M	7½	Wolves	Jan. 1969	Large lump in left diastema apparently from healed fracture
M-206	M	8½	Wolves	Jan. 1969	Light necrosis around base of teeth
M-218	M	3½	Wolves	Feb. 1969	Large lump in left diastema apparently from healed fracture
M-228	F	11½	Wolves	Mar. 1969	Heavy necrosis around molars and extending into bone; half of each M_3 destroyed, both roots and crown
M-236	F	14½	Wolves	Feb. 1969	Light necrosis around base of teeth
M-402	F	10½	Hunters	Nov. 1968	Heavy necrosis and lumps on both sides of mandible

1/ Not including dental abnormalities, which are described in table 5.

animals, normal); heart (seven animals, normal); liver (four animals, one small unidentified tapeworm cyst). Twin fetuses were found in each of two adult does examined.

Twelve deer were checked for body fat in one or all of the following areas: back (subcutaneous), kidneys, heart, omenta. Of these animals, seven had large amounts of fat, but five were almost depleted of fat from these stores. These five were all killed in February or March 1969; three were fawns, and two were yearlings that had not yet shed their deciduous premolars.

Of 69 animals examined for femur marrow condition, two had fat-depleted marrow. One was a fawn killed in March 1969 that had not shed its deciduous first incisors, and the other was a 5½-year-old buck killed in February 1966.

A fawn and a yearling that had died in February 1969 from unknown causes also had fat-depleted marrow. These animals might have been killed by wolves, for wolves had fed on them. However, they could have died from malnutrition and been eaten as carrion.

Table 7.—Pathological conditions in the lower limbs of deer killed by wolves or hunters

Specimen number	Sex	Age	Cause of death	Condition
		Years		
M-28	M	5½	Wolves	Right hind foot: "Old healed ankylosis of the pastern joint ... a spontaneously healed bacterial arthritis with the destroyed joint cavity filled in by solid bone. This deer probably had defective gait"[1] (fig. 14).
M-29	F	5½	Wolves	Front foot: "A 3x4x5 cm. fibrous mass in the subcutis about the digital flexor tendon on the volar surface of the metacarpus. The surface was denuded, ulcerated, and superficially infected by surface bacteria. ... Probably did detract from the animal's speed of flight"[1] (fig. 15).
M-37	F	7½	Wolves	Hind foot: "Probable that the lesion was at one time an active bacterial bone marrow infection that had eventually fistulated to the skin. ... Regional tendons and their sheaths were also present among this inflammation and scarring, and it would be fair to assume that the animal's agility was impaired to some extent."[1]
M-115	M	4½	Hunter	Right front hoof: Broken at tip.
M-196	F	4½	Wolves	Left front foot: "Two severe transverse lacerations on the volar surface. Each was approximately 4 cm. in length. One was located at the margin of the heel, and the other was located several cm. proximad. The more proximal wound had severed the flexon tendons, and the consequent uselessness of the limb was suggested by the splayed toes, the unmarred hoof wall and unworn soles"[2] (fig. 16).
M-227	M	9½	Wolves	Left hind leg: "A diffuse swelling of the distal metatarsal bone, the surface of which was studded with small osteophytic spicules. The major flexor and extensor tendons were forced to assume a convex course over the summits of the dorsal and plantar surfaces of the defect, but the tendon sheaths were clean and the normal wear on soles of the involved toes suggested that functional deficit and pain were probably minimal. ... quite certainly a callus from previous fracture"[2] (fig. 17).

[1] D. M. Barnes. Personal correspondence to L. D. Mech, April 11, 1967.

[2] D. M. Barnes. Undated laboratory report transmitted to L. D. Mech in 1969.

DISCUSSION AND CONCLUSIONS

It has been established that wolves hunting Dall sheep (Murie 1944), caribou (Crisler 1956), moose (Mech 1966a), and other species usually have a low percentage of success. In the case of a pack of 15 wolves hunting moose on Isle Royale during winter, only 4.6 percent of all the moose detected by the pack were killed; considering only the moose that the wolves caught up to or held at bay, the kill rate was 7.6 percent (Mech 1966a).


Figure 11.—The jaws and legs of kills were inspected closely for abnormalities. (Photo courtesy of L. D. Frenzel.)

What little evidence there is about wolves hunting deer indicates that the success rate is also low with this prey species, at least in winter. The senior author has now observed a total of 14 deer being chased by wolves in northeastern Minnesota, mostly by packs of five, seven or eight wolves (Mech 1966b, and see Mech *et al.*, p. 1). In only one case (6.7 percent) did the wolves (a pair) succeed in catching their prey.

Low hunting success rates imply that the circumstances influencing hunts are seldom favorable enough, or the prey animals encountered are seldom vulnerable enough for the wolves to succeed. When the evidence cited earlier that most wolf-killed animals are inferior members of their populations is considered, the most cogent explanation for the low hunting success of wolves is that relatively few prey animals are vulnerable.

Table 8.—Incidence of various abnormalities and pathological conditions in wolf-killed deer compared with that in hunter-killed deer

Condition	Wolf-kills			Hunter-kills			Level of significance
	Deer in sample	Deer with condition		Deer in sample	Deer with condition		
	Number	Number	Percent	Number	Number	Percent	Percent
Dental abnormalities	142	8	5.6	259	5	1.9	[2]90
Jaw necrosis, lumps, or fractures[1]	142	6	4.2	259	1	0.4	[2]95
Pathology of lower limbs	75	5	6.7	126	1	0.8	95

[1] Two mandibles from wolf-killed deer had large lumps from healed fractures in the region of the diastemas.

[2] If all dental and jaw abnormalities are pooled, the difference between the incidence in the wolf-kill sample (9.8 percent) and that in the hunter-kill (2.3 percent) is significant at the 99 percent level.

Figure 12.—When internal organs were present in kills, they were examined in the field. (Photo courtesy of L. D. Mech.)

Age Structure

Our data strongly indicate that in northeastern Minnesota wolves prey much more heavily on the older members of the deer population, at least during winter (fig. 7). Substantial vulnerability to wolves seems to begin at about the age of 5 years (fig. 13), because the percentage of wolf-killed deer in each year class increases from 9 percent for 4½-year-old animals to 15 percent for 5½-year-olds (table 4). Indeed, 48 percent of the wolf-kills were aged 5½ and over, which compares favorably with the Ontario figure of 58 percent for these age classes (Pimlott *et al.* 1969).

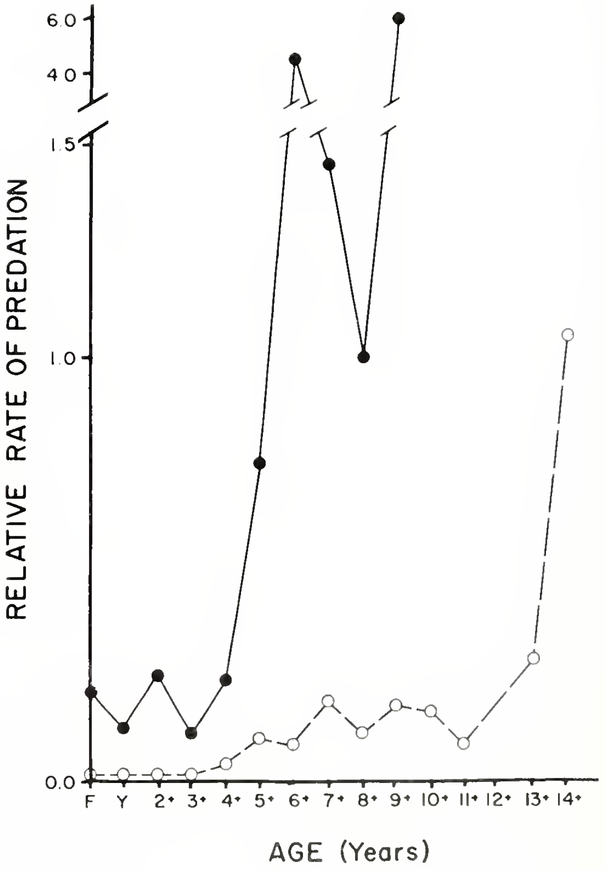

Figure 13.—Relative rates of predation on deer of various ages, based on comparisons of the ages of wolf-killed deer with those of a theoretical population (dashed line) and those of the hunter-killed population. See figure 7.

These figures assume added significance when compared with a sample of deer killed by hunters in the same general area (fig. 1). Only 10 percent of the hunter-killed deer were 5½ years old or older, and the percent killed in each year class dropped off suddenly from 13 percent aged 4½ to 6 percent aged 5½. If the age structure of the hunter-kill sample is reasonably representative of the age structure of the population at large, the wolf-kill data show that wolf predation in our study area during winter has a definite selective effect on the deer population.

There is no direct way of knowing that the age structure of the hunter-killed deer represents the age structure of the deer population at large. However, sampling hunter-kills is the most practical means available for gaining an index to the age structure of the existing herd. Further, there are three indirect pieces of evidence indicating that the hunter-kill sample represents the actual age structure of the population, just as Maguire and Severinghaus (1954) found in New York. First, our sample has the basic theoretical form expected of a stable deer herd; i.e., the youngest year class contained the most members, and each older cohort included fewer (fig. 7). Second, the age structure of our sample has the same form as most other deer age structures from widely diverse areas, (Ontario, Pimlott *et al.* 1969; southern Minnesota, Erickson *et al.* 1961; Massachusetts, Shaw 1951). Third, there is no reason to believe that in our area rifle hunting is especially selective for any particular age classes. In talking with large numbers of hunters, we have learned that most shoot at any and all deer they happen to see.

Even if the age structure of the hunter-kill sample did not approximate that of the actual herd, the comparison of the wolf-kill with the theoretical population dictates the same conclusion: the rate of kill of older deer by wolves was several times greater than that of younger deer, excluding fawns (fig. 13). In any case, if the actual deer population in our study area had an age structure similar to that of our sample of wolf-kills (which would be the only age structure that would contradict our conclusion), its numbers would be declining by orders of magnitude each year, and there would now be only a remnant population. Such obviously is not the case.

The only other question that might arise from a comparison of the age structure of our wolf-killed deer with that of the hunter-killed deer concerns the area from which each sample was taken. Fifty of our wolf-kills came from a region almost inaccessible to hunters (fig. 1). However, the other 92 came from the same general area as the hunter-kills. Nevertheless, there was no statistically significant difference in age structure between the wolf-kills from the wilderness versus those from the hunted area (table 1). This fact also suggests that the human hunting in the area is relatively light and has little effect on the age structure of the deer population in the area.

Wolves may also be taking a disproportionately high number of fawns, although our data do not show this. Nevertheless, there may be a bias against fawns in our method. It is not unusual to discover the remains of a wolf-killed deer so completely eaten that there is no indication left of the animal's age. Because fawns often are only about half the size of adult deer, and their skeletons have not yet completely ossified, the chances are better that fawns will be more completely eaten. Pimlott *et al.* (1969) also recognized this possible bias, although their data did indicate that wolves were killing a higher percentage of fawns than occurred in the population.

Our study does support the other conclusion of Pimlott *et al.* (1969), based on a study of 331 kills, that wolf predation on deer during winter shows a definite selection for older animals. It does not agree with the tentative conclusion of Stenlund (1955) that wolves in the Superior National Forest do not prey disproportionately on old deer. However, Stenlund's conclusion was based on 36 kills and on the assumption that only deer at least 7 years old were "old." Deer 5 years old and older composed 33 percent of Stenlund's sample, a figure considerably higher than the 10 percent in these age classes in our hunter-kill sample (table 4). Thus Stenlund's data do not contradict our conclusion.

The age of 5 years seems to be the beginning of the period of vulnerability for adult deer. Although 5 years might not seem especially old, there are two aspects of significance concerning deer of this age and older. First, they are in the second half of the life span for most members of the species, and their alertness and ability to bolt quickly away might be expected to decline. It is of interest in this regard that Klein and Olson (1960, p. 87) believed 5 years of age to be "the upper limit of physiological efficiency" of black-tailed deer (*Odocoileus hemionus*) in Alaska. Second, up to the age of at least 4½ years, and perhaps beyond, the apparent weight-load-on-track of deer increases with age (Kelsall 1969). Thus older deer would sink farther into the snow than younger ones, and their escape might be slowed and hindered more. For further discussion of the effect of snow on the vulnerability of deer, see Mech *et al.* (p. 51).

Sex Ratio

Statistical tests comparing a number of subsamples of both wolf-killed deer and hunter-killed deer showed a series of significantly different sex ratios (tables 1-3). The ratio of males to females in the fawn cohort of the hunter-kill, which is probably the most representative of the actual fawn sex ratio, was even (table 2). With wolf-kills, however, a significantly higher percentage of females was taken in the fawn subsample (59 percent) than in the adult subsample (46 percent). These results compare favorably with those of Stenlund (1955), who found that from 1948 to 1953 in the same area as the present study 68 percent of 19 sexable fawn wolf-kills were females and 44 percent of 63 sexable adult wolf-kills were females.

If the sex ratio of fawns began even, and more females than males were killed by wolves, then a higher proportion of males would be left in the adult population, unless some other mortality factor kills more male fawns. Thus it is not surprising that in the wildnerness area, where little or no hunting is done, the sex ratio of wolf-kills in the adult cohort is significantly heavy toward males (71 percent : 29 percent). This was also true of the wolf-kills in Algonquin Provincial Park, where males made up 57 percent of the total sexable wolf-kill (Pimlott *et al.* 1969). The latter figure may even have been higher if calculated for adults alone, for a preponderance of female fawns in the Algonquin Park data (such as occurred in our and Stenlund's samples) would tend to obscure the preponderance of males in the adult sample.

The adult subsample of hunter-kills also contained a higher percentage of males (66 percent : 34 percent). Although this might also reflect the influence of wolf predation on female fawns, it probably is more a result of the greater movement of bucks during the hunting season, which overlaps with the rutting season. Even the sex ratio of adult deer killed in wolf-free areas shows a preponderance of males (Erickson *et al.* 1961).

However, it appears that the higher harvest of bucks by human hunters does markedly affect the sex ratio of the deer population in the hunted area, for the wolf-kill of adults in that area contained a significantly higher percentage of does (56 percent) than did the wolf-kill of adults in the wilderness area (29 percent).

Evidently the hunter harvest is not heavy enough to affect the age structure of the deer population to any marked degree, for no significant difference in age structure was found between the wolf-kill in the hunted area and that in the wilderness area (table 1). This does not conflict with the conclusion that hunting affects the sex ratio of the deer herd, because it would take much less to influence a population characteristic having two classes (sex) than one having 14 (age).

One additional difference in the sex ratio was found between two other subsamples of the wolf-kill – that is, the wolf-kill before and after an unusually high snow accumulation, which reached its peak about February 1, 1969 (table 1). Of a total of 77 animals killed before this snow condition occurred (including those from previous years), 38 percent were females. Of 44 animals killed after the heavy accumulation, 57 percent were females. One possible explanation for this is that females may normally be less vulnerable to wolf predation, for Kelsall (1969) has shown that they probably have a lighter weight-load-on-track than males. Thus when snow conditions changed greatly, making deer generally much more vulnerable to wolves (see Mech *et al.*, p. 35), a preponderance of does suddenly might have become available. There is some evidence that does may be generally less vulnerable under most conditions, for all seven of our wolf-killed deer over 10 years old were females, and the oldest was over 14.

Condition of Wolf-Killed Deer

Because the data show that wolves in our study area tend to kill a disproportionate number of older deer, it is not surprising to discover that wolves also tend to capture a disproportionate number of individuals with abnormalities and pathological conditions (table 8). The explanation for such selection is obvious in regard to the abnormalities of the lower limbs (figs. 14-17): deer with injured or abnormal limbs simply cannot run as fast or as agilely as normal animals (table 7). Our observations show that deer usually depend on their alertness and speed to escape approaching wolves (Mech 1966b, Mech *et al.*, p. 1). Any trait or condition that tended to interfere with either alertness or speed would decrease an individual's chance of escape.

It is more difficult to explain how dental abnormalities or pathological conditions of the mandible (figs. 8-10) would predispose an individual to wolf predation. However, in the case of dental abnormalities the genetic or environmental conditions that caused the abnormality might also have caused some other trait that increased the animal's vulnerability. Or the abnormal condition itself may have caused a further, more critical, disruption of the animal's physiology or behavior, which in turn predisposed it to wolf predation.

The finding of several wolf-kills with poor fat stores could indicate that primary or secondary malnutrition was a factor in the animals' deaths. However, it would take a statistical comparison between the fat stores of the deer at large and those of the wolf-kills to establish this.

The discovery that 13 percent of the fawns and 84 percent of the yearlings killed during January, February, and March had not yet shed their deciduous incisors and premolars, respectively, also fits well with the rest of our information. Evidently some unusual factor had caused the delay in tooth development and replacement. One possibility is that the animals were born in August or September, much later than normal. Although most deer in Minnesota are born in May and June, there are records of births in July and August. In addition, a fetus 181 to 200 days old was found in a doe killed on September 26 (Erickson *et al.* 1961).

An alternate explanation for the delay in tooth replacement is that the animals were suffering from malnutrition or nutrient deficiency. Severinghaus[2] has evidence that yearling bucks that have not replaced their deciduous premolars during November, and thus are aged at 17 months (Severinghaus 1949), generally have shorter, narrower antlers and fewer points than 18- and 19-month-old individuals. Degree of antler development in turn is considered related to nutritional state (Latham 1950). Thus it is reasonable to conclude that animals behind in tooth development and replacement, whether this is caused by age or diet, are physiologically inferior.

Most of the abnormal conditions discussed above pertain to the skeletal parts of wolf-kills. If the soft parts of a large number of kills could be examined thoroughly, one might discover a much higher incidence of diseases and other pathological conditions.

In conclusion, our data on both age and condition of wolf-killed deer show that at least during winter, wolves in our study area usually do not kill just any deer they discover, although they do try to. Evidently, most deer can usually escape wolf predation. The most frequent exceptions are those 5½ years old and older, those born late, those suffering from poor nutrition, those with abnormalities or pathological conditions, and possibly fawns.

The above conclusions parallel those of Murie (1944), Crisler (1956), Mech (1966a), and Pimlott *et al.* (1969) for wolves preying on Dall sheep, caribou, moose, and deer respectively, and further substantiate the claim by Mech (1970) that they can be extended to wolves preying on most, if not all, species of large mammals under most conditions. It is also apparent from the data presented above that deer over 5 years of age and those with abnormalities of the jaw or lower limbs represent such a small percentage of the total population that they are seldom taken by human hunters. In this respect, competition between timber wolves and human hunters appears to be minimal in the study area.

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[2] *C. W. Severinghaus. Unpublished data.*

Figure 14.—Arthritis in right hind foot of specimen M-28. (Photo courtesy of University of Minnesota Veterinary Diagnostic Laboratory.)

Figure 15.—Infection and fibrous mass in a front foot of specimen M-29. (Photo courtesy of University of Minnesota Veterinary Diagnostic Laboratory.)

Figure 16.—Injury to left front foot of specimen M-196. (Photo courtesy of L. D. Mech).

Figure 17.—Healed fracture of left hind leg of specimen M-227. (Photo courtesy of University of Minnesota Veterinary Diagnostic Laboratory.)

SUMMARY

White-tailed deer (*Odocoileus virginianus*) killed by wolves (*Canis lupus*) during winter in a relatively unhunted wilderness area and in an immediately adjacent hunted area of Minnesota were compared with deer killed by hunters in the same general area, and with a hypothetical population. Deer killed by wolves were significantly older. Statistical comparisons also showed the following: (1) hunters generally killed an even sex ratio of fawns, and a disproportionate number of adult bucks, (2) wolves took a higher percentage of female fawns than female adults, a disproportionate number of bucks in the wilderness area, and a higher percentage of does in the hunted area. The latter fact evidently reflects the higher hunter success on males in the hunted area. Significantly higher incidences of abnormalities and pathological conditions of both mandibles and lower limbs were found in wolf-killed deer than in hunter-killed deer, and these conditions are described. It is concluded that wolf predation on white-tailed deer in the study area during winter generally is selective in that it tends to remove members of the prey population that are old, debilitated, or abnormal. Apparently these classes of deer represent such a small percentage of the population that they are seldom taken by human hunters.

ACKNOWLEDGMENTS

This study was supported by Macalester College, the New York Zoological Society, the Minnesota Department of Conservation, the USDA Forest Service, and the U.S. Bureau of Sport Fisheries and Wildlife. Pilots Robert Hodge, Pat Magie, John Winship, Jack Burgess, Don Murray, and Walt Neumann aided substantially in obtaining jaws from wolf-killed deer. Students from the Macalester College Biology Department and personnel of the USDA Forest Service and the Minnesota Department of Conservation helped secure mandibles from both wolf-killed and hunter-killed deer. The interest of Mr. John E. Peninger and of many deer hunters in contributing the jaws is also greatly acknowledged.

Mr. David W. Kuehn sectioned the incisors of the deer jaws and determined their ages. Dr. Donald M. Barnes of the University of Minnesota Veterinary Diagnostic Laboratory examined the abnormal lower limbs, described their pathology, and provided photos of specimens used herein.

Mr. Wallace C. Dayton and Miss Elizabeth Dayton and the Quetico-Superior Foundation, all of Minneapolis, financed Mech during the preparation of this paper.

The following individuals read the manuscript and offered many helpful suggestions: Mr. R. L. Downing, Mr. C. W. Severinghaus, Mr. J. M. Peek, Dr. C. T. Cushwa, Mr. M. H. Stenlund, and Dr. R. R. Ream.

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THE EFFECT OF SNOW CONDITIONS ON THE VULNERABILITY OF WHITE-TAILED DEER TO WOLF PREDATION

L. David Mech, L. D. Frenzel, Jr., and P. D. Karns

Wolves (*Canis lupus*) and deer (*Odocoileus virginianus*) having evolved together, no doubt have become adapted to contending with each other's physical abilities. Thus it is not surprising to learn that deer which succumb to wolf predation are generally weaker, older, or abnormal compared with the total deer population (Pimlott *et al.* 1969, also see Mech and Frenzel, p. 35).

However, the structural and behavioral adaptations of both species must have evolved under environmental conditions that are average or usual; otherwise, an adjustment of wolf to deer populations, and vice versa, could not have been maintained over long periods. This implies that extreme or unusual conditions might sometimes occur, to which either the wolf or the deer is poorly adapted.

One of the most important environmental factors that can influence the interactions of wolves and deer is snow. The total fall, depth on the ground, and the density are all aspects of snow that may vary considerably and affect the ability of wolves to capture deer. Recent studies of wolves and deer in northeastern Minnesota (see Mech *et al.*, p. 1, also Mech and Frenzel, p. 35) afforded us opportunities to investigate the relationships between snow and the interactions of wolves and deer.

METHODS

Two principal methods of study were used in this investigation. The first involved recording the snow depth and support quality ("penetrability") in feet and tenths of feet (Verme 1968). Snow measurements were taken during the winters of 1966-67, 1967-68, and 1968-69, in which large differences in snow conditions existed. Ten such measurements were made weekly near Isabella, Minnesota, in an open aspen (*Populus tremuloides*) stand away from influences that might have caused drifting or other unusual snow conditions; the measurements were averaged.

Penetrability was determined with Verme's snow-compaction gauge — a 3-foot piece of 1⅛-inch (outside diameter) copper tube filled with lead to total 3 pounds, which gives a weight per area of 211 gm./cm.^{2}. To obtain a measurement, the pipe is held vertically with its lower end just flush with the snow, and then is released. The depth to which it sinks is considered the penetrability of the snowpack by a walking deer.

Although the snow conditions measured at Isabella are not representative of the entire study area, year-to-year comparison in the Isabella area should also apply generally throughout the region.

The second technique used in this study was observing the movements of wolves and deer. This was usually done from low-flying aircraft, and was facilitated by the use of radiotracking, as described by Mech *et al* (p. 1). Close inspection of wolf-killed deer was made from the ground (Mech and Frenzel, p. 35).

RESULTS AND OBSERVATIONS

Snow measurements for each winter are shown in figures 1 through 3. The winter of 1968-69 was the most extreme of the three in terms of accumulated snow, and was generally regarded as having one of the heaviest snowfalls and accumulations on record for the study area. Snow depth on the level near Isabella reached 3.9 feet at one time, and from January 3 to April 4 it exceeded 2.4 feet. The highest snow level reached during 1966-67 was 2.4 feet, and the highest level reached during 1967-68 was 1.4 feet. In the vicinity of Ely, some 30 miles from Isabella, the 1968-69 peak accumulation was 39 inches, the highest accumulation since 1948-49 when records were first kept.[1] Thus we consider the winters of 1966-67 and 1967-68 to be within the normal range for the study area, and the 1968-69 winter as being most unusual (fig. 4).

The snow penetrability in 1966-67 remained high throughout January, February, and March. During the following winter, penetrability fluctuated more, but even at its greatest, it was relatively unimportant to deer because the total

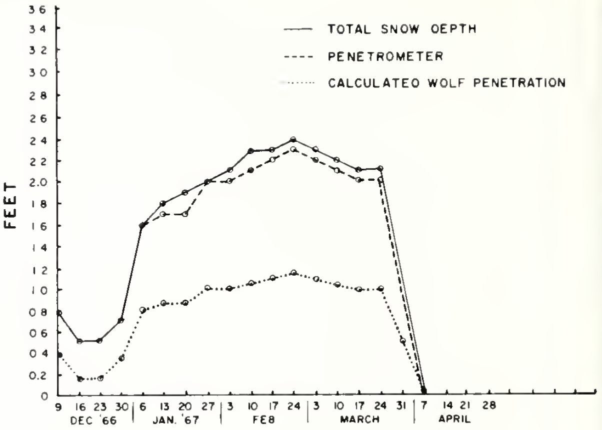

Figure 1.—Snow depth and penetrability by deer and wolves near Isabella, Minnesota, 1966-67.

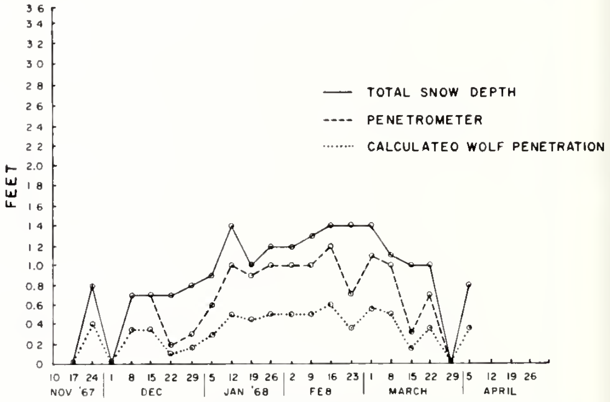

Figure 2.—Snow depth and penetrability by deer and wolves near Isabella, Minnesota, 1967-68.

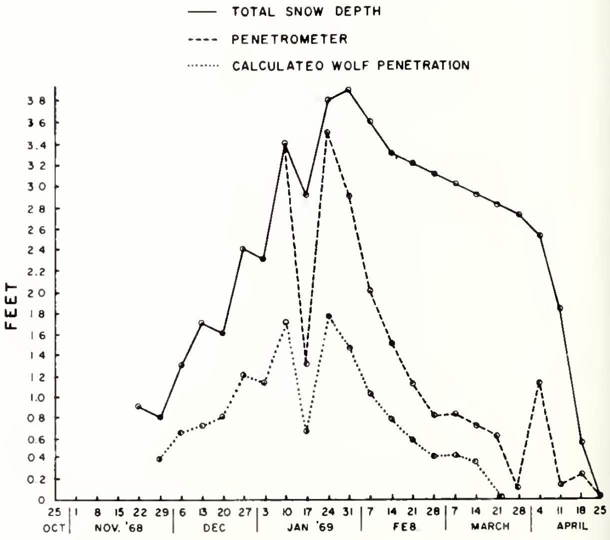

Figure 3.—Snow depth and penetrability by deer and wolves near Isabella, Minnesota, 1968-69.

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[1] *M. H. Stenlund. Personal correspondence to L. D. Mech, Oct. 10, 1969.*

Figure 4.—During the winter of 1968-69, the snow was unusually deep in the study area. (Photo courtesy of L. D. Frenzel.)

snow depth was so low. During 1968-69, however, penetrability was a very important aspect of snow condition. It was so high during late January and early February, when snow accumulation was also at its peak, that a walking deer would be expected to sink in 2.5 to 3.5 feet. Snow penetrability then decreased through February and March to a point where a walking deer would sink in approximately 0.6 foot on March 21. However, because snow accumulation remained so high through February and March, the lower penetrability during late February and March still afforded no relief to running deer, because they must exert forces several times as great as when walking. On the contrary, the low penetrability (which is an indirect measure of density) could be expected to hinder a running deer in deep snow, for it would cause much more resistance.

Deer movements, like snow conditions, varied greatly during the three winters of the study. During the first two winters, deer were generally found singly and in groups of two to six, often around the shores of lakes but also scattered about inland. In late January and February 1967, running deer were observed sinking deeply into snow, but their movements still did not seem to be hindered, no doubt because of the high penetrability (low density) of the snow that year (fig. 1).

However, during late January, February, and March of 1969 the deer were much more concentrated, mostly in conifer swamps, along southwest-facing slopes, or on lakes. Although groups of two or three animals could be found in scattered inland "pockets" throughout the winter, groups of five or six were not uncommon on lakes during January. The tendency to concentrate continued to increase, and on February 6, as many as 11 deer were observed on one lake; by March 13, group size had increased to as high as 22 deer in the same area. Throughout February and March, heavy concentrations of deer tracks covered most wilderness lakes, further evidencing much greater use of shorelines than had occurred in the two previous winters (fig. 5).

No doubt deer tended to concentrate on lakes because travel inland became so difficult. On January 28, two deer were seen plowing through snow up to their necks. Although the snow began settling in February, and the penetrability decreased, by late February running deer still plunged chest-deep and had to hesitate at every bound. These conditions persisted until about March 26, by which time a surface crust strong enough to hold a running deer had formed.

Figure 5.—Under unusually deep snow conditions, deer used lake shores heavily. (Photo courtesy of L. D. Mech.)

In considering wolf mobility in snow, two types of movement must be recognized: the trot used during general travel, and the bounding used while chasing prey. The trot is an easy gait of about 5 m.p.h. on firm footing (Mech 1970), and can be continued for hours at a time. During periods of deep snow and high penetrability, most wolf travel is on frozen waterways, roads, snowmobile trails, and animal trails, including the wolves' own pathways, which become well packed with frequent use (fig. 6, 7A, B). Such travel was observed during each of the three winters of this study.

The second type of wolf movement affected

Figure 6.—Wolves travel single file in deep snow. (Photo courtesy of L. D. Mech.)

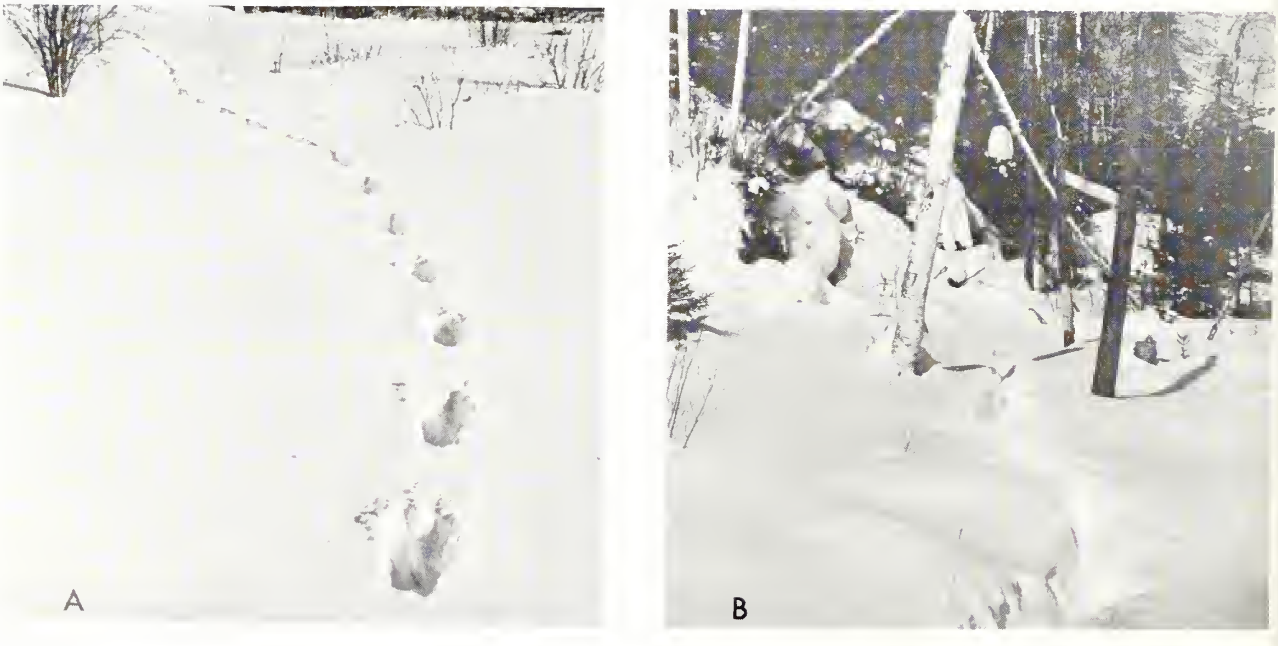

Figure 7.—(A) A single wolf must break his own trail through the snow. (Photo courtesy of L. D. Frenzel.) (B) Regular use by a pack keeps trails open. (Photo courtesy of L. D. Mech.)

by snow is the leaping and bounding associated with chasing prey. The shallower angle of the wolf's bound (fig. 8) (compared with that of the deer) often causes the wolf to flounder in snow that presents little hinderance to deer (Mech 1970). Such was the case in January and February 1967 in our study area. During 1967-68 no observations of wolves chasing deer were made by the authors, but reports by other field workers indicated that running conditions were similar to those of 1967.

During the winter of 1968-69, wolves also bogged down a great deal in snow when chasing deer. However, after January 1969 the snow was so deep that deer were floundering even more than wolves in many cases. The fact that wolves could run in the trail broken by deer probably also gave the wolves an advantage under the conditions that severely restricted deer movements.

The above observations of snow conditions, deer movements, and wolf movements during the three winters of the study are in accord with observations made on the differences in the ability of the wolves to capture deer during the same period. Two indices support the conclusion that wolves had a much easier time catching deer during February and March 1969 than earlier in the winter and in the two previous winters: (1) the degree of utilization of wolf-killed deer, and (2) the kill rate of radiotagged wolves.

During the winters of 1966-67 and 1967-68, and in December and early January1968-69, most wolf-killed deer found had been thoroughly eaten, and the bones – if present at all – were well chewed and scattered at each kill (fig. 9). All skin and flesh from the skull were eaten, and the mandible was usually separated from the skull. During late February and early March 1967, few fresh kills were even found, and wolves were returning several times to old kills that had been cleaned up many days before.

However, in late January 1969 a substantial change began taking place. The skeletons of most kills found were almost intact, the flesh having been eaten from around the bones (fig. 10). Appreciably more skin was usually left on the carcass, especially on the side lying on the snow, and the neck and head were generally intact. This was true even of fawns, which in the past often were almost completely consumed.

In several cases, only about half of the flesh had been eaten from the carcasses. On February 2, 1969, four deer recently killed by wolves were found along a 1½-mile stretch of Birch Lake

Figure 8.–Wolves run at a shallow angle, thus hindering them in deep snow. (Photo courtesy of D. H. Pimlott.)

Figure 9.—Usually the remains of a wolf kill are well chewed and scattered before the wolves abandon them. (Photo courtesy of L. D. Mech.)

Figure 10.—During a period of especially deep snow, wolves abandoned many kills before pulling apart the skeletons. (Photo courtesy of L. D. Mech.)

and nearby Polaris Lake (Minnesota-Ontario border). One large doe was completely uneaten and remained so for at least 24 hours after discovery from the air. Further, one fawn had only a few pounds of flesh eaten, a yearling doe was half eaten, and another fawn was about 75 percent eaten. Hazardous landing conditions during this period severely limited the number of carcasses that could be examined from the ground, but on February 6 a yearling doe was discovered that had only about 5 to 10 pounds of flesh eaten, and on February 8 an adult doe was found that was completely intact except for wounds.

In past winters some kills had been located that had been only partly eaten, but in each case the carcasses were soon revisited and cleaned up (Mech 1970). This was often not the case in 1969. For the rest of the winter most of the deer killed by wolves in our study area were not as completely consumed as in previous winters. Pimlott *et al.* (1969) found a similar relationship between the severity of the winter and the degree to which wolf-killed deer were utilized.

Correlated with the above information was the kill history of our radiotagged wolves (Mech *et al.*, p. 1). From December 1968 through January 1969 No. 1051 had killed three or possibly four deer, and generally had spent 6 or 7 days feeding on each. However, throughout most of February this animal visited a new deer carcass (which presumably he killed) every 3 days, and he spent only 1 or 2 days at each. In two cases two new carcasses were found in the immediate vicinity of this animal during the same day, and in each case the wolf spent only 1 day in the area. A second wolf (1053) which had spent most of December and January scavenging on the remains of both deer and moose (*Alces alces*) that had died long before, made her first known kill of a deer on January 31, 1969. The kill rate of the other three radiotagged wolves also increased, although the data for them are less complete. The average kill rate for all radiotagged wolves and their associates was one deer per wolf per 16 to 20 days before February 1, and one per 8 to 12 days after February 1 (see Mech *et al.*, p. 1).

DISCUSSION AND CONCLUSIONS

Under usual snow conditions throughout most of the range of the white-tailed deer, healthy vigorous individuals can probably escape most attacks by wolves. Observations by Mech (1966), Rutter and Pimlott (1968), and Mech *et al* (p. 1) indicate that a high percentage of attempts by wolves to kill deer during winter are unsuccessful. This is further implied by the figures of Pimlott *et al.* (1969) and Mech and Frenzel (p. 35) showing that at least during winter wolves tend to kill a disproportionate number of old deer as well as those with various abnormalities and pathological conditions.

However, during a winter with extremely deep snow, the usual relationships seem to change somewhat. Fewer deer are able to escape wolves, and a surplus is killed. This means that some individuals not vulnerable under the usual snow conditions become vulnerable during extreme conditions. There are two main possible reasons for this, the effect of the extreme weather conditions on the health and vigor of the deer, and the physical effect of the snow on the escapability of the deer.

In regard to the first possibility, there was limited evidence that during February and March 1969 some fawns and yearlings in our study area were losing their fat stores. Two of three yearlings, and both fawns intact enough for examination during this period lacked back fat, and the marrow in one of six fawn femurs was partly fat depleted. Nevertheless, the third yearling inspected still had back fat, and a 3½-year-old doe had heavy omental, renal, heart, and back fat during the same period. Thus, although an abnormal decline in the physical condition of some deer in the late winter might partly account for the increased kill by wolves during February and March 1969, the effect of snow on the escapability of the deer probably was also involved.

The key difference in snow conditions between the two periods — (1) the winters of 1966-67, 1967-68, and December-January 1968-69, and (2) February and March 1969 — was the heavy, persisting accumulation of snow during the latter period, combined with the increasing density of the snow. As our observations show, this greatly hindered the movements of deer fleeing from wolves.

Under more usual conditions, a running deer might sink through the snow to the ground and thus obtain a firm footing from which to spring again. In discussing wolf-caribou relations in snow, Kelsall (1968, p. 249) stated the following: "While caribou (*Rangifer tarandus*) will sink into snow even deeper than wolves, their longer legs permit them to run efficiently where a wolf will bog down. Nasimovich (1955) considered that roe deer and sika deer could be taken by wolves when snow was not more than 30 cm. (11.8 inches) in depth. At depths above that their pursuit becomes difficult or fruitless."

However, it appears that when snow becomes extremely deep, wolves then gain the advantage. With 22 to 48 inches or more of snow to plow through, a deer would have trouble even touching a firm foundation. According to Kelsall (1969), deer measure only 20 to 24 inches from hoof tip to chest, with legs extended.

It is true that wolves stand even shorter than deer and so might be expected to flounder even more. However, this is where another factor becomes important, the "weight-load-on-track" or total weight per area of track. As Kelsall (1969) has pointed out, the mean weight-load-on-track for deer is extremely difficult to measure directly, because the actual under-surface of the deer's foot slants vertically, and a much greater area may be used to support an animal in snow than on a hard surface. This probably explains the discrepancy between Kelsall's measurements and work done by Verme (1968) in Michigan. According to Kelsall, deer weight-load-on-track (hoof only) varies between 431 and 1,124 gm/cm.^{2}. However, Verme stated that his compaction gauge (with a weight load of about 211 gm./cm.^{2}, described earlier in this paper) sank in virtually the same amount in snow as did deer. Under the snow conditions in our study area, we found that the same type of compaction gauge generally penetrated to a depth within a half inch of that to which deer were sinking. On this basis, it seems reasonable to suggest that a deer in snow is supported by more of its foot than just the hoof, and that the actual weight-load-on-track of deer in snow is about 211 gm./cm.^{2}.

For wolves, this measure varies from 89 to 103 gm./cm.^{2} (Foromozov 1946). This means that for the same amount of force applied during running, a wolf would have twice as much support as a deer. It also means that in deep snow a walking wolf generally is much less restricted than a walking deer. Late in February 1969, for example, when deer were seriously limited in their ability to travel, wolves were able to travel widely (Mech *et al.*, p. 1).

Even though wolves have much greater support than deer, when running they still sink into the snow almost as much as deer under most conditions, probably because both run with such force that snow usually offers little support. Nevertheless, with extremely deep snow, the difference in support factor between wolves and deer could become critical, and this is probably what happened during February and March 1969. With deer seriously restrained by the deep snow, even a slight advantage in favor of the wolf could increase hunting success. A high snow density during that period would accentuate this advantage. This is because until the snow becomes dense enough to hold a running deer, each increase in density would further the advantage of the wolf, which would require only half the density to support it, while it would hinder the deer.

One result of the extreme snow conditions of early 1969 was that deer tended to gravitate to lakes, where snow was shallow and footing was firm. Initially upon disturbance by human beings, and probably by wolves, these deer usually headed inland, but it is apparent from a number of kills examined that when pressed hard by wolves inland, deer headed out onto lakes where possible. Apparently they could run there with better footing. However, frozen lakes also provide wolves with good running conditions, and even seem to give them an advantage (Rutter and Pimlott 1968, Mech 1970), so many of these deer were killed (fig. 11).

Stenlund (1955, p. 44) reported as follows on years of low snowfall, the opposite condition, which demonstrated the same relationship between snow depth and kills on lakes: "The winters of 1951-52 and 1952-53 were abnormally mild with little early snow. As a result, few wolf-killed deer appeared on the lakes and most deer attempted to outrun wolves in the woods."

Figure 11.—On frozen lakes, wolves often seem to have the advantage over deer, such as in this case where the wolf (center) has just killed a deer and is trying to discourage a raven from joining him in the feed. (Photo courtesy of L. D. Frenzel.)

Thus it appears that extreme snow conditions in our study area increase the vulnerability of deer to wolf predation in three ways: (1) by causing a decline in the health and nutritional state of some members of the deer population; (2) by hindering the escapability of the deer; and (3) by causing deer to congregate on frozen lakes where wolves have the advantage in running.

SUMMARY

During the winters of 1966-67, 1967-68, and 1968-69, the interactions of wolves (*Canis lupus*) and white-tailed deer (*Odocoileus virginianus*) were observed in northeastern Minnesota from aircraft. Snow depth and supporting ability were also measured during these winters, and the ability of wolves to capture deer was compared for a period of usual snow conditions versus a period of extreme snow conditions.

It was found that during February and March 1969, when snow remained from 2.5 to 3.9 feet deep and failed to support running deer, wolves were able to capture deer more easily. This was evidenced by kills that were left partly or completely uneaten, and by a higher rate of predation by radiotagged wolves and their associates.

Although both wolves and deer floundered in the extremely deep snow, the relatively lighter weight-load-on-track of wolves evidently gave

them a greater advantage than under the usual snow conditions, when wolves were observed floundering more than deer. This factor, plus a decline in the health and vigor of some segments of the deer population and a tendency for deer to congregate on frozen lakes, where wolves have an advantage, help explain the increased vulnerability of deer to wolf predation during the winters of deep snow.

ACKNOWLEDGMENTS

This study was supported by Macalester College, the Minnesota Department of Conservation, the USDA Forest Service, the U.S. Bureau of Sport Fisheries and Wildlife, and the New York Zoological Society. Pilots John Winship, Pat Magie, Jack Burgess, and Don Murray flew the observation planes during radiotracking. Miss Elizabeth Dayton, Mr. Wallace C. Dayton, and the Quetico-Superior Foundation, all of Minneapolis, financed Mech during the writing of this report.

Thanks are also due L. J. Verme, J. P. Kelsall, and J. M. Peek for their helpful reviews.

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THE POSSIBLE OCCURRENCE OF THE GREAT PLAINS WOLF IN NORTHEASTERN MINNESOTA

L. David Mech and L. D. Frenzel, Jr.

The timber wolf (*Canis lupus*) of northeastern Minnesota occupies an area within the range given by Goldman (1944) for the eastern timber wolf (*C. 1. lycaon* Schreber). However, this area is within 150 miles of the eastern edge of the former range of the Great Plains wolf (*C. 1. nubilus* Say), and there is some question as to whether the Minnesota wolf is really an intergrade between these two subspecies. Writing of *nubilus*, Goldman (1944, p. 444) stated: "Specimens from eastern Minnesota and Michigan seem more properly referable to *lycaon*, but relationship to *nubilus* is shown in somewhat intermediate characters."

In describing *lycaon* as basically a gray wolf, Goldman made no mention of the occurrence of black or white color phases in that subspecies. However, in discussing *nubilus*, Goldman (1944, p. 442) wrote the following: "Many color variations are presented. Individuals may be nearly white at any season, except for a sprinkling of black hairs over the back, a small, narrow, but conspicuous, black patch over the tail gland, and a more or less distinctly black tip. Black individuals may occur in the same litter with those normally colored." Goldman also referred to *nubilus* as "now probably extinct."

In the eastern part of the range of *lycaon*, color phases other than gray appear to be rare as Rutter and Pimlott (1969, p. 188) attest: "The uniformity of the color of timber wolves in many areas is evidenced by the work in Algonquin Park, in Ontario. There, over the past eight years, dozens of packs have been observed from the air. However, we have never been able to discriminate between any of them on the basis of the color variation of individual animals."

Thus it seems significant to report on incidences of black and white color phases in wolves that we have observed in northeastern Minnesota during some 480 hours of flying associated with wolf research (Mech *et al.*, p. 1). The observations took place in the Superior National Forest, in northern Cook, Lake, and St. Louis Counties during the winters of 1966-67, 1967-68, and 1968-69. A total of 309 sightings were made of wolves that could be classified by color; of these, 11 (3.6 percent) were jet black (fig. 1) and two (0.6 percent) were creamish white, with the cream color the most intense on the back. No doubt some of the grays, and perhaps the blacks and whites, were repeated observations, but the figures should provide a reasonable approximation of the incidence of these color phases in this area. All black or white animals except one were observed with gray wolves (table 1 and fig. 2).

Figure 1.—A few wolves observed in the study area were jet black. (Photo courtesy of L. D. Mech.)

A number of black wolves, and a few white wolves, have been seen by other observers, all in the three counties listed earlier. To gain some idea of the past incidence of these color phases in the same general area, we asked Conservation Officers Robert Hodge, Robert Jacobsen, and Frank Baltich of the Ely, Minnesota, area about the numbers of each phase that they took before 1960. They reported killing an approximate total of 580 wolves, of which four were black and three were white or creamish white.

Because black and white color phases have rarely if ever been reported for *lycaon*, yet were

Table 1.—Observations of wolves of black and white color phases

Date	Location	Color combinations within each pack
Feb. 24, 1967	T64N-R8W-S1 Vera Lake	3 grays; 1 black; 1 white
Mar. 4, 1967	T63N-R9W-S27 Lake Two	3 grays; 2 blacks
Dec. 18, 1968	T63N-R8W-S35 Lake Insula	2 grays; 2 blacks[1]
Jan. 17, 1969	T65N-R8W-S27 Carp Lake	1 gray; 1 white
Feb. 1, 1969	T63N-R8W-S13 Lake Insula	4 blacks; 2 grays[2]
Feb. 5, 1969	T63N-R8W-S8 Benezie Lake	1 black
Feb. 6, 1969	T63N-R10W-S33 Clear Lake	3 grays; 1 black

[1] These animals were near the shore of the lake, so others may have been inland where they could not be seen.

[2] This group might well have been the same as that seen on Dec. 18, 1968.

Figure 2.—A pack of four blacks with two grays (first and third). (Photo courtesy of John Winship.)

well known for *nubilus*, it is not unreasonable to conclude that the race of wolves now occupying northeastern Minnesota does show strong *nubilus* influence. Goldman examined the skulls only of 10 Minnesota specimens assignable to *lycaon* and only one referable to *nubilus*. Because wolves in the known range of *nubilus* are thought to be extinct, and because the animals in northeastern Minnesota are legally unprotected and subject to a control program, it seems highly desirable that the question of their taxonomy be studied intensively while specimens are still available.

ACKNOWLEDGMENTS

This study was supported by Macalester College, the New York Zoological Society, the Minnesota Department of Conservation, the U.S. Bureau of Sport Fisheries and Wildlife, and the USDA Forest Service. Mr. Wallace C. Dayton and Miss Elizabeth Dayton, and the Quetico-Superior Foundation, all of Minneapolis, financed Mech during the preparation of this paper. We would also like to thank Dr. J. L. Paradiso, Dr. H. L. Gunderson, and Mr. M. H. Stenlund for reviewing this manuscript.

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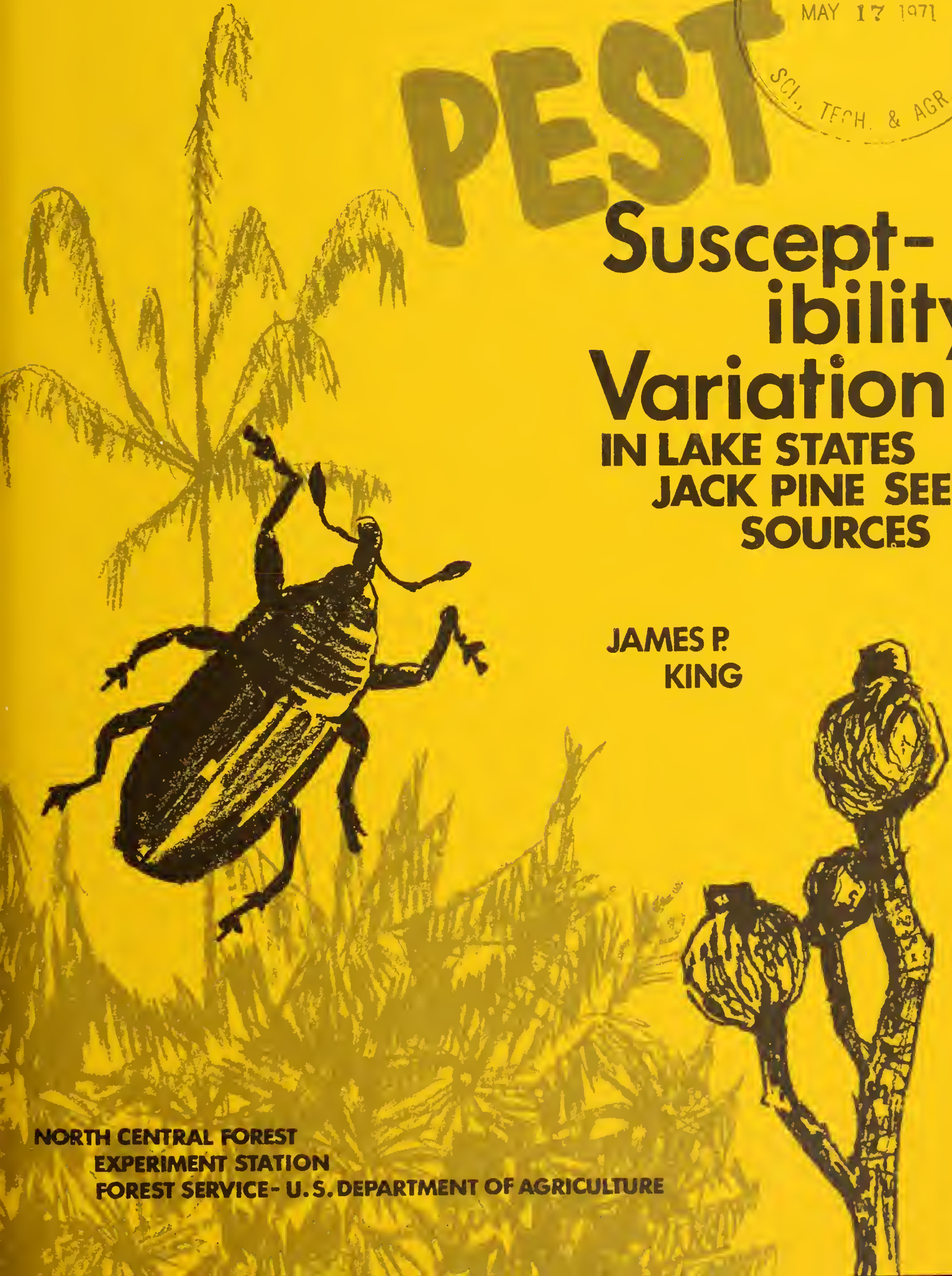
1971

PEST Suscept-ibility Variation
IN LAKE STATES JACK PINE SEED SOURCES

JAMES P. KING



NORTH CENTRAL FOREST
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CONTENTS

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PEST SUSCEPTIBILITY VARIATION IN LAKE STATES JACK PINE SEED SOURCES

James P. King

Development of pest-resistant tree varieties in a species can be undertaken only after useful levels of genetic variation have been shown to exist. The North Central (then Lake States) Forest Experiment Station and the University of Minnesota initiated a provenance study of Lake States jack pine (*Pinus banksiana* Lamb.) in 1951. Various Federal, State, and private forestry agencies collected seed from natural stands in Minnesota, Wisconsin, and Michigan. The seedlings were used to establish 17 permanent test plantations in the three States. Data from these plantations have shown jack pine to vary genetically in resistance to some, but not all, of the pests encountered.

PREVIOUS WORK

Three reports on insects or diseases in some of these plantations have already been published. Batzer (1962) described differences among seed sources in incidence of white-pine weevil (*Pissodes strobi* (Peck)) on the Chippewa and Superior National Forests in northern Minnesota. He measured weeviling incidence in 1958 and 1959 on the Chippewa and in 1960 and 1961 on the Superior. On the Chippewa National Forest, trees from the following seed sources showed significantly more weeviling than those from the local source: Pine County (1595, 1596), Minnesota; Douglas (1604), Burnett (1608), Marinette (1609), Oneida (1610), and Wood (1611) Counties, Wisconsin; and Gogebic (1612) County, Michigan. On the Superior National Forest trees from the following sources were more weeviled than those from the local source: Cass (1589, (1600), Pine (1595), and Becker (1597) Counties, Minnesota; and Douglas (1604), Burnett (1608), Marinette (1609), Oneida (1610), and Wood (1611) Counties, Wisconsin.

Arend *et al.* (1961) measured three test plantations in Lower Michigan 5 years after planting and found differences among provenances in susceptibility to white-pine weevils, bark beetles (*Pityophthorous* spp.) and the red-headed pine sawfly (*Neodiprion lecontei* (Fitch)).

There were striking similarities in weevil incidence in Michigan (Arend *et al.* 1961) and in Minnesota (Batzer 1962). The 7 most weeviled sources in the Michigan plantings were among the 8 most weeviled sources on the Chippewa and the 12 most weeviled sources on the Superior.

Provenance differences in jack pine needle cast (*Hypodermella ampla* Dearn.) infection in a southern Wisconsin and a western Upper Michigan planting were described by King and Nienstaedt (1965). Trees from Lower Michigan seed sources showed the least infection and those from northeastern Minnesota sources the most.

METHODS

Seed was collected in 1951 from 29 jack-pine stands in Minnesota, Wisconsin, and Michigan. Each collection was made from dominant and codominant trees in a stand considered good for its locality.

In the spring of 1952, seed from all 29 stands was sown in both the General Andrews State Nursery at Willow River, Minnesota, and in the Hugo Sauer Nursery at Rhinelander, Wisconsin.

Two-year-old seedlings were used to establish 17 test plantations throughout the three States. Seedlings from the General Andrews Nursery were used in the six Minnesota plantings and two western Wisconsin plantings, while seedlings from the Hugo Sauer Nursery were used to establish four Wisconsin and five Michigan plantations.

A four-replicated, randomized, complete-block design was used at each location. Each replication contained trees from each experimental seed source, plus trees from a local source furnished by a commercial nursery in the area of the plantation. Each seed source was represented by a square 64 tree plot in each replication. Because of shortages within various seedlots, substitution had to be made in several plantings. Nevertheless, there were still 26 sources (fig. 1) common to all plantings.

In the fall of 1958, trees in all the plantations were measured for total height, d.b.h. and causes of loss or current injury. All 64 trees on each plot were measured. Trees in 11 test plantations (fig. 1) were again scored in 1963 for total height, d.b.h., form, and the presence or absence of seven insects and two diseases. The 1963 measurement included only 16 trees per plot,

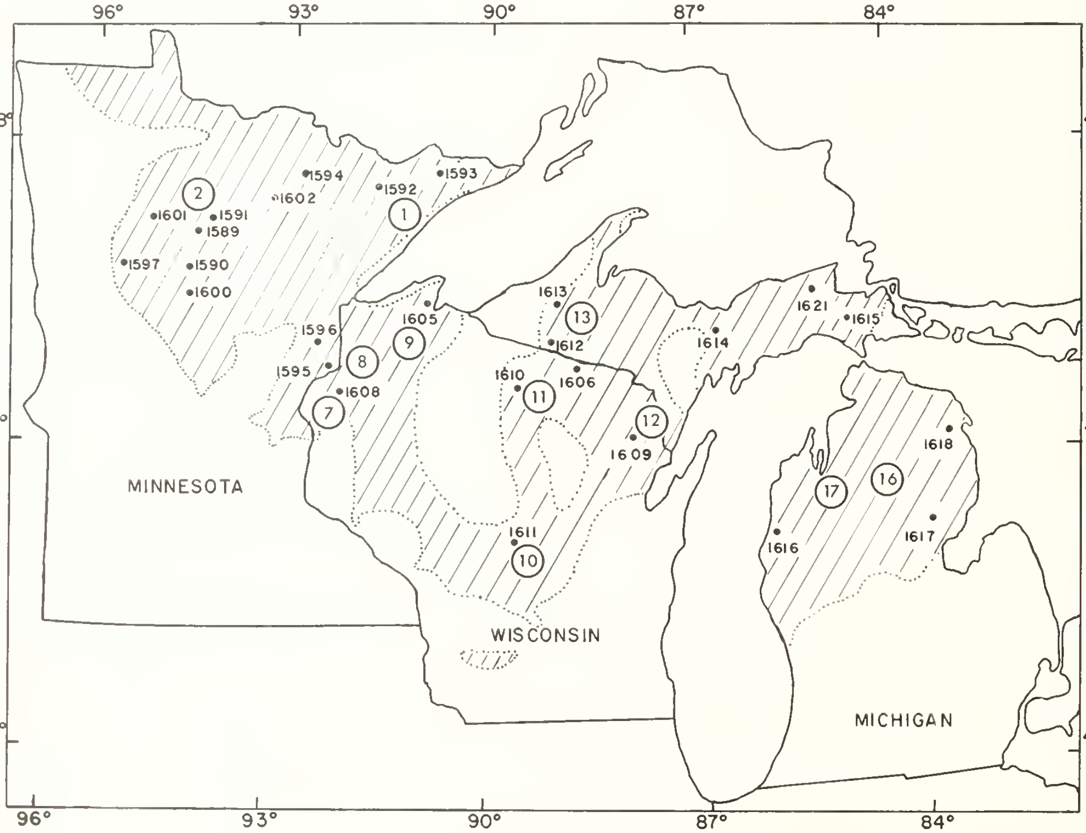

Figure 1.—Location of seed sources and plantations used in regional study. Dots show seed source locations; encircled numbers show plantation locations. Shaded areas show natural range of jack pine.

which were systematically selected and measured. Results of the 1963 height-growth measurements have been reported by King (1966).

Only the 16 trees per plot and the 11 plantings that were measured in both 1958 and 1963 were used in this analysis. Moreover, only the 26 sources common to the 11 plantings are discussed.

ANALYSES

Insect or disease incidence was often low. As a result, many of the plot mean distributions were badly skewed. Few transformations were tried as most of the skewness was due to the large number of zero values for plot means.

When infestations were such that 20 percent or more of all the measured trees within a planting were attacked, the plot mean distribution tended to be normal or nearly so, and analysis of variance was used.

When between 5 and 20 percent of all measured trees were infected, only the seed source totals over all replications were used in the analyses. In this situation the seed source totals from several plantings were combined into a single analysis, using the normally distributed seed source totals as the basic data. This provided a valid test of seed source differences when the seed source x plantation terms were used as the estimate of error variance. However, no within-planting error terms were available for testing the significance of the seed source x plantation interaction.

When less than 5 percent of all the trees in a planting were damaged, the data were not analyzed.

RESULTS

White-Pine Weevil

After five growing seasons in the field, more than 20 percent of the measured trees in three plantings had been attacked by white-pine weevil (table 1). These plantings were at Burnett County Forest, Wisconsin; Au Sable State Forest, Michigan; and Fife Lake State Forest, Michigan. Seed source differences in weeviling incidence could be found at the Burnette and Au Sable plantings, but not at the Fife Lake planting (table 2). When the data from the Burnett County and the Au Sable plantings were combined, highly significant seed source differences were found, but the seed source x plantation interaction was not significant. Significant seed source differences in 5-year height were also noted and the seed source x plantation height growth interaction was nonsignificant. The correlation between seed source tree height at 5 years and weeviling incidence was positive, but just short of significant at the 5-percent level (calculated r = 0.375; tabular r for 0.05 — 0.388 with 24 degrees of freedom (d.f.)).

Table 1.—Incidence of white-pine weevil on jack pine in 11 test plantations 5 and 10 years after establishment

Plantation number	Name and location	Percent of trees attacked in	
		1958	1963
1	Superior National Forest Minnesota	1.4	8.1
2	Chippewa National Forest Minnesota	5.6	6.3
7	Burnett County Forest Wisconsin	23.5	2.5
8	Mosinee Industrial Forest Wisconsin	4.4	7.2
9	Chequamegon National Forest Wisconsin	6.7	1.3
10	Nepco Industrial Forest Wisconsin	1.3	1.0
11	Argonne Experimental Forest Wisconsin	6.7	1.0
12	Marinette County Forest Wisconsin	8.4	1.6
13	Ottawa National Forest Michigan	3.8	1.0
16	Au Sable State Forest Michigan	32.2	49.5
17	Fife Lake State Forest Michigan	23.1	7.3

Even though the height-weeviling correlation was not significant, trees from the five least-weeviled sources were all below average in height and those from the four most-weeviled sources were all above average in height. Thus, the 5-year data suggest that height differences do make a small contribution to weeviling differences but factors other than height are primarily responsible for weeviling differences at that age.

Table 2.—Incidence of white-pine weevil on jack pine in 1958 and 1963

Seed source	State	1958		1963				Au Sable and Fife Lake (combined)	
		Burnett	Au Sable	Superior	Chippewa	Au Sable	Fife Lake	1963 height	1963 weeviling (Adjusted for height)
		Percent	Percent	Percent	Percent	Percent	Percent	Feet	Percent
1589	Minn.	27	27	3	3	47	8	10.5	24.2
1590	Minn.	19	38	8	6	52	5	10.4	25.9
1591	Minn.	14	22	8	5	47	6	10.2	26.5
1592	Minn.	3	25	5	5	38	5	9.9	23.3
1593	Minn.	9	23	2	0	34	3	9.6	24.5
1594	Minn.	23	38	8	2	27	0	9.2	22.8
1595	Minn.	30	22	11	5	56	13	10.5	31.1
1596	Minn.	20	31	5	11	63	8	10.2	34.9
1597	Minn.	20	31	5	5	50	2	10.1	25.9
1600	Minn.	32	25	5	5	52	8	10.0	31.6
1601	Minn.	22	23	9	3	39	8	10.6	19.6
1602	Minn.	17	34	6	5	36	8	9.8	25.2
1605	Wisc.	28	39	3	6	53	11	9.7	36.3
1606	Wisc.	20	22	6	5	36	3	9.8	22.6
1608	Wisc.	30	55	16	16	59	9	10.5	31.5
1609	Wisc.	41	38	17	16	52	20	10.1	36.4
1610	Wisc.	41	42	20	8	55	5	10.6	25.2
1611	Wisc.	56	27	14	13	67	17	10.1	43.1
1612	Mich.	23	38	13	13	52	19	10.0	37.1
1613	Mich.	17	23	9	9	56	3	9.7	33.8
1614	Mich.	19	45	6	5	41	3	9.4	29.2
1615	Mich.	14	28	8	9	45	6	9.6	31.6
1616	Mich.	30	44	2	0	61	9	11.6	21.4
1617	Mich.	20	41	13	6	59	6	11.2	22.5
1618	Mich.	14	39	5	3	56	2	11.6	15.2
1621	Mich.	20	22	6	2	56	3	9.5	35.6
Seed source F value		4.94	1.86	1.80	2.05	1.88	1.94	--	--

The sources from Lower Michigan seem to be the most notable exceptions to the height-weeviling relationship. Sources 1617 and 1618 were two of the three sources producing the tallest trees, and yet both were slightly below average in weevil incidence. Trees from source 1616 were the tallest in the plantings, but were the sixth most weeviled. Trees from source 1605 (northeastern Wisconsin), on the other hand, were the next to the shortest in the planting but the fifth most weeviled.

In the 1963 measurements, only one of the plantings, Au Sable State Forest, showed a high degree of weevil infestation —about 50 percent of the tree were attacked (table 1). The differences among sources were highly significant. (table 2).

Incidence of white-pine weeviling in the two Minnesota plantings was low (table 2). When weeviling incidence from both Minnesota and Michigan plantings are combined, there is a significant seed source x planting interaction between the Minnesota and Michigan plantings.

In most cases differences in weeviling incidence between the Minnesota plantings and the Michigan plantings paralleled height growth differences. For example, trees from two of the Lower Michigan seed sources (1616 and 1618) were among the fastest growing in the Michigan plantations and among the slowest growing in the Minnesota plantations. They were well above average in weeviling incidence in Michigan but below the plantation average in Minnesota. In other words, changes in white-pine weeviling incidence could be accounted for mainly by changes in height growth.

In the Minnesota plantings where the weeviling incidence was low there was no correlation between 10-year height and 10-year weeviling. But in the Au Sable, Michigan, planting there was a significant correlation ($r = .53$ with 24 d.f.). Covariance analysis established that there were weeviling differences independent of height. Moreover, in the Minnesota plantations, sources 1606, 1589, and 1616 averaged 3 percent of their trees weeviled, while sources 1609 and 1611 averaged 15 percent weeviled trees. Yet these five sources did not differ significantly in the 10-year tree height growth. Clearly not all differences in weeviling are the result of height growth differences.

If 10-year weevil incidence in the two Lower Michigan plantings is adjusted for height differences (via linear covariance), trees from source 1618 and source 1601 are taller and relatively less weeviled in the Lower Michigan plantings. Tree from sources 1609, 1611, and 1610 had the highest (adjusted) weevil incidence although these made about average height growth.

In the Minnesota plantings trees from sources 1609 and 1611 were below average in height, but were among the most heavily weeviled. Trees from a number of other sources were low in weeviling incidence, and no source could be singled out as showing exceptional resistance to weeviling in the Minnesota plantings.

There was no obvious relationship between weeviling and seed source latitude. This same lack of geographic pattern in weevil incidence has been reported for eastern white pine (*Pinus strobus*)by Garrett.[1]

In summary, it is clear that there are real differences among seed sources in white-pine weevil resistance. But these differences can be and are often obscured by the varying height growth among seed sources. Trees from the southernmost Wisconsin seed sources showed a consistently high incidence of weeviling, both in this study and in those reported by Batzer (1962) and Arend *et al.* (1961). Trees from the northern Minnesota seed sources show a consistently low incidence of weeviling, but trees from these sources are usually the slowest growing in the Wisconsin and Michigan plantations. When a statistical adjustment is made for seed source tree height differences, the Lower Michigan sources appear quite low in weevil preference. Because trees from these Lower Michigan sources (1616, 1617, and 1618) are the fastest growing in Wisconsin and Michigan (King 1966), they would appear to be the best sources to use as a starting point in a white-pine weevil resistance breeding program.

Eastern Pine-Shoot Borer

Eastern pine-shoot borer (*Eucosma gloriola* Heinrich) was found in 8 of the 11 plantations after five growing seasons in the field (table 3). Although the incidence of this insect in the plantations ran as high as 38 percent, no significant seed source differences could be shown.

Table 3.—Incidence of eastern pine-shoot borer on jack pine in 11 test plantations 5 and 10 years after establishment

Plantation number	Name and location	Percent of trees attacked in:	
		1958	1963
1	Superior, Minn.	0.4	2.1
2	Chippewa, Minn.	10.0	2.7
7	Burnett, Wisc.	10.6	4.9
8	Mosinee, Wisc.	6.5	23.5
9	Chequamegon, Wisc.	21.4	2.9
10	Nepco, Wisc.	37.9	20.9
11	Argonne, Wisc.	5.3	18.8
12	Marinette, Wisc.	.6	1.2
13	Ottawa, Mich.	.0	24.8
16	Au Sable, Mich.	.0	1.3
17	Fife Lake, Mich.	.0	1.2

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[1] *Garrett, Peter W. Resistance of eastern white pine (Pinus strobus L.) provenances to the white-pine weevil (Pissodes strobi Peck). (Manuscript in preparation for publication.)*

In the 1963 measurements some shoot borer damage was found in all the plantings. Of the four most heavily attacked plantings, seed source differences were found at the Nepco Industrial Forest, Argonne Experimental Forest, and the Ottawa National Forest but not at the Mosinee Industrial Forest (table 4).

When the insect data from the Nepco, Argonne, and Ottawa were combined into a single analysis, a significant seed source x plantation interaction was found. And while there was a negative correlation (r = −.67 with 24 d.f.) between shoot borer incidence and 10-year height over the three plantings, a linear covariance analysis showed that variation in shoot borer incidence could not be entirely attributed to height growth variation.

Based on the three-plantation average and on a covariance adjustment for 10-year height, trees from sources 1609, 1596, 1612, and 1616 showed the lowest overall shoot borer incidence (table 4). Trees from sources 1592, 1597, and 1615 had the highest incidence.

Sources contributing most of the seed source x plantation interaction were 1606 and 1621, which had relatively higher insect incidence at the Argonne Experimental Forest than at the Nepco or Ottawa plantings; sources 1595 and 1610, which had relatively high insect incidence at the Ottawa National Forest; and sources 1592 and 1614, which both responded differently at the Nepco than at either of the other two plantings.

Table 4.—Incidence of eastern pine-shoot borer on jack pine in 1963

Seed source	State	Nepco	Argonne	Ottawa	All three plantings combined	
					Mean height	1963 incidence (adjusted for height)
		Percent	Percent	Percent	Feet	Percent
1589	MN	19	17	19	13.5	20.5
1590	MN	25	17	33	12.9	25.3
1591	MN	27	25	22	12.4	22.8
1592	MN	20	39	44	12.0	31.1
1593	MN	33	34	39	11.5	25.0
1594	MN	20	19	25	11.7	16.9
1595	MN	17	16	33	12.9	22.1
1596	MN	8	9	25	12.5	12.7
1597	MN	22	33	39	12.9	31.5
1600	MN	19	25	25	12.9	23.2
1601	MN	28	19	30	13.0	26.0
1602	MN	25	19	36	12.6	25.7
1605	WS	27	16	22	12.6	20.5
1606	WS	13	22	16	12.5	16.7
1608	WS	16	13	14	13.7	17.1
1609	WS	11	6	17	13.1	12.4
1610	WS	13	13	28	13.7	20.8
1611	WS	25	16	27	12.4	20.7
1612	MC	17	8	11	13.5	14.4
1613	MC	17	16	22	12.8	17.8
1614	MC	33	17	22	12.9	24.0
1615	MC	31	36	28	12.7	31.0
1616	MC	14	5	13	14.0	14.9
1617	MC	20	6	17	13.4	16.8
1618	MC	19	5	19	13.8	17.7
1621	MC	25	39	22	12.1	25.7
Seed source F value		1.87	4.05	2.91	--	--

The heights of the trees in each seed source may be the most important factor in shoot borer incidence. In 1958, when seed source tree height averaged 4 to 6 feet, the insects showed no seed source preferences. By 1963, when the mean height of the test plantation trees exceeded 12 feet, the insects preferred the seed sources with shorter trees. However, the fact that height variation could not entirely account for variation in shoot borer incidence suggests that other unknown factors may also play a role in insect resistance, and possibly these factors are not present in younger trees.

There was a tendency for trees from the more northerly seed sources to have a higher level of pest incidence than those from the southern seed sources. That is, there was a positive correlation between eastern pine-shoot borer incidence and the latitude of the seed source. However, this is probably a reflection of the fact that the trees from Minnesota sources were shorter than those from the Michigan sources in these plantations. That is, insect resistance appears related to latitude only because height growth is related to latitude.

Eastern Gall Rust

In the 5-year plantation measurements 15 percent of the trees in the Mosinee Industrial Forest plantation and 14 percent of the trees in the Chequamegon National Forest planting had eastern gall rust (*Cronartium quercum* (Berk.) Miyabe Ex Shirai) cankers (table 5). Gall rust was also found in the Superior National Forest, Chippewa National Forest, Burnett County Forest, and Nepco Industrial Forest and the Au Sable State Forest plantings, but in none of these was the overall rust incidence greater than 3 percent. There were significant differences in rust incidence among seed sources on both the Mosinee Industrial Forest and Chequamegon National Forest (table 6).

In the 10-year measurements gall rust cankers were found in every test plantation. There were significant seed source differences in every planting where more than 15 percent of the trees were infected (table 6).

Table 5.—Incidence of eastern gall rust on jack pine in 11 test plantations 5 and 10 years after establishment

Plantation number	Name and location	Percent of trees attacked in: 1958	1963
1	Superior, Minn.	0.1	3.1
2	Chippewa, Minn.	.4	26.1
7	Burnett, Wisc.	3.3	38.8
8	Mosinee, Wisc.	14.7	80.8
9	Chequamegon, Wisc.	14.1	70.4
10	Nepco, Wisc.	2.4	32.8
11	Argonne, Wisc.	.0	.7
12	Marinette, Wisc.	.0	1.1
13	Ottawa, Mich.	.0	2.8
16	Au Sable, Mich.	.7	44.7
17	Fife Lake, Mich.	.0	15.1

There was a significant negative correlation between 1963 height and 1963 rust incidence in some of the more heavily infested plantings ($r = -.66$ with 24 d.f.). But height differences alone could not account for all the differences in rust incidence and vice versa.

A combined analysis of the 10-year data showed significant seed source x plantation interaction. When the 5- and 10-year data from the Mosinee and Chequamegon plantings were used in a combined analysis, a significant year x seed source interaction was indicated. However, these interactions arose mainly from large between-plantation differences in the overall level of infection rather than from increased (or decreased) susceptibility of any seed source. The ranking of the seed sources between plantations and years remains quite consistent, with very few exceptions.

Trees from seed source 1611 (Wood County, the southernmost source in this test) had the lowest rust incidence in both the Mosinee and Chequamegon plantations at age 5. At age 10, trees from this source had the lowest rust incidence in five of the seven plantations showing significant seed source differences, and were never significantly poorer than the best in any of the plantings. Sources 1595, 1600, and 1608 were also consistently among the five sources having the lowest incidence.

Table 6.—Incidence of eastern gall rust on jack pine in 1958 and 1963

(Percent of trees infected)

Seed source	State	1958				1963				
		Mosinee	Chequamegon	Chippewa	Burnett	Mosinee	Chequamegon	Nepco	Au Sable	Fife Lake
1589	MN	3	11	16	13	75	63	22	28	11
1590	MN	2	8	22	34	89	58	19	28	8
1591	MN	20	6	30	50	86	77	30	42	13
1592	MN	49	25	39	59	100	80	59	58	20
1593	MN	27	17	41	44	97	92	45	56	16
1594	MN	33	39	55	64	97	92	75	63	31
1595	MN	3	3	19	17	66	33	11	20	5
1596	MN	11	13	23	42	84	77	19	39	9
1597	MN	9	5	30	44	75	53	20	25	16
1600	MN	2	6	13	20	70	53	28	19	2
1601	MN	17	14	36	38	83	80	19	44	11
1602	MN	30	19	52	67	94	88	67	75	27
1605	WS	6	8	13	36	69	55	11	27	9
1606	WS	20	17	44	38	91	80	42	69	23
1608	WS	3	13	8	17	56	45	11	28	5
1609	WS	6	9	13	27	72	55	19	14	2
1610	WS	23	31	33	36	98	89	38	59	28
1611	WS	2	3	14	3	22	28	3	14	8
1612	MC	28	23	25	47	95	92	39	56	27
1613	MC	6	23	17	50	88	88	47	39	19
1614	MC	23	11	23	52	92	86	36	66	16
1615	MC	33	27	39	63	100	92	52	61	27
1616	MC	9	8	14	30	70	52	13	42	11
1617	MC	6	14	14	28	59	69	42	64	16
1618	MC	14	13	22	34	80	67	30	63	17
1621	MC	17	11	27	58	92	89	55	64	20
Seed source F value		3.69	2.65	2.98	6.65	14.75	11.36	6.81	6.98	3.22

Within the portion of the jack-pine range that was sampled in this study, trees from the northernmost sources showed the highest rust incidence, while those from the southern sources showed the lowest rust incidence. It seems possible that the seed sources from the southern portion of the range have been subjected to more intense gall rust infections, and hence have developed some resistance to it, while sources from the more northernly areas (where the alternate hosts are not as plentiful) have not been subjected to as severe selection for resistance.

Anderson (1965) has reported both eastern gall rust and western gall rust (*C. coleosporioides* Arth.) in the Lake States. These two rusts are morphologically indistinguishable, and it is possible that some plantings were infested with eastern gall rust, and others with western gall rust. If this was the case, the consistent variation pattern throughout the planting suggests that resistance to both rust species is governed by the same factors.

Assuming that various races of these rusts may have developed, the consistent performance of the sources also suggests that a source possessing resistance to one race of eastern gall rust will possess resistance to other races of rust.

Other Pests

The northern pitch-blister moth (*Petrova albicapitana* (Busck.)) was found in all of the plantations in either 1958 or 1963. The Nepco Industrial Forest plantation was the most heavily attacked. In both years about 20 percent of all the trees showed symptoms of this insect. And yet, significant differences among sources were not found in any of the plantings. Moreover, Arend *et al.* (1961) reported 30 percent of the trees damaged in an identical planting (not covered in this paper) in Lower Michigan without finding seed source differences. If genetic variation in resistance to this insect exists in the material studied, it could not be detected by this test.

Pine tortoise scale (*Toumeyella numismaticum* (Pettit and McDaniel)) was present in the 1963 examination on 25 percent of the trees in the Argonne Experimental Forest planting. No significant seed source differences were found. Although the insect was found in several plantings, at no other planting did the overall incidence exceed 4 percent.

In 1958 other insects and diseases noted in the plantings included: Aphids, Saratoga spittlebug (*Aphrophora saratogenis* (Fitch)), pine root-collar weevil (*Hylobius radicis* Buchanan), jack-pine sawfly (*Neodiprion pratti banksianae* Rohwer), red-headed pine sawfly, pine webworm (*Tetralopha robustella* Zeller), jack-pine budworm (*Choristoneura pinus* Freeman), Zimmerman pine moth (*Dioryctria zimmermani* (Grote)), jack pine needle cast and sweetfern rust (*Cronartium comptoniae* Arth.).

In 1963 the presence of jack-pine sawfly, jack-pine budworm, Saratoga spittlebug and sweetfern rust were also noted.

None of these pests occurred on more than 1 percent of the trees in any plantation. No seed source differences could be detected for any of these pests.

DISCUSSION

The results showed that trees from a particular seed source may grow faster than average and be less susceptible to several forest pests, but are not necessarily resistant to all pests. Sources 1609 (Marinette County, Wisconsin) and 1611 (Wood County, Wisconsin) showed a higher than average incidence of white-pine weeviling in the study as well as in Batzer's (1962). Yet these same two sources showed a relatively low incidence of eastern pine-shoot borer. Source 1611 was also among the lowest in eastern gall rust incidence.

Certainly a breeding program intended to produce resistant varieties should not be confined to only a single insect or disease. For in doing so one may increase resistance to one pest while increasing susceptibility to another. On the other hand, if a pest resistance breeding program tries to take into account the full spectrum of forest insects and diseases, the program may be diluted to the point where no progress is made in increasing resistance to any pest. Thus, the tree breeder, in close consultation with forest entomologists and pathologists must carefully limit the choice of insects and diseases to be considered in a resistance breeding program.

Pests that do not have an important economic impact on forest productivity should be ignored. The tree breeder must accept the presence of forest pests that do not reach epidemic proportions or that cause essentially aesthetic damage to forest stands. Pests that can be controlled through silvicultural means should also be ignored. Obviously, any insects or diseases that do not occur in the areas where the resistant varieties are to be used commercially should not be considered.

In this and other studies (Arend *et al.* 1961, Garrett[2]), the relationship between the geographic location of the seed origin and pest incidence has been random. It is not possible to predict the sources that will produce the least susceptible trees. Thus, a resistance breeding program should begin with a range-wide seed source study of the tree species in question. If a seed source study already exists, then the breeder might begin by either selecting parents from trees that are part of the existing study or by returning to the areas of the faster growing provenances and selecting several hundred new parents for more intensive studies of resistance.

Comparing the present pest data with 10-year height growth variation (King 1966), it is evident that the height growth ranking of the seed sources is little affected by the presence of heavy pest infestations. This is probably the result of the laterals in jack pine quickly assuming dominance then the terminal shoot is damaged. Thus, there is little loss in height growth

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[2] *Ibid.*

from white-pine weevil or eastern pine-shoot borer attack. There would, therefore, be little natural selection either for or against insect resistance. This may account for the random nature of the geographic distribution of resistance to these two insects.

The insect and disease incidence reported in this study was entirely the result of natural infestations. If the trees could have been artificially infested, a more uniform level of pest incidence could probably have been achieved. This would have increased the precision of the test and probably revealed a greater number of statistically significant seed source differences. This has already been demonstrated with white-pine weevil (Soles and Gerhold 1968). Moreover, as noted in this study, insect attack was often influenced (either positively or negatively) by relative tree height. It could not be determined from the present data whether the relationship between tree height and insect incidence was due to a direct link between growth rate and insect success or whether growth rate merely affected the availability of the tree to the insect. Clearly, the development and use of controlled infestation techniques is a prerequisite to an efficient pest resistance breeding program.

CONCLUSIONS

1. Jack pine seed source variation in incidence of white-pine weevil, eastern pine-shoot borer, and eastern gall rust has been demonstrated.

2. Resistance to one pest does not imply resistance to other pests. New improved varieties must be tested against several carefully chosen destructive pests. A thorough understanding of host-pathogen relationships will be needed to insure that selection for resistance to one pest does not increase suspectibility to other pests.

3. Insect attack can be strongly influenced, either positively or negatively, by relative tree height. In an insect resistance breeding program, artificially inducing insect attack through caging or carrying live insects to every tree may help to eliminate the confounding effect on growth rate. But in any case, more must be known about factors influencing movement, oviposition, dispersal and feeding habits of each pest.

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NFLUENCE OF STAND DENSITY ON STEM QUALITY IN POLE-SIZE NORTHERN HARDWOODS

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INFLUENCE OF STAND DENSITY ON STEM QUALITY IN POLE-SIZE NORTHERN HARDWOODS

Richard M. Godman and David J. Books

The objective in managing most northern hardwood stands is to produce high-value saw logs and veneer logs in the shortest possible time. Nearly two-thirds of the northern hardwood area in the Lake States is in young, second-growth stands approaching the need for their first commercial thinning (Cunningham and Survey Staff 1956). Most trees are still too small to have established their grade potential. Stem quality must be developed through a series of intermediate cuttings that also stimulate the growth of individual trees.

Generally, the improvement of stem quality results from a gradual process of natural pruning because live limbs and limb-related defects are the prime causes of degrade in potential crop trees. Many trees with other types of defects causing degrade and volume loss can be removed in intermediate cuttings to enhance the development of high quality stems (Jacobs 1966).

One of the primary means available to forest managers for improving bole quality is regulating residual stand density. Both stocking guides and criteria for tree selection have been prepared for Lake States stands (in which sugar maple is the principal species) to assure continuous growth and provide for desirable reproduction (Arbogast 1957, Eyre and Zillgitt 1953). Although these guides stress that moderate to high basal area is necessary to develop quality, particularly in older stands, their recommendations have not been evaluated for second-growth stands where trees are smaller and more uniform in diameter distribution. Nor do these guides indicate the range of residual stand density that may be acceptable for quality improvement.

To learn more about the effect of residual basal area on the total number and distribution of live limbs and limb-related defects, the first two logs (33 feet) of trees in pole-sized northern hardwood stands were studied 15 years after cutting. Only trees expected to be in the final crop were selected for study. Tree quality change will be further quantified by future measurements.

METHODS

Study Area

Three 40-acre tracts of second-growth northern hardwoods on the Argonne Experimental Forest in northeastern Wisconsin were selected for study. These stands, which consisted mainly of trees 5 to 8 inches in diameter, originated from a commercial cut in about 1905. Stocking of all trees 4.6 inches and larger averaged more than 90 square feet of basal area and about 240 trees per acre before cutting. A few holdover saw-log-sized trees ranging up to 25 inches in diameter were randomly distributed throughout the stand.

Sugar maple was the predominant species both in numbers and basal area. Basswood, white ash, yellow birch, and red maple were common associates, although none of these constituted more than 15 percent of the total basal area. Species distribution tended to be uniform, with basswood the most abundant of the associated species. All three stands are located on well drained, silt-loam soils with boulders in the surface layer.

Treatment

Six 2½-acre treatments, begun in 1951, left a uniformly distributed stand of the more desirable trees at specified residual basal areas:

Initial treatment	*Residual basal area per acre*	*Subsequent treatment*	*Basal area per acre after 15 years*
	Sq. Ft.		*Sq. Ft.*
Check	93	None	128
Improvement cutting	90	Recut after 10 years	101
Improvement cutting	75	Do	87
Improvement cutting	60	Do	76
Crop-tree release	60	None	105
8-inch stump diameter limit	20	None	69

The improvement-cutting stands were recut in the fall of 1961 on a planned 10-year cutting interval.

The crop-tree-release stands contained about 40 uniformly distributed trees per acre that were selected as potential crop trees. The stand residual basal area averaged 60 square feet per acre, although basal area around individual crop trees was slightly lower. Because the initial release of crop trees was considered fairly heavy, no additional treatments have been made.

The 8-inch stump diameter limit cutting did not result in uniform stocking throughout the compartment because of the arbitrary removal of all trees 8 inches d.b.h. and larger. However, stocking on and adjacent to the 1/10-acre sample plots was relatively uniform and the treatment is therefore included in this report.

Although the cutting methods varied widely, the primary difference in the residual stands was in basal area stocking. In all treatments the proportion of the stand made up by trees in the 5- to 8-inch diameter class increased after initial cutting to nearly 75 percent of the residual basal area. Trees of this initial size may eventually make up an even larger proportion of the stand, and thus are the most important trees to follow in determining the influence of residual basal area on the development of bole quality.

Sampling Procedures

Each treatment was replicated three times. Sample trees were located on five 1/10-acre plots established in each treatment area. Sample trees were selected from among the better trees on each plot that were between 4.5 and 8.6 inches d.b.h. when the study was established 15 years ago. Three trees of each of the major species — sugar maple (*Acer saccharum* Marsh.), red maple (*Acer rubrum* L.), yellow birch (*Betula alleghaniensis* Britton), basswood (*Tilia americana* L.), and white ash (*Fraxinus americana* L.) — were sampled on each plot, when they were available, for a maximum of 45 trees per treatment. Only sugar maple was present on all plots, so it was the only species consistently sampled.

Tree-Quality Measurements

Four classes of stem defects were recognized: live limbs, dead limbs, epicormic sprouts, and bumps. Live limbs included epicormic sprouts larger than ½ inch in diameter. Dead limbs included knots with the dead stub still visible. Bumps were swelling or protrusions rising more than ½ inch above the stem surface, and included open knots with no visible limb stub, overgrown knots, and other limb-related bumps.

The study zone extended from 1 foot above the ground to 33 feet in all sample trees. Defects were recorded by 4-foot height zones.

Crown diameter and growth rate of individual trees were measured, because these factors influence improvement in stem quality. The height of forks[1] on the main stem was recorded to the nearest foot.

RESULTS

Frequency of Bole Defects by Species

Sugar maple consistently had the greatest number of stem defects, while basswood and white ash had the fewest defects (table 1). Thus species composition alone can influence stand

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[1] *A fork was recorded whenever the diameter of the stem above the juncture was at least 25 percent smaller than the diameter of the main stem below the juncture and/or when the smaller of the fork members was at least two-thirds the diameter of the larger.*

Table 1.—Defects in the first two logs of northern hardwood trees 15 years after initial cutting

(In number of defects per tree)

Species	Treatment and residual basal area in sq. ft./acre						No. trees
	8 In. dia. limit	Improvement cuttings			Crop tree	Check	
	20	90	75	60	60	93	
Sugar maple	20	14	16	16	16	21	212
Yellow birch	--	8	8	12	14	12	54
Red maple	--	11	--	11	12	12	40
White ash	--	6	9	--	--	6	29
Basswood	--	8	9	6	5	4	70

quality and subsequent value. This partially explains the variation in quality within and between second-growth northern hardwood stands. Although several factors could account for the difference in number and retention of defects among species, shade tolerance appears to have the primary influence: the most tolerant species appear to have the greatest number of defects. The average number of defects per tree was not significantly different among residual basal area levels 15 years after initial cutting, although the sugar maple, yellow birch, and red maple in the check treatment usually had more defects than trees in the cut stands. Differences in the type and position of defects on the bole, however, were found to be associated with residual basal area density in all five hardwood species studied, although only those for sugar maple are reported here.

Effect of Basal Area on Defects on Sugar Maple

Frequency of the four types of defects influencing bole quality varied with residual basal area (fig. 1, table 2). Live limbs, which are the greatest deterrent to tree quality improvement, were much more abundant at the lower densities and in the second log (fig. 2). For example, the stands cut to residual densities of 60 square feet had nearly twice as many live limbs after 15 years as the untreated stand. Although most of the live limbs occurred on the second log, almost

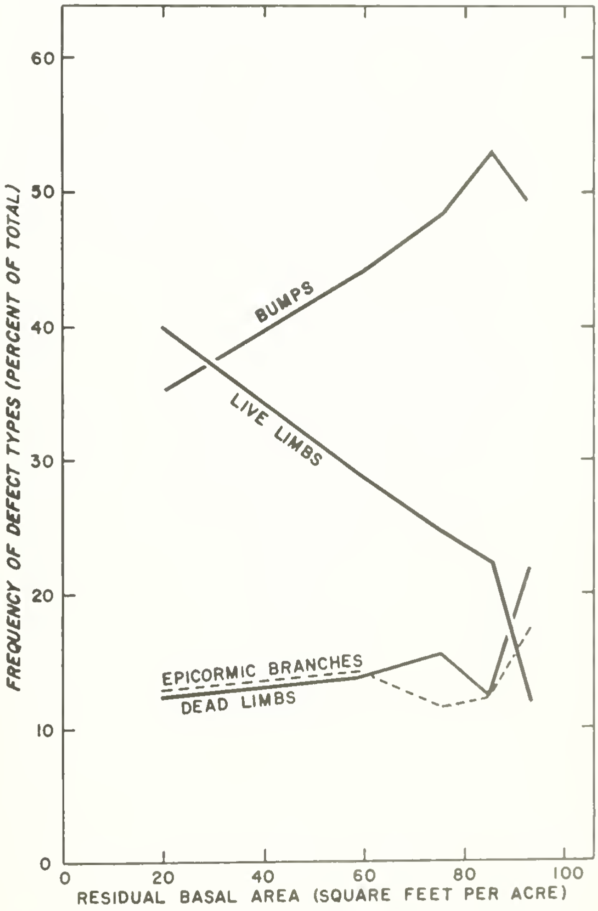

Figure 1.—Frequency of defect types in first 33 feet of stem 15 years after cutting to different residual basal area densities. Sugar maple trees originally 5 to 8 inches d.b.h.

Table 2.—Defects in sugar maple 15 years after cutting to different residual basal areas

(In number of defects per tree)

Defect type	16-Ft. Log Position	Treatment and residual basal area in sq. ft./acre					
		8-In. dia. limit	Improvement cuttings			Crop tree	Check
		20	90	75	60	60	93
Live limbs	First	0.9	0.3	0.4	0.4	0.3	0.2
	Second	7.0	3.0	3.6	4.2	4.5	2.2
	Total	7.9	3.3	4.0	4.6	4.8	2.4
Epicormic branches	First	.6	.6	.5	.7	.8	1.1
	Second	1.9	1.1	1.4	1.5	1.4	2.5
	Total	2.5	1.7	1.9	2.2	2.2	3.6
Dead limbs	First	.6	.1	.4	.3	.5	.6
	Second	1.8	1.7	2.1	1.9	2.2	3.9
	Total	2.4	1.8	2.5	2.2	2.7	4.5
Bumps	First	3.1	2.0	2.4	2.3	1.8	3.2
	Second	3.9	5.7	5.5	4.8	4.7	6.9
	Total	7.0	7.7	7.9	7.1	6.5	10.1
All defects	First	5.2	3.0	3.7	3.7	3.4	5.1
	Second	14.6	11.5	12.6	12.4	12.8	15.5
	Total	19.8	14.5	16.3	16.1	16.2	20.6

half the tree still had at least one live limb in the first log in the partially cut stands. Although the frequency of large limbs was not significantly greater in the heavier cuttings, most limbs in these cuttings can be expected to persist longer and become larger because of the lack of side competition. Under the improvement cutting to 60 square feet, approximately 30 percent of the limbs were considered large; however this percentage could be influenced by epicormic shoots that have since become small branches.

Epicormic sprouts were most abundant in the high density stands where the growing space between tree crowns was most restricted, particularly the check and 8-inch diameter limit cut. They were also more abundant in the second log than the first. While there were fewer epicormic sprouts in the intermediate density treatments, this was not necessarily because fewer epicormic sprouts developed. Possibly some sprouts grew into small live limbs, or partially developed and then died from suppression.

Dead limbs were most abundant on the trees in the untreated stand, probably due to the greater amount of competition for light, greater persistence due to the lack of disturbance in the stand, and a difference in moisture conditions at the branch that could influence the rate of decay (Heikinheimo 1953, Peace 1962). Trees in the crop-tree-release plots had a few more dead limbs 15 years after cutting than trees in the improvement cutting to the same stand basal area, perhaps because the crop-tree-release stand was not recut. In all treatments, most dead limbs were on the second log.

Bumps, which are the final stage in defect recovery, were the most abundant defect — more abundant than live and dead limbs combined in most treatments. The number of bumps per tree was greatest in the untreated stand, partly due to slower growth, and decreased gradually as residual stand density decreased. There were at least twice as many bumps in the second log as in the first log in all treatments except one.

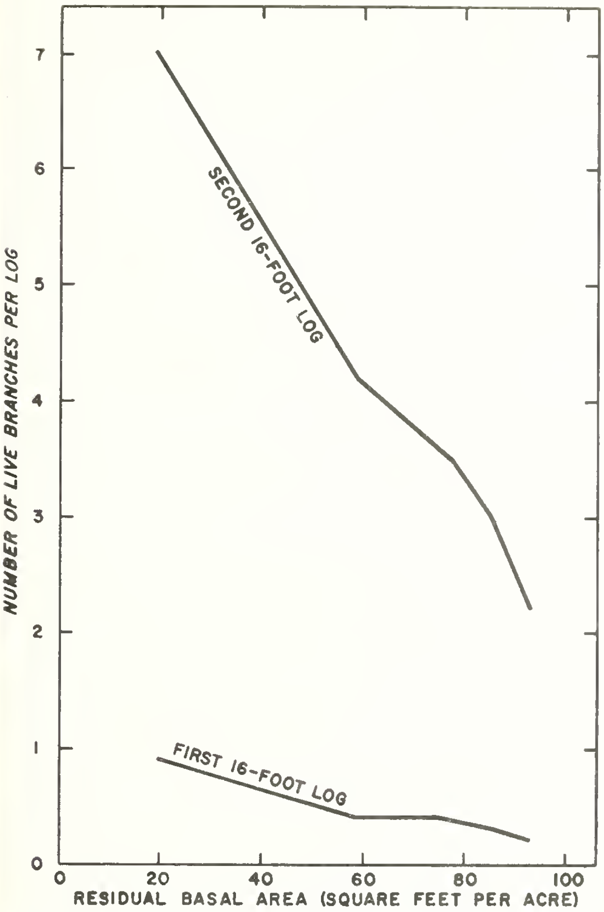

Figure 2.—Number of live limbs by log position 15 years after cutting to different residual basal area densities. Sugar maple trees originally 5 to 8 inches d.b.h.

While some differences in quality development may be due to the method of cutting (as indicated by the difference between the improvement cutting to 60 square feet and the crop tree release to the same density) this appears far less important than residual basal area density.

Other Characteristics Influencing Bole Quality

Forking, crown size, and diameter growth rate are also related to residual stand density and affect stem quality (table 3).

Forking is common in even-aged sugar maple stands and tends to increase after cutting (Conover and Ralston 1959, Godman 1968). In this study nearly three-fourths of the sample trees had forks.

In general, more trees in the thinned stands forked above 37 feet than trees in the check stands, probably due to the reduced crown competition. The crop-tree release, in which only about 40 carefully selected trees were released per acre, had the least forking (58 percent). But three-fourths of the forks in this treatment were above 37 feet. Forks in this position will probably prevent additional increase of merchantable bole length.

In addition to the reduction in merchantable length, these forks constitute a risk to tree survival and long-term growth rate. Numerous trees in the stand had split at the base of V-shaped forks and both members had broken out in some trees. Most of the breakage appeared to be old forks with large, heavy limbs that had long and apparently weak crotch seams (Eames and McDaniels 1925).

Height to the base of the live crown tends to vary because of differences in stocking within a stand. In all cuttings, both exceptionally poor and good trees were found. However, the proportion of trees with a height-to-live-crown of 25 feet or less is a good indication of the influence of basal area on natural pruning. Height-to-live-crown was greatest in the denser stands and decreased at lower basal areas. Although trees in the crop-tree release treatment had the greatest average height-to-live-crown, this advantage was offset by an increase in number of live branches in the second log (table 2).

Crown diameter in relation to stem diameter tends to increase in the less dense stands. However, sugar maples apparently develop into wolf-trees only in rather open stands.

Average diameter growth ranged from 1.8 inches per 15-year period in the untreated stand to 3.6 inches in the stand cut to 20 square feet of residual basal area. The maximum growth rate observed for any sugar maple tree was 6.0 inches in 15 years; this occurred in the crop-tree release treatment. The proportion of the desirable trees growing more than 2.0 inches per decade increased as the residual stand density decreased. This study indicates that with good tree selection and stand conditions favorable for improvement of bole quality following partial cuttings, sugar maple is capable of a rapid increase in diameter.

Table 3.—General form and growth characteristics of potential sugar maple crop trees 15 years after cutting to different residual basal area densities. Trees were originally 5 to 8 inches d.b.h.

Tree Characteristic	Treatment and residual basal area					
	8-In. dia. limit	Improvement cuttings			Crop tree	Check
	20	90	75	60	60	93
Stem Form:						
Percent of trees with forks	77	64	78	72	58	73
Percent of forks at 37 ft. +	57	68	59	64	76	45
Crown Size:						
Percent of trees with height to live crown of 25 ft. or lower	57	14	21	26	10	15
Diameter Growth:						
Average 15-year increase in inches	3.6	2.1	2.7	2.9	3.0	1.8
Greatest 15-year increase in inches	5.4	4.9	4.0	4.5	6.0	2.9
Percent growing 2.0 in. + per decade	70	8	39	52	54	0

CONCLUSIONS

Stem qualilty improvement during the 15 years after initial cuttings in pole-size northern hardwoods was related to the residual stand density. While there was no significant reduction in the average number of defects per tree during this period, there was a difference in the type and position of the defects on the lower bole where the greatest volume and value of the stem occurs. This trend appears to be applicable to all northern hardwood species, but sugar maple generally has the greatest number of defects and retains them longest.

Because live limbs are the greatest deterrent to stem quality, they should be considered first in managing a stand for maximum quality development. The number of live limbs per tree decreases as residual stand density increases in pole-sized stands. Thus, the most rapid improvement in stem quality during the initial cuttings will occur at higher densities. Once the limbs have died, the rate of healing can be stimulated (hence the time required to produce clear wood shortened) by heavier cuttings.

Epicormic sprouting was variable within treatments, possibly because of differences in crown size and competition among trees. Forking is a common defect at all densities in even-sized stands of northern hardwood. At the lower stand densities there appears to be little chance of fork correction because of the lack of competition between crowns.

The relations between tree quality and stand density found in this study show that an acceptable compromise between quality improvement and growth rate can be obtained by thinning to 85 square feet of basal area per acre. This recommendation should be applicable for initial cuttings in pole-sized stands managed for production of high-quality saw logs. This guide could probably be modified to some degree, depending on management objectives or intensity. For instance, combining pruning with thinning, or thinning after an acceptable clear merchantable length has developed, would normally favor management at lower stand densities to further stimulate diameter growth and volume production.

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USDA FOREST SERVICE
RESEARCH PAPER NC-55
1971

The Dynamic Forces & Moments Required in Handling Tree-Length Logs

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THE DYNAMIC FORCES AND MOMENTS REQUIRED IN HANDLING TREE-LENGTH LOGS

John A. Sturos

The trend toward mechanized logging in recent years has resulted in the availability of a number of commercial harvesting machines, each with its own specific method of handling full trees and tree-length logs. Development of many of these machines has been on a trial and error basis, primarily due to lack of information on the forces required to handle the trees and logs. Consequently, machine manufacturers are interested in knowing the relative order of magnitude of the dynamic peak forces and moments required in handling full trees and tree-length logs. Not only could this dynamic handling data prevent costly failures of prototype equipment, but also it could prevent unnecessary overdesign.

The primary objective of this study was to determine the peak forces and moments required to handle tree-length logs of various weights. Both lifting and swinging handling modes were studied. Also, the effect of grab-point location was investigated.

EXPERIMENTAL PROCEDURE

The test involved moving a tree-length log with a commercially-made hydraulic loader through specific handling modes, while measuring and recording continuously the linear acceleration of the center of gravity of the log in three orthogonal directions and the angular accelerations of the log. The peak values of linear acceleration obtained from the recordings were then used to determine the peak handling forces and moments by a computer program.

Three preliminary steps in the procedure were determining (1) the weight of the log, (2) the center of gravity of the log, and (3) the mass moment of inertia of the log about a transverse axis through the center of gravity of the log. A cedar log 35.4 feet long and 17 inches in diameter at the butt end was used. The weight and center of gravity were determined by placing a transducer between the boom and a chain sling suspending the log[1]. The sling connections included a swivel and uniball joint to allow the log to hang freely. A point on the log vertically below the apex of the sling was marked as the center of gravity. Two log weights were used in the field tests – 505 and 1,188 pounds. The center of gravity was 13.3 feet from the butt end for both logs, because the heavier log was contrived by strapping weights at the center of gravity of the lighter log.

The mass moment of inertia of the 505-pound log was determined experimentally by using the principle of the pendulum. The log was suspended horizonally by nylon rope on a pulley. It was oscillated through a small angle (±5°), and the period of oscillation was measured accurately by using a photoelectric cell, flashlight, mirror,

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[1] *Steinhilb, H. M. and John R. Erickson. Weights and centers of gravity for quaking aspen trees and boles. USDA Forest Serv. Res. Note NC-91, 4 p., illus. N. Cent. Forest Exp. Sta., St. Paul, Minn.*

and a Sanborn recorder with a time marker (fig. 1).[2] Knowing the weight of a log (pounds), the distance from the center of gravity of the log to the axis of oscillation (inches), and the period of oscillation (seconds), the moment of inertia about a transverse axis through the center of gravity of the log was calculated using the following equation (refer to Appendix A for the derivation of the equation):

$$\bar{I} = (T^2Wh/4\Pi^2)-(Wh^2/g)$$

The moment of inertia of the 505-pound log was 1,449 pounds-feet-seconds^2.

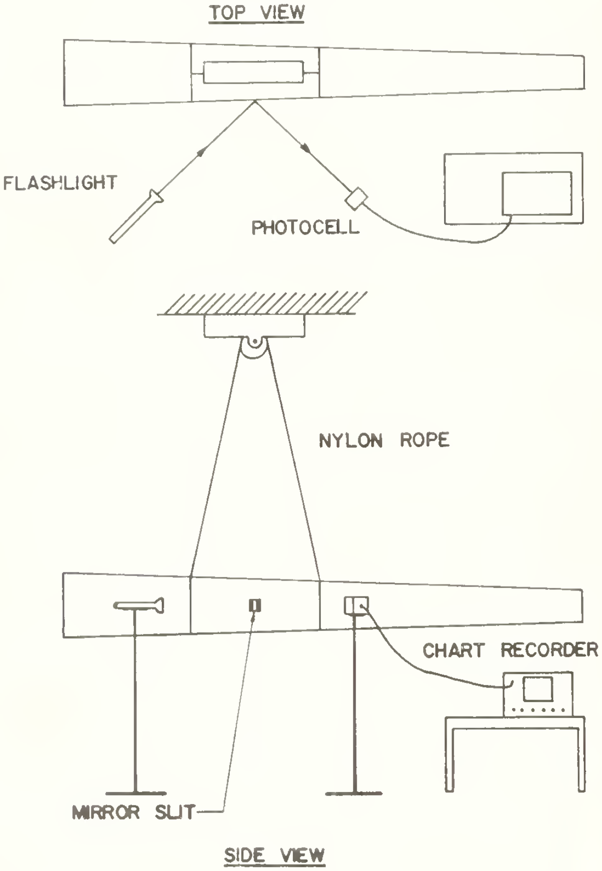

Figure 1.—Experimental set-up used in determining the moment of inertia of the log.

Because the heavier log was contrived by strapping weights at the center of gravity of the 505-pound log, the moment of inertia of an actual 1,188-pound log could not be determined. In a separate field study, the results of which have not yet been published, the moments of inertia of 13 red pine and 12 aspen trees and tree-length logs were determined by an experimental procedure similar to that mentioned above. The results showed the average moment of inertia of a 1,188-pound red pine log to be 2.95 times that of a 505-pound log, while the moment of inertia of a 1,188-pound aspen log was 4.05 times greater. Therefore, to obtain approximate dynamic moment values, it was assumed that the moment of inertia of the 1,188-pound log is three times that of the 505-pound log, or 4,347 pounds-feet-seconds^2.

Five Statham Model A5 linear accelerometers were used in this study. Three accelerometers were mounted in three orthogonal directions on a steel collar at the center of gravity of the log (fig. 2). These accelerometers gave the translational acceleration of the log in the vertical lateral, and longitudinal directions. Also, two accelerometers were mounted in the vertical and lateral directions on another collar 10 feet from the center of gravity toward the small end of the log. The difference between the two vertical acceleration readings divided by the distance between them (10 feet) was equal to the angular acceleration of the log in the vertical plane.

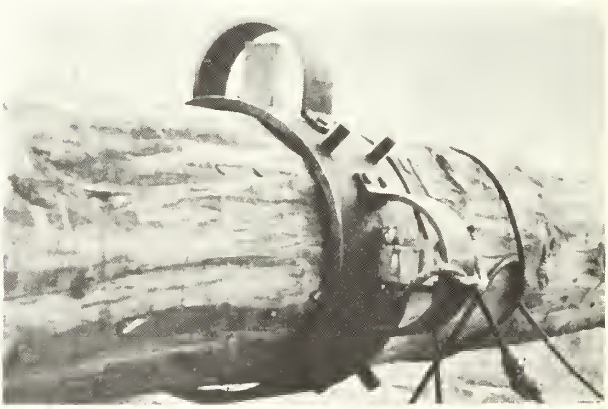

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Figure 2.—The accelerometers mounted on a steel collar at the center of gravity of the log.

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[2] *The names of manufacturers and models of equipment are mentioned in this paper for identification only, and no endorsement by the USDA Forest Service is implied.*

Likewise, the two lateral acceleration readings were used to determine the angular acceleration in the horizontal plane. The accelerometers were connected by low-noise extension cables through a CEC Type 1-118 carrier amplifier to a portable CEC Type 5-124 recording oscillograph. Fluid-damped galvanometers (CEC Type 7-316) with a frequency range from zero to 1,200 c.p.s. were used in the oscillograph, along with five static reference trace galvanometers (CEC Type 7-002). The acceleration-time curves were recorded as permanent records for all of the test runs.

A Model HOBC Prentice truck-mounted hydraulic loader was equipped with a pull-type jib and a heeling boom for handling tree-length logs. The loader was operated at maximum speed for all test runs. For the swinging mode this meant approximate 2.5 r.p.m. Pointed metal studs were welded on the grapple jaws to prevent the log from rotating relative to the grapple. Before each test run, the truck stabilizers were checked to make sure they were grounded.

A level was used to position the accelerometer mounting collars on the log, thus assuring that the respective mounting surfaces on the two collars were parallel. The accelerometers and their protective covers were then mounted on the collars and the cables connected to the amplifiers. The accelerometers were balanced and zeroed with the log in the horizontal position, which was checked with a level.

ANALYSIS OF DATA

The equations of motion were derived by using free-body diagrams of the log. (Refer to Appendix B for the derivations and legend of symbols.) The equations derived for the lifting handling mode (translation plus rotation in the vertical plane) are as follows:

$$F_z = m\ddot{z} + mg\cos\theta$$

$$F_y = m\ddot{y} + mg\sin\theta$$

$$F_R^2 = F_z^2 + F_y^2$$

$$M_x = \bar{I}\ddot{\theta}_x + F_z d$$

The equations of motion derived for the swinging handling mode in addition to the equations above are as follows:

$$F_x = m\ddot{x}$$

$$F_R^2 = F_x^2 + F_y^2 + F_z^2$$

$$M_z = \bar{I}\ddot{\theta}_z - F_x d$$

$$M_R^2 = M_x^2 + M_z^2$$

A computer program was written for a digital computer to perform the calculations in the above equations of motion. The input data consisted of the weight and moment of inertia of the logs, the distance from the log center of gravity to the grab point, and the acceleration readings from the five accelerometers.

A typical recording of a vertical acceleration of the log is shown in figure 3. Free-body diagrams of the log at two specific instants within the same lifting cycle are compared: when the log was horizontal ($\theta = 0°$) but just after the log was accelerated vertically, and when the log was at a 30° angle to horizontal ($\theta = 30°$) and moving at constant velocity. The peak accelerations occur the instant the log is moved. The data from the oscillograms plus the other input data were recorded on prepared cards for the computer program.

The effect of the location of the grab point was also studied by repeating the handling modes with the 505-pound log after the grab point was shifted 4 feet toward the log center of gravity. The computer output gave the component and resultant handling forces and moments in tabular form for each handling mode, log weight, and grab-point location.

RESULTS

The major objective of this study was to determine the dynamic forces and moments required at the grab point on the log for the lifting and swinging handling modes. The forces and moments required for the lifting mode are greater than those required for the swinging mode (tables 1, 2, 3 and 4). The results given in the tables summarize the tabular computer output for each handling mode and log weight.

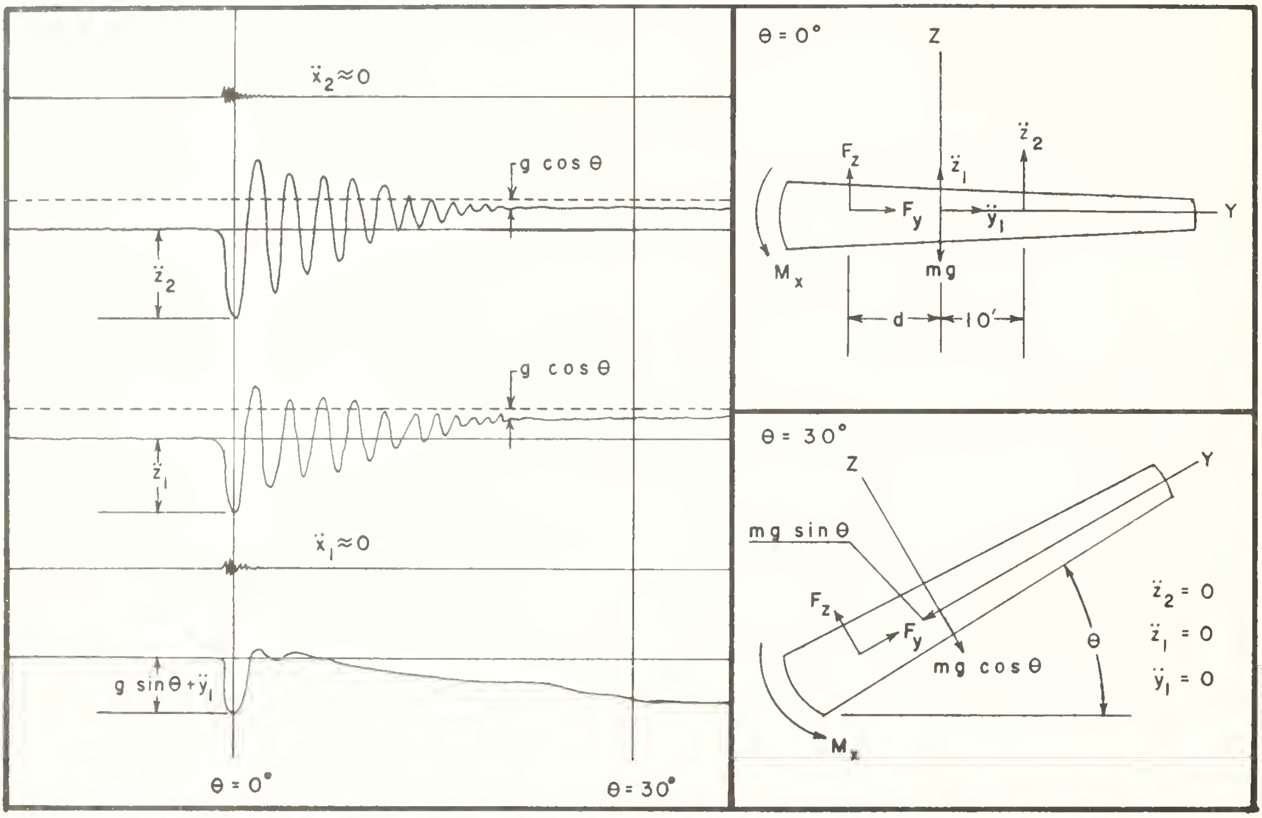

Figure 3.—A typical recording of the vertical acceleration of the log plus free-body diagrams of the log at two specific instants, namely when $\theta = 0°$ and 30°.

The computer program calculated the maximum component forces and moments for each test run. The average of these values is the mean value given in the tables. The 95-percent upper confidence limit (U.C.L.) on the mean can be explained as follows: If a 505-pound log was lifted up and down a number of times (in this case 15 times) and the maximum vertical force noted each time, it can be concluded with 95-percent confidence that the average maximum vertical force would be no greater than 1,563 pounds. In the field tests with the 505-pound log, vertical forces as high as 2,358 pounds were encountered. The total or resultant force or moment can be calculated by adding vectorally the component forces or moments. The dynamic moments in table 4 are approximate values because the moment of inertia of the 1,188-pound log was assumed to be three times that of the 505-pound log.

Another method of expressing the dynamic handling requirements is by the dynamic force and moment factors. These dynamic factors are defined as the ratios of the maximum dynamic force and moment to the required static force and moment. For a 505-pound log, the mean dynamic force factor is 2.64 and the mean dynamic moment factor is 2.90 (table 5). The values in table 5 were obtained by comparing the required static values (vertical force and moment about x-axis) with the dynamic values taken from tables 1, 2, 3 and 4. These dynamic factors show that it is possible to encounter dynamic forces and moments three to five times greater than the corresponding static values.

Because the accelerations of the logs were recorded continuously throughout the handling cycle, the forces and moments were determined

Table 1.—Dynamic forces required to handle a 505-pound log

Component force	Handling mode	Dynamic force (pounds)			
		Mean value	Standard deviation	95 percent U.C.L.1/ on mean	Maximum experimental value
Vertical	Lifting	1,332	417	1,563	2,358
	Swinging	730	158	875	1,010
Longitudinal	Lifting	349	70	388	480
	Swinging	281	109	383	414
Lateral	Swinging	337	116	444	505

1/ U.C.L. = Upper confidence limit

Table 2.—Dynamic moments required to handle a 505-pound log

Component moment	Handling mode	Dynamic moment (foot-pounds)			
		Mean value	Standard deviation	95 percent U.C.L. on mean	Maximum experimental value
Moment about x-axis	Lifting	15,059	4,054	17,304	24,664
	Swinging	8,304	1,610	9,793	10,403
Moment about z-axis	Swinging	5,002	1,648	6,527	7,534

Table 3.—Dynamic forces required to handle a 1,188-pound log

Component force	Handling mode	Dynamic force (pounds)			
		Mean value	Standard deviation	95 percent U.C.L. on mean	Maximum experimental value
Vertical	Lifting	3,135	861	3,682	4,752
	Swinging	1,485	425	2.161	1,984
Longitudinal	Lifting	1,551	742	2,023	3,374
	Swinging	329	110	601	392
Lateral	Swinging	469	97	624	594

Table 4.—Dynamic moments required to handle a 1,188-pound log

Component moment	Handling mode	Dynamic moment (foot-pounds)			
		Mean value	Standard deviation	95 percent U.C.L. on mean	Maximum experimental value
Moment about x-axis	Lifting	57,268	21,215	70,747	93,846
	Swinging	14,841	3,706	20,738	18,618
Moment about z-axis	Swinging	19,222	1,140	21,036	20,276

Table 5.—Dynamic force and moment factors determined for the vertical force (F_z) and the moment about the x-axis (M_x) for the lifting handling mode

Log weight (pounds)	Dynamic factors					
	Mean value		95 percent U.C.L. on mean		Maximum experimental value	
	Force	Moment	Force	Moment	Force	Moment
505	2.64	2.90	3.10	3.33	4.67	4.74
1,188	2.64	4.68	3.10	5.78	4.00	7.67

at those instants when acceleration spikes occurred. For the lifting handling mode, the acceleration spikes occurred at the following instants: (1) upward acceleration of the log at the beginning of the cycle (log horizontal); (2) deceleration at the top of the cycle (log at about 60°); (3) downward acceleration at the top of the cycle; and (4) deceleration of the log at the end of the cycle (log horizontal). As would be expected, the dynamic forces and moments varied considerably during this handling cycle. The maximum values occurred either during the acceleration at the beginning, or the deceleration at the end of the cycle.

The effect of the location of the grab point on the dynamic moments was investigated only with the 505-pound log. The lifting and swinging test runs were repeated after the grab point was moved 40 percent closer to the center of gravity (from 10.3 to 6.3 feet). The average decreases in the dynamic moments for the lifting and swinging handling modes are as follows: 21 percent decrease in M_x for the lifting mode, and 20 percent decrease in M_x and 46 percent decrease in M_z for the swinging mode (fig. 4).

CONCLUSIONS

Instead of designing a tree or log harvesting device statically, realistic dynamic loading requirements have been determined on which a design should be based. This study has shown that dynamic forces and moments four times as great as those required statically can occur in the field. A designer of timber harvesting equipment must consider this information in his stress analysis work in order to prevent fatigue and shock-load failures, and also to design a machine with the highest possible strength-to-weight ratio.

Though the tests were made with a specific hydraulic loader, and certain simplifying assumptions were made, the resulting dynamic loading data can be considered typical for original design purposes. Particularly important are the dynamic load factors that indicate much error can be made if the design is based on static analysis. In conventional static design a safety factor would normally be applied to cover the "unknowns"; however, with the information contained herein, the designer is better equipped to specify a more optimum design with the first prototype.

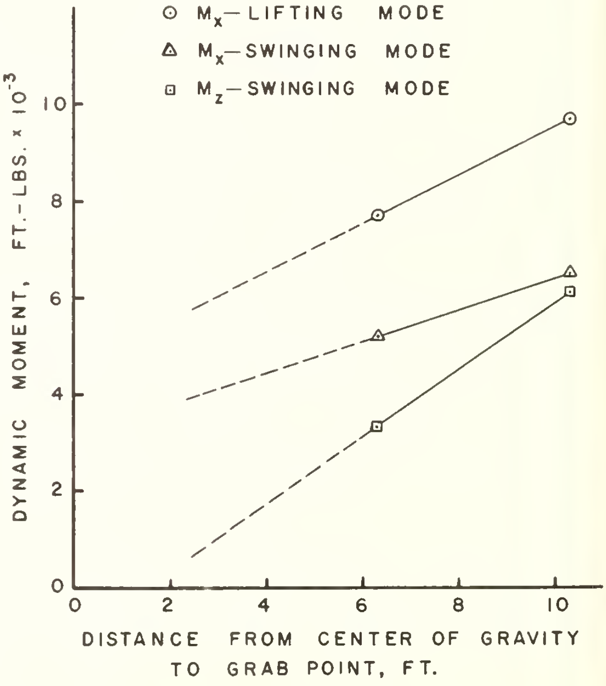

Figure 4.—The effect that grab point location has on the dynamic moments required in handling a 505-pound log.

APPENDIX A

The moment of inertia of a compound pendulum about its axis of oscillation (I') is given by the following equation:[3]

$$I' = T^2Wh/4\pi^2$$

where

T = period of oscillation (seconds)
W = weight of pendulum (pounds)
h = distance from the center of gravity of the pendulum to the axis of oscillation (feet).

Therefore, by using the parallel-axis theorem, the moment of inertia of a pendulum about its center of gravity ($\bar{I}$) is given by the following equation:

$$\bar{I} = I' - Wh^2/g = (T^2Wh/4\pi^2) - (Wh^2/g),$$

where

g = acceleration due to gravity (feet/second^2).

The effect of the amplitude of the oscillations and the damping at the point of suspension are assumed negligible.

APPENDIX B

Legend of Symbols

$\bar{a}$ = Resultant acceleration (ft./sec.^{2})
g = Acceleration due to gravity (32.2 ft./sec.^{2})
d = Distance between the grab point and the center of gravity (feet)
m = Mass of the log (lbs.-sec.^{2}/ft.)
$\ddot{x}_1$ = Acceleration of the center of gravity of the log along the x-axis (ft./sec.^{2})
$\ddot{y}_1$ = Acceleration of the center of gravity of the log along the y-axis (ft./sec.^{2})
$\ddot{z}_1$ = Acceleration of the center of gravity of the log along the z-axis (ft./sec.^{2})
$\ddot{x}_2$ = Acceleration in the x-direction of a point on the log 10 feet toward the small end from the center of gravity (ft./sec.^{2})
$\ddot{z}_2$ = Acceleration in the z-direction of a point on the log 10 feet toward the small end from the center of gravity (ft./sec.^{2})
θ_x = Angle of the log in the vertical plane measure from the horizontal position (radians)
$\ddot{\theta}_x = \dfrac{\ddot{z}_2 - \ddot{z}_1}{10} =$ Angular acceleration of the log about the x-axis (radians/sec.^{2})
$\ddot{\theta}_z = \dfrac{\ddot{x}_2 - \ddot{x}_1}{10} =$ Angular acceleration of the log about the z-axis (radians/sec.^{2})
C = Center of gravity of the log
P = Grab point of the log
$\bar{I}$ = Mass moment of inertia of the log about the x-axis or z-axis (lbs.-ft.-sec.^{2})
F_x = Force in the x-direction applied at the grab point (lbs.)
F_y = Force in the y-direction applied at the grab point (lbs.)
F_z = Force in the z-direction applied at the grab point (lbs.)
F_R = Resultant force applied at the grab point (lbs.)
M_x = Moment about the x-axis applied at the grab point (ft.-lbs.)
M_z = Moment about the z-axis applied at the grab point (ft.-lbs.)
M_R = Resultant moment applied at the grab point (ft.-lbs.)

Derivation of the Equations of Motion

The equations of motion for the lifting handling mode were derived by using a free-body diagram of the log in the vertical plane (fig. 5). The coordinate system is a right-handed rectangular XYZ system fixed at the center of gravity of the log with the Y-axis in the longitudinal direction of the log. Therefore, the coordinate system rotated with the log. F_y and F_z are the instantaneous forces required at the grab point (P) to accelerate the log in the Y-Z plane, with $\ddot{y}$ and $\ddot{z}$ being the respective accelerations at the center of gravity (C). M_x is the instantaneous moment required at the grab point in addition to the moment $F_z d$ to rotate the log about the center of gravity at an angular acceleration equal to $\ddot{\theta}_x$. The moment of inertia of the log about its center of gravity is denoted by $\bar{I}$, and the weight of the log is equal to mg. One assumption that is made in writing the equations of

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[3] *E. Hausmann and E. P. Slack. Physics. (Ed. 2) New York, D. Van Nostrand Co., p. 159, 1939.*

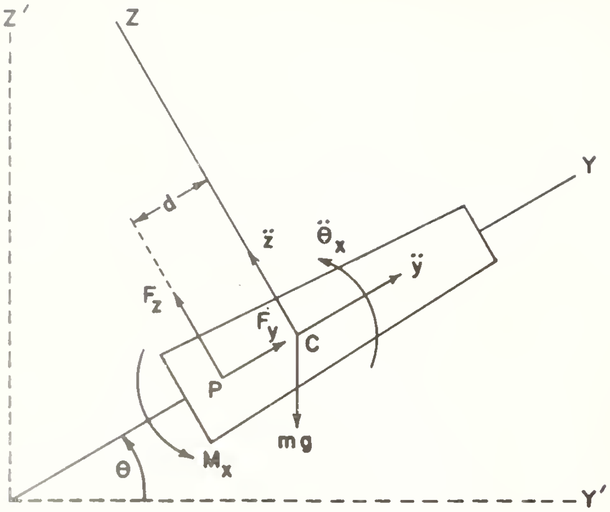

Figure 5.—Free-body diagram of a log being translated and rotated in the vertical plane.

motion is that the theoretical grab point is one-half the distance between the grapple and the heel boom. Therefore, the external forces and moments are assumed to be applied at one point.

The derivation of the equations of motion for the lifting handling mode (translation plus rotation of the log in the vertical plane) is as follows:

From Newton's Second Law: $F_R = m\bar{a}$,

therefore:
$$\Sigma F_z = m\ddot{z} = F_z - mg\cos\Theta$$
$$F_z = m\ddot{z} + mg\cos\Theta$$
$$\Sigma F_y = m\ddot{y} = F_y - mg\sin\Theta$$
$$F_y = m\ddot{y} + mg\sin\Theta$$
$$F_R^2 = F_z^2 + F_y^2$$
$$\Sigma\bar{M} = \bar{I}\ddot{\Theta}_x = M_x - F_z d$$
$$M_x = \bar{I}\ddot{\Theta}_x + F_z d\ .$$

A free-body diagram of the log in the horizontal plane was used to derive the equations of motion necessary for the swinging handling mode in addition to the equations above (fig. 6). F_x and F_y are the instantaneous forces required at the grab point to accelerate the log in the X-Y plane, with $\ddot{x}$ and $\ddot{y}$ being the respective accelerations at the center of gravity. M_z is the instantaneous moment required at the grab point in addition to the moment $F_x d$ to rotate the log about the center of gravity at an angular acceleration equal to $\ddot{\theta}_z$.

The derivation of the equations of motion for the swinging handling mode (translation plus rotation of the log in the horizontal plane) as as follows:

$$\Sigma F_x = m\ddot{x} = F_x$$
$$\Sigma F_y = m\ddot{y} = F_y$$
$$\Sigma\bar{M} = \bar{I}\ddot{\Theta}_z = M_z + F_x d$$
$$M_z = \bar{I}\ddot{\Theta}_z - F_x d\ .$$

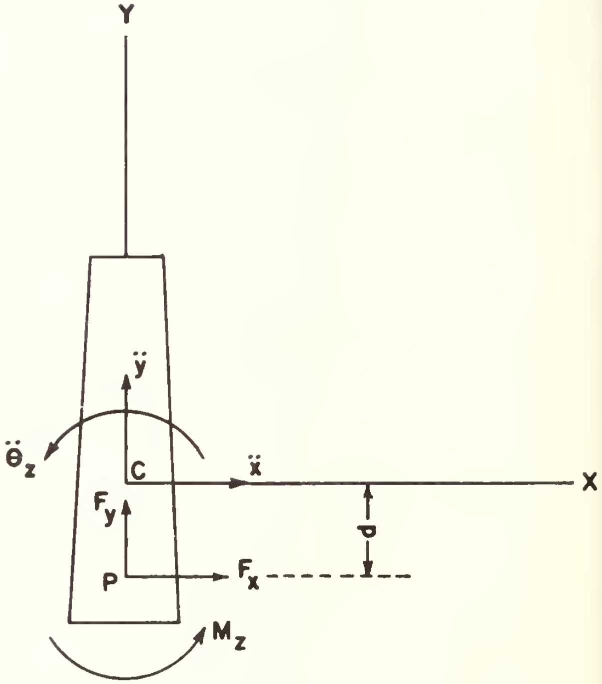

Figure 6.—Free-body diagram of a log being translated and rotated in the horizontal plane.

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- Managing and protecting the 187-million acre National Forest System.

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